1.-

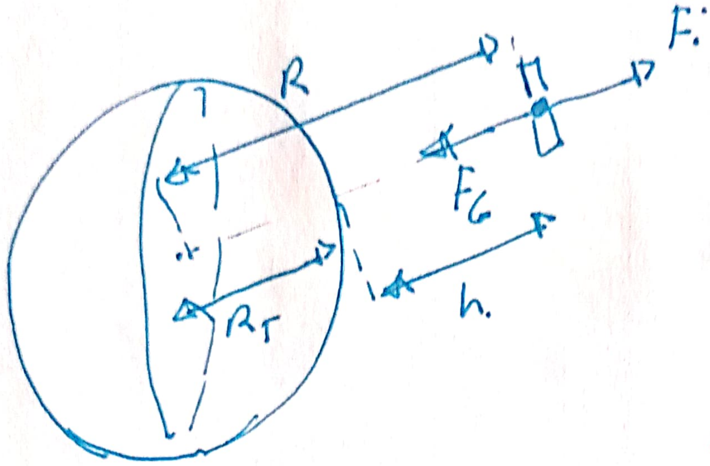
$m = 200\ Kg$

$V = 7'00\ Km.s^{-1}$

DATOS

$g = 9'81\ m.s^{-1}$ // $R_T = 6'37.10^6 m.$

a)



$$F_G = F_i$$

$$G\frac{M_T m}{R^2} = m\frac{v^2}{R}$$

$$G\frac{M_T}{R^2} = \frac{v^2}{\cancel{R}}$$

$$G\frac{M_T}{R} = v^2$$

$$v^2 = G\frac{M_T}{R} \quad ; \quad G??$$

$$g = G\frac{M_T}{R_T^2} \Rightarrow G = \frac{g.R_T^2}{M_T}$$

$$v^2 = \frac{g\ R_T^2}{\cancel{M_T}} \cdot \frac{\cancel{M_T}}{R} \Rightarrow v^2 = \frac{g.R_T^2}{R}$$

$$R = \frac{g.R_T^2}{v^2} = \frac{(6'37.10^6 m)^2 \cdot 9'81\ m/s^2}{(7.10^3 m/s)^2} = 8'124.10^6 m.$$

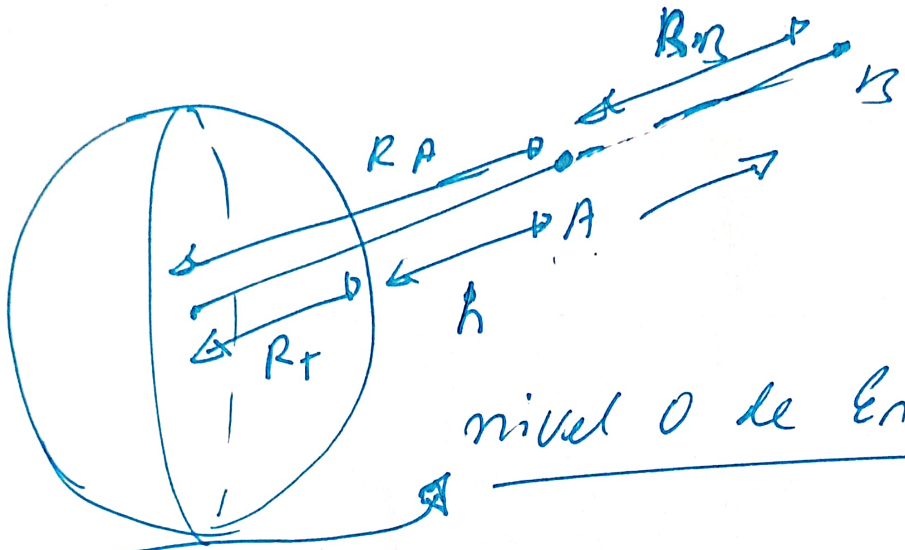
$$R = R_T + h \Rightarrow h = R - R_T = 8'124.10^6 m - 6'37.10^6 m.$$

$$h = 1'75.10^6 m.$$

$$R_1 = \frac{1}{\frac{1}{8'124\cdot 10^6 m} - \frac{(7\cdot 10^3 m/s)^2}{9'81 \frac{m}{s^2}\cdot (6'37\cdot 10^6 m)^2}} = - \;??$$

Moito máis axeitado

b)



nivel 0 de Enerxia potencial A

Enerxia A = Enerxia B

$$(E_p)_A + (E_c)_A = (E_p)_B + (E_c)_B$$

$(E_c)_B = 0$ non continua o movemento

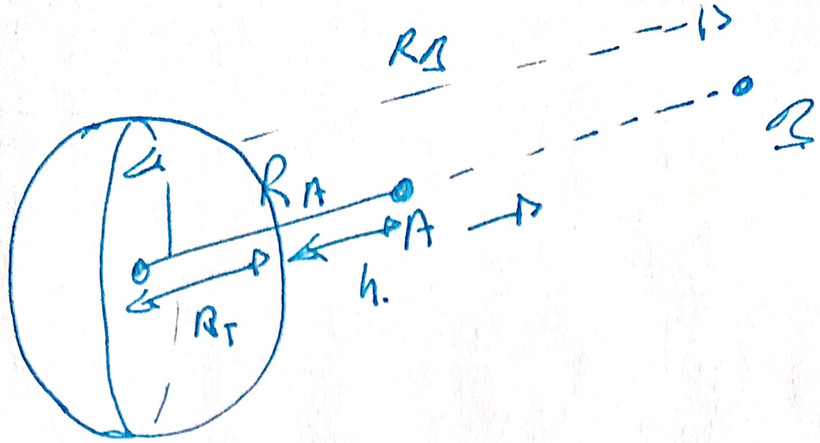
$(E_p)_A = 0$

$$(E_c)_A = -(E_p)_B \;;\; \frac{1}{2} m v_A^2 = -G\frac{M\cdot m}{R_B}$$

Valor de R_B negativo.

$r = \infty$

b)



Enerxia A = Enerxia B

$$(E_p)_A + (E_c)_A = (E_p)_B + (E_c)_A$$

$$\frac{1}{2} m v_A^2 - G\frac{Mm}{R_A} = \frac{1}{2} m v_B^2 - G\frac{M.m.}{R_B}$$

$$\frac{1}{2} \cancel{m} v_A^2 - G\frac{M\cancel{m}}{R_A} = \frac{1}{2} \cancel{m} \underbrace{\cancel{v_B^2}}_{0} - G\frac{M\cancel{m}.}{R_B}$$

porque non sigue. Calcular $\Rightarrow R_B$?

$$\frac{1}{2} v_A^2 - \frac{GM.}{R_A} = -G\frac{M.}{R_B} \; ; -\frac{1}{2} v_A^2 + G\frac{M.}{R_A} = G\frac{M.}{R_B}$$

$$\left(-\frac{1}{2} v_A^2 + \frac{GM.}{R_A}\right) R_B = GM. \; ;$$

$$\frac{1}{\cancel{GM.}}\left(-\frac{1}{2} v_A^2 + \frac{GM.}{R_A}\right) R_B = \cancel{GM.} \frac{1}{\cancel{GM}}$$

$$\left(-\frac{v_A^2}{GM} + \frac{1}{R_A}\right) R_B = 1 \Rightarrow R_B = \frac{1}{\frac{1}{R_A} - \frac{v_A^2}{GM}}$$

$$R_B = \frac{1}{\frac{1}{R_A} - \frac{v_A^2}{\frac{g R_T^2}{\cancel{M_T}} \cancel{M_T}}} \; ; \; R_B = \frac{1}{\frac{1}{R_A} - \frac{v_A^2}{g . R_T^2}}$$

2.-

DATOS

$T_X = 12\, T_T$

a)? $R_X = n\, T_T$

relación

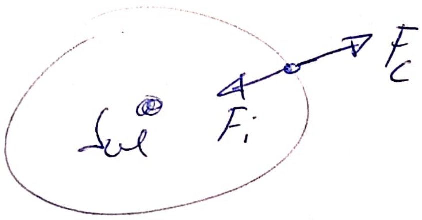
lei de Kepler $\frac{T^2}{R^3} = cte$: $\frac{T_X^2}{R_X^3} = \frac{T_T^2}{R_T^3}$

$R_X^3 \frac{T_T^2}{R_T^2} = T_X^2 \Rightarrow R_X^3 = \frac{T_X^2}{T_T^2} \cdot R_T^3$

$R_X^3 = \frac{(12)^2 T_T^2}{T_T^2} \cdot R_T^3 \Rightarrow R_X = 5'24\, R_T$

b) aceleracións nas órbitas?

a aceleración normal de xiro en torno a Xupiter.



$F_c = m \cdot \frac{V^2}{r} \Rightarrow a_T = \frac{V_T^2}{R_T}$; $a_X = \frac{V_X^2}{R_X}$

$\frac{V^2}{R} = \frac{w^2 \cdot R^2}{R} = \frac{4\pi^2 R}{T^2}$; $a_X = \frac{4\pi^2 R_X}{T_X^2}$

$a_X = \frac{4\pi^2 (5'24 R_T)^2}{(12\, T_T)^2} = 4\pi^2\, 0'036 \frac{R_T}{T_T^2}$ | $a_X = 0'036\, a_T$

$a_T = 4\pi^2 \frac{R_T}{T_T^2}$

3.-

$R = 1850 km = 1'850 \cdot 10^6 m$

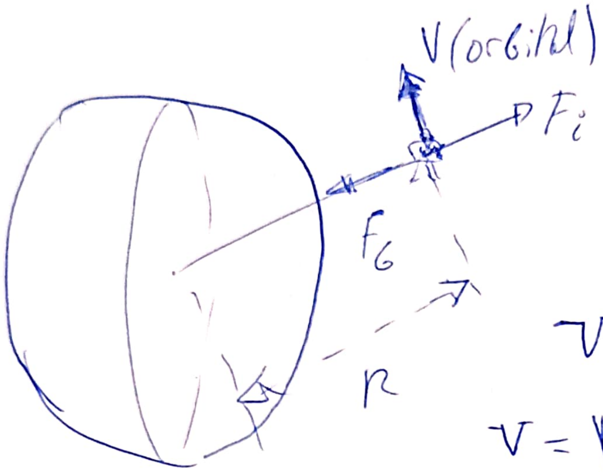
$M_L = 7'36 \cdot 10^{22} kg$

a) v?

b) T?

Dato $G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$

a)



$|F_G| = |F_i|$

$G \frac{M \cancel{m}}{R^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{R}}$

$v^2 = \frac{GM}{R} = \frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 7'36 \cdot 10^{22} kg}{1'850 \cdot 10^6 m}$

$v = \sqrt{\frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 7'36 \cdot 10^{22} kg}{1'850 \cdot 10^6 m}}$

$v = 1629 \frac{m}{s}$

b)

$v = \omega \cdot R \Rightarrow v = \frac{2\pi}{T} \cdot R \Rightarrow T = \frac{2\pi R}{v}$

$T = \frac{2 \cdot \pi \cdot 1'850 \cdot 10^6 m}{1629 \frac{m}{s}} = 7136 s$

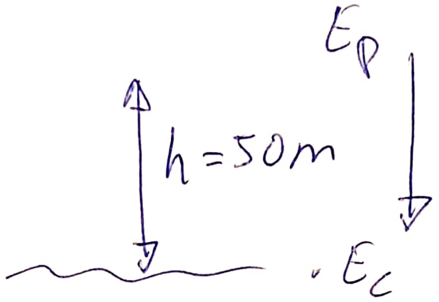
4.-

$M_M = 0'107\, M_T$

$R_M = 0'533\, R_T$

DATOS: $g_T = 9'81\, m.s^{-1}$; $R_T = 6'37 \cdot 10^6 m$.

a) $h = 50\,m$. tiempo que tarda t



$$E_c = E_p \Rightarrow \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh \Rightarrow v = \sqrt{2gh}$$

$$g_M = \frac{G M_M}{R_M^2} \; ; \; g_T = \frac{G M_T}{R_T^2} \; ; \; \frac{g_M}{g_T} = \frac{\frac{\cancel{G} M_M}{R_M^2}}{\frac{\cancel{G} M_T}{R_T^2}} \Rightarrow \frac{g_M}{g_T} = \frac{R_T^2 M_M}{R_M^2 M_T}$$

$$g_M = g_T \frac{M_M}{M_T} \frac{R_T^2}{R_M^2} \; ; \; g_M = g_T \frac{0'107\, M_T}{M_T} \frac{\cancel{R_T^2}}{(0'533)^2 \cancel{R_T^2}}$$

$$g_M = 0'377\, g_T \Rightarrow g_M = 3'69 \frac{m}{s^2}$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 3'69 \frac{m}{s^2} \cdot 50m} = 19'59\, m/s$$

t?

$$v = v_0 + gt \; ; \; \cancel{e = v_0 t - \frac{1}{2} g t^2} \quad e = e_0 + v_0 t - \frac{1}{2} g t^2$$

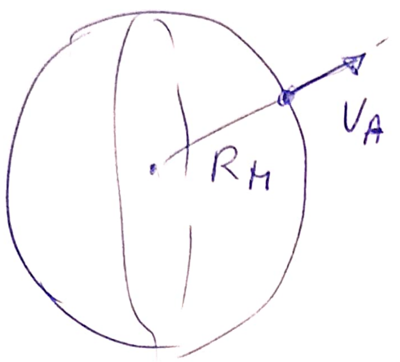
$$-e_0 = -\frac{1}{2} g t^2 \Rightarrow t = \left(\frac{2e_0}{g}\right)^{1/2} ; \; t = \left(\frac{2 \cdot 50m}{3'69 \frac{m}{s^2}}\right)^{1/2}$$

$$t = 5'21\, s$$

b) V de escape

∞

$V_B = 0$



$E_A = E_\infty$; $(E_c)_A + (E_p)_A = (E_c)_\infty + (E_p)_\infty$

$$\frac{1}{2} m v_A^2 - G\frac{M_M \cdot m}{R_M} = 0$$

$$v = \left(\frac{2}{\cancel{m}} \frac{G M_M \cdot \cancel{m}}{R_M}\right)^{1/2} = \sqrt{\frac{2 \cdot G M_M}{R_M}}$$

$$G?? \Rightarrow g_M = G\frac{M_M}{R_M^2} \Rightarrow G = \frac{g_M \cdot R_M^2}{M_M}$$

$M_M = 0'107 M_T$; $R_M = 0'533 \cdot R_T$

$$v = \left(\frac{2 \cdot \frac{g_M \cdot R_M^{\cancel{2}}}{\cancel{M_M}} \cdot \cancel{M_M}}{\frac{\cancel{R_M}}{1}}\right)^{1/2} = (2 \cdot g_M \cdot R_M)^{1/2}$$

$$v = (2 \cdot 3'69\ m/s^2 \cdot 0'533 \cdot 6'37 \cdot 10^6\ m)^{1/2}$$

$$v = 2'242 \cdot 10^3\ m/s$$

5.-

$g_{planeta} = 7'2 . m . s^{-2}$

$R_{planeta} = 4100 Km = 4'1 \cdot 10^6 m.$

Datos

$G = 6'67 . 10^{-11} \frac{N . m^2}{Kg^2}$

a) M_p ?

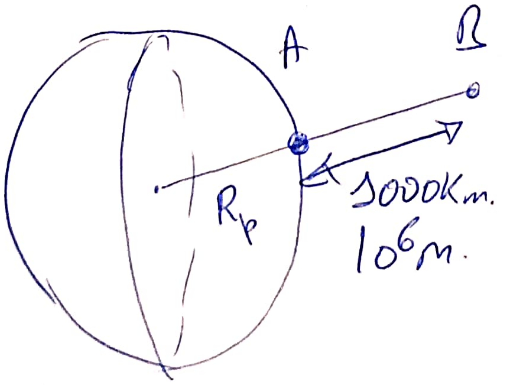
$$g_p = G \frac{M_p}{R_p^2} \Rightarrow M_p = \frac{g_p R_p^2}{G}$$

$$M_p = \frac{7'2 \, m . s^{-2} (4'1 . 10^6 m)^2}{6'67 . 10^{-11} \frac{N . m^2}{Kg^2}} = 1'81 \, 10^{24} Kg.$$

Unidades

$$\frac{m . s^{-2} \, m^2}{\frac{N . m^2}{Kg^2}} = \frac{\frac{m . s^{-2} . \cancel{m^2}}{1}}{\frac{Kg . m . s^{-2} . m^2}{Kg^2}} = \frac{\cancel{m} . \cancel{s^{-2}} . \cancel{m^2} \, Kg^2}{Kg \, \cancel{m} . \cancel{s^{-2}} . \cancel{m^2}} = Kg$$

b)



Enerxia para ir de A. B.

é a enerxia cinética

$$(E_c)_A + (E_p)_A = \cancel{(E_c)_B} + (E_p)_B$$

$\cancel{\Delta}$ 0

cuando chegue a B non ten velocidade porque non segue movendose

$$(E_c)_A = (E_p)_B - (E_p)_A$$

$$(E_c)_A = -\frac{G M . m}{r_B} - \left(-\frac{G M . m}{r_A}\right)$$

$$(E_c)_A = G M . m . \left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$(E_c)_A = 6'67 . 10^{-11} \frac{N . m^2}{\cancel{Kg^2}} 3 Kg . 5'81 \, 10^{24} Kg \left(\frac{1}{4'1 \, 10^6 m} - \frac{1}{5'1 \, 10^6 m}\right)$$

$$(E_c)_A = 1'73 \, 10^7 J$$

Unidades

$$N . \frac{m^{\cancel{2}}}{\cancel{Kg^2}} . \cancel{Kg} . \cancel{Kg} \, \cancel{m^{-1}} = N . m = J$$

6.-

$m = 150 \, Kg$

$V = 30 \, km \cdot s^{-1} = 3 \, 10^{4} \, m \, s^{-1}$

DATOS

$G = 6'67 \cdot 10^{-11} \, \frac{N \cdot m^2}{Kg^2}$

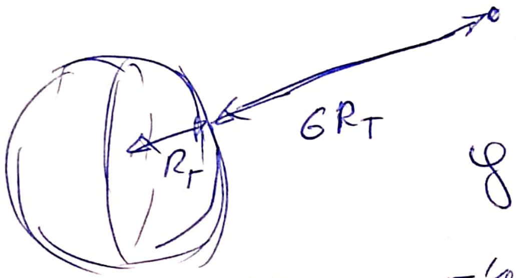
$M_T = 5'98 \, 10^{24} \, Kg$

$R_T = 6'37 \cdot 10^{6} \, m$

a) Peso ?

b) Em ?

a) Peso ; $Peso = m \cdot g$



$$g = G \frac{M}{R^2}$$

$$g = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{5'98 \, 10^{24} \, Kg}{(7 \cdot 6'37 \, 10^{6} \, m)^2} = 0'20 \, m/s^2$$

$$Peso = m \cdot g = 150 \, Kg \cdot 0'20 \, \frac{m}{s^2} = 30'1 \, N$$

b) $$E_m = (E_c + E_p) = \frac{1}{2} m v^2 + \left(-G \frac{M \cdot m}{r}\right)$$

$$E_m = m \cdot \left(\frac{1}{2} v^2 - G \frac{M}{r}\right) = 150 \, Kg \left[\frac{1}{2}\left(3'0 \, 10^{4} \, \frac{m}{s}\right)^2 - \frac{6'67 \cdot 10^{-11} \, N \cdot m^2/Kg^2 \cdot 5'98 \, 10^{24} \, Kg}{7 \cdot 6'37 \, 10^{6} \, m}\right]$$

$$E_m = 6'6 \, 10^{10} \, J$$

7.-

$m_1 = 2\ Kg \qquad x = 0\ m$

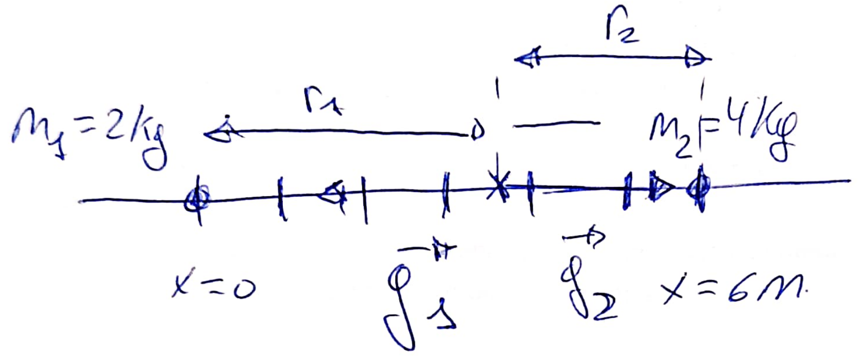
$m_2 = 4\ Kg \qquad x = 6\ m.$

a) $g = 0$? onde

b) V? en $x=2$

c) $W_{x=2}^{\infty}$ desde $x=2m$ a infinito

a)



de onde estará máis cerca o punto da masa de 4 Kg ou da de 2 Kg

$$\vec{g}_1 = -G\frac{m_1}{r_1^2}\vec{i} \qquad |\vec{g}_1| = |\vec{g}_2|$$

$$\vec{g}_2 = G\frac{m_2}{r_2^2}\vec{i}$$

$$\cancel{G}\frac{m_1}{r_1^2} = \cancel{G}\frac{m_2}{r_2^2} \Rightarrow \frac{m_1}{r_1^2} = \frac{m_2}{r_2^2}\ ;\ r_1 + r_2 = 6m.$$

$\dfrac{m_1}{r_1^2} = \dfrac{m_2}{r_2^2} \Rightarrow$ extraigo a raiz para non ter ecuacións de 2º grado

$$\frac{m_1^{1/2}}{r_1} = \frac{m_2^{1/2}}{r_2}\ ;\ m_1^{1/2}\, r_2 = m_2^{1/2}\cdot r_1\ ;\quad r_1 = 6 - r_2$$

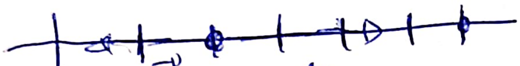
$$\frac{m_1^{1/2}}{m_2^{1/2}} r_2 = 6 - r_2 \Rightarrow \left(\frac{m_1^{1/2}}{m_2^{1/2}} + 1\right) r_2 = 6m.$$

$$r_2 = \frac{Gm}{\left[\left(\frac{m_1}{m_2}\right)^{1/2}+1\right]} \Rightarrow r_2 = \frac{Gm}{\left[\left(\frac{2}{4}\right)^{1/2}+1\right]} =$$

$r_2 = 3'51\,m.$; $r_1 = 2'49\,m.$

b) g en $x = 2\,m.$

$g?$



$m_1 = 2\,Kg$ $\quad \vec{g}_1 \quad \vec{g}_2 \quad$ $m_2 = 4\,Kg$

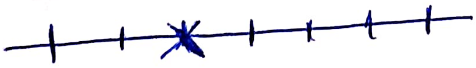
$$\vec{g}_1 = -G\frac{m_1}{r_1^2}\,\vec{i} \; ; \; \vec{g}_2 = +G\frac{m_2}{r_2^2}\,\vec{i}$$

$$\vec{g}_T = G\left(\frac{m_2}{r_2^2} - \frac{m_1}{r_1^2}\right)\vec{i}$$

$$\vec{g}_T = 6'67 \cdot 10^{-11}\,\frac{N \cdot m^2}{Kg^2}\left(\frac{4\,Kg}{(4\,m)^2} - \frac{2\,Kg}{(2\,m)^2}\right)\vec{i}$$

$$\vec{g}_T = -1'67 \cdot 10^{-11}\,\frac{N}{Kg}\,\vec{i}$$

V en $x = 2\,m.$



$m_1 = 2\,Kg$ $\qquad$ $m_2 = 4\,Kg$

$$V_1 = -G\frac{m_1}{r_1} \; ; \; V_2 = -G\frac{m_2}{r_2}$$

$$V_T = V_1 + V_2 = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$V_T = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}\left(\frac{2\ Kg}{2m} + \frac{4\ Kg}{4m}\right)$$

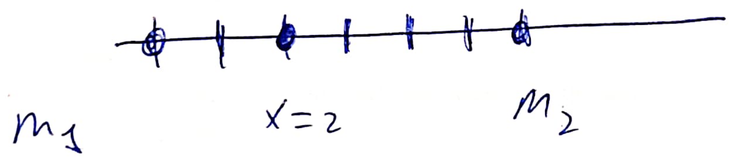
$$V_T = -1'33\ 10^{-10} \frac{J}{Kg}$$

unidades

$$\frac{N \cdot m^2}{Kg^{\cancel{2}}} \cdot \frac{\cancel{Kg}}{m} = \frac{N \cdot m}{Kg} \quad [N \cdot m = J \text{ (julio)}]$$

o traballo $W = F \cdot e$ $(N \cdot m)$

c) $W = -\Delta E_p$; $m' = 6\ Kg$



$$U = E_p = V \cdot m'$$

o potencial gravitatorio pola masa é a enerxía potencial

$$W_{x=2}^{\infty} = -\Delta E_p = -\left[\cancel{(E_p)_{\infty}} - (E_p)_{x=2}\right]$$

0

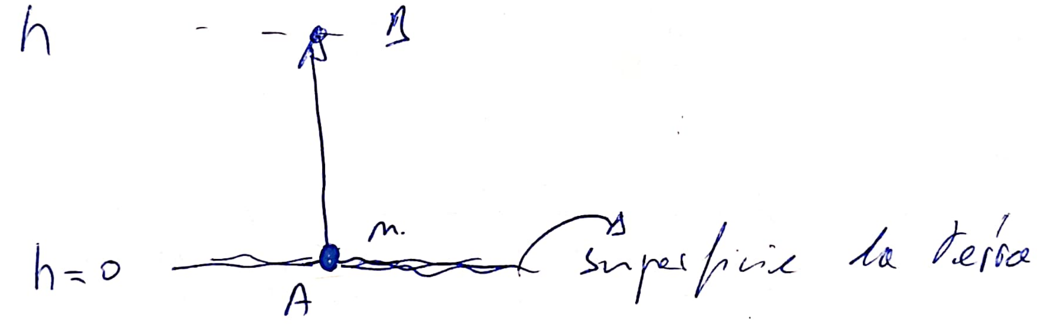
$$W_{x=2}^{\infty} = (E_p)_{x=2} = -V_{x=2} \cdot m'$$

$$\downarrow$$

$$m' = 6\,Kg$$

$$W_2^{\infty} = -1'33 \cdot 10^{-10} \frac{J}{Kg} \cdot 6\,Kg = -7'98 \cdot 10^{-10}\,J$$

interpretación do signo, Dependendo de si o signo do traballo é positivo o negativo o traballo é realizado polo campo ou contra o campo.



Si subimos unha masa do punto A o punto B, temos que facelo co noso traballo, imos ver o signo

$$W_A^B = -\Delta E_p = -(U_B - U_A) = -\left[-\frac{M.m.}{R_T+h} - \left(-\frac{GMm}{R_T}\right)\right]$$

$$\left|\frac{Mm.}{R_T+h}\right| < \left|\frac{GMm.}{R_T}\right|$$

$W_A^B = (-)$ negativo, feito contra o campo

8.- $h = 350\ Km = 3'5\,10^5\,m.$

a) v?

b) T?

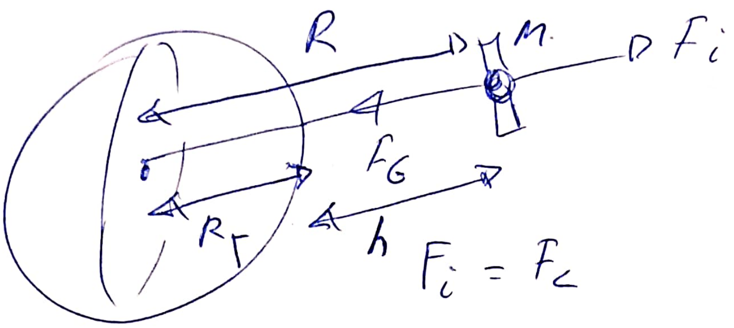
c) comparar a_c con g_T

DATOS

$r_T = 6'37\,10^6\,m.$

$g_0 = g_T = 9'81\ \frac{m}{s^2}$

a)



$R = 3'5\,10^5\,m + 6'37\,10^6\,m$

$R = 6'72.\,10^6\,m.$

$F_i = F_c$

$$G\frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{R}} \Rightarrow v^2 = \frac{G M_T}{R}$$

G en función de g_0; $g_0 = g_T$

$$g_0 = G\frac{M_T}{R_T^2} \Rightarrow G = \frac{g_0 R_T^2}{M_T}$$

$$v^2 = \frac{g_0 R_T^2 \cancel{M_T}}{R \cancel{M_T}}\ ;\ v = \left(\frac{9'81\,m/s^2 \cdot (6'37\,10^6\,m)^2}{6'72\,10^6\,m}\right)^{1/2}$$

$$v = 7'7.\,10^3\,m/s$$

b)

$$v = \omega . R \Rightarrow v = \frac{2\pi}{T}.R \Rightarrow T = \frac{2\pi R}{v}$$

$$T = \frac{2.\pi.\,6'72.\,10^6\,m}{7'7.\,10^3\,\frac{m}{s}} = 5'45\,10^3\,s$$

c) $$a_c = \frac{v^2}{R} = \frac{(7'7\,10^3\,m/s)^2}{6'72\,10^6\,m} = 8'82\,m/s^2$$

9.- $\frac{2 \text{ voltas}}{24h}$

DATOS

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$M_T = 5'98 \cdot 10^{24}\ Kg$

$R_T = 6'37 \cdot 10^6\ m$

$m_{sate} = 150\ Kg$

a) h?

b) Em?

c) T? para 2h.

a)

$$w = \frac{2 \text{ voltas}}{24h} \frac{2\pi \text{ rad}}{1 \text{ volta}} \frac{1h}{3600s} = 1'45 \cdot 10^{-4} \frac{rad}{s}$$

$$G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{R}} ; \quad G \frac{M_T}{R} = w^2 R^2$$

$$R^3 = \frac{G M_T}{w^2}$$

Comprobamos se esta ben despexado, UNIDADES

$$\frac{G M_T}{w^2} = \frac{\frac{N \cdot m^2}{Kg^{\cancel{2}}} \cdot \cancel{Kg}}{s^{-2}} = \frac{N \cdot m^2}{Kg\ s^{-2}} = \frac{\cancel{Kg}\ m \cdot \cancel{s^{-2}} \cdot m^2}{\cancel{Kg}\ \cancel{s^{-2}}} = \frac{m^3}{R^3}$$

$$R = \left(\frac{6'67 \cdot 10^{-11} N \cdot m^2/Kg^2 \cdot 5'98 \cdot 10^{24} Kg}{\left(1'45 \cdot 10^{-4} \frac{rad}{s}\right)^2} \right)^{1/3}$$

$R = 2'67 \cdot 10^7\ m$.

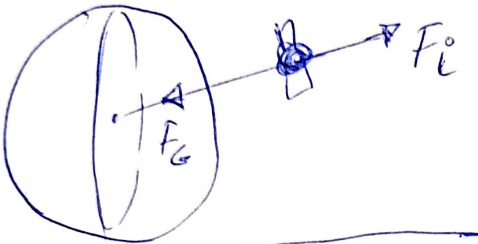
$h = R - R_T = 2'03 \cdot 10^7\ m$.

b)

$$E_m = E_c + E_p$$

$$E_c = \frac{1}{2} m v^2 \; ; \; E_p = U = -G \frac{M \cdot m}{r}$$

para un obxecto que xira en torno a outro



$$G \frac{M m}{R^{\cancel{2}}} = m \frac{v^2}{\cancel{R}}$$

$$m v^2 = G \frac{M m}{R}$$

$$\frac{1}{2} m v^2 = \frac{1}{2} G \frac{M \cdot m}{R}$$

$$E_m = E_c + E_p = \frac{1}{2} m v^2 - G \frac{M m}{R}$$

$$E_m = \frac{1}{2} G \frac{M m}{R} - G \frac{M \cdot m}{R} = -\frac{1}{2} G \frac{M m}{R}$$

$$E_m = -\frac{1}{2} 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{5'98 \, 10^{24} Kg \cdot 150 Kg}{(2'67 \cdot 10^7 m}$$

$$E_m = -1'12 \, 10^9 J$$

c) $h' = 2 \times 2'67 \cdot 10^7 m = 5'34 \cdot 10^7 m$

$$R = R_T + h' = 5'98 \cdot 10^7 m.$$

$$G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{R}} \; ; \; G \frac{M_T}{R} = \frac{4\pi^2}{T^2} \cdot R^2 \; ; \; T^2 = \frac{4\pi^2 R^3}{G M_T}$$

$$T = \left(\frac{4\pi^2 (5'98 \cdot 10^7 m)^3}{6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98 \, 10^{24} Kg} \right)^{1/2} = 1'45 \, 10^5 s$$

$T = 40'41 h.$

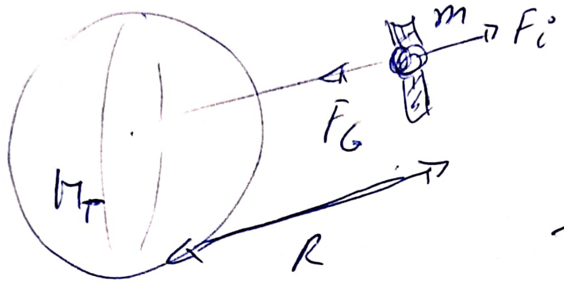
10.- $R = 2R_T$

DATOS

$R_T = 6370\ km = 6'37 \cdot 10^6\ m$

$g = 9'81\ m/s$; $g_T = g_0 = 9'81\ \frac{m}{s^2}$

a) v ?



$F_G = F_i$

$\cancel{m} \frac{v^2}{R} = G \frac{M_T \cancel{m}}{R^2}$

$v^2 = \frac{G M_T}{R}$

$g_T = G \frac{M_T}{R_T^2} \Rightarrow v^2 = \frac{G M_T}{2R_T}$

$G = \frac{g_T R_T^2}{M_T}$

$v^2 = \frac{g_T R_T^{\cancel{2}}}{\cancel{M_T}} \frac{\cancel{M_T}}{2\cancel{R_T}} = \frac{g_T R_T}{2}$

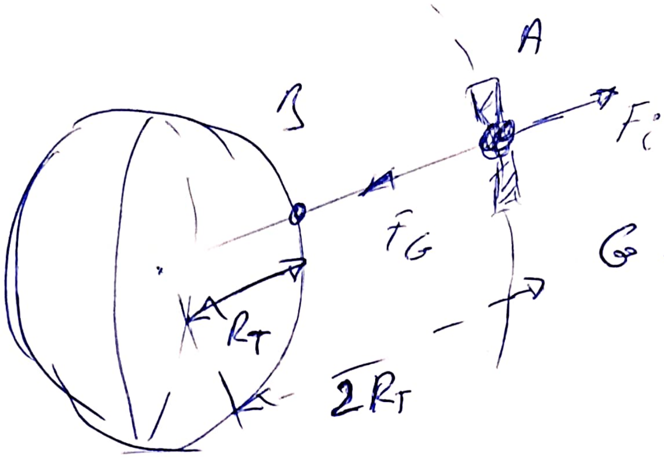
$v^2 = \frac{9'81\ m/s^2 \cdot 6'37 \cdot 10^6\ m}{2}$; $v^2 = 3'124 \cdot 10^6\ \frac{m}{s^2}$

$v = 5'589 \cdot 10^3\ m/s$

b) $g_{2R} = G \frac{M_T}{(2R_T)^2}$; $g_{2R} = \frac{g_T}{4} = \frac{9'81\ m/s^2}{4}$

$g_{2R} = 2'45\ \frac{m}{s^2}$

c)



$$G\frac{M}{R^2} = m\frac{v^2}{R}$$

Enerxía A = Enerxía B

$$(E_c)_A + (E_p)_A = (E_c)_B + (E_p)_B$$

$$\frac{1}{2}\cancel{m}v_A^2 - G\frac{M\cancel{m}}{2R_T} = \frac{1}{2}\cancel{m}v_B^2 - G\frac{M\cancel{m}}{R_T}$$

$$\frac{1}{2}v_A^2 - G\frac{M}{2R_T} = \frac{1}{2}v_B^2 - G\frac{M}{R_T}$$

$$v_B^2 = 2\left[\frac{1}{2}v_A^2 + \frac{GM}{R_T}\left(-\frac{1}{2}+1\right)\right] \Rightarrow -\frac{1}{2}+1 = +\frac{1}{2}$$

$$v_A^2 = v_B^2 + \frac{GM}{R_T} \Rightarrow G = \frac{g_T R_T^2}{M_T}$$

$$v_A^2 = (40\,m/s)^2 + \frac{g_T R_T^{\cancel{2}}}{\cancel{M_T}}\frac{\cancel{M_T}}{\cancel{R_T}}$$

$$v_A^2 = (40\,m/s)^2 + 9'81\,m/s^2 \cdot 6'37\cdot10^6\,m$$

$$v_A^2 = 6'25\cdot10^6\,m^2/s^2 \quad ; \quad v_A = 7'905\cdot10^3\,m/s$$

11.- $m = 10^2\,kg$

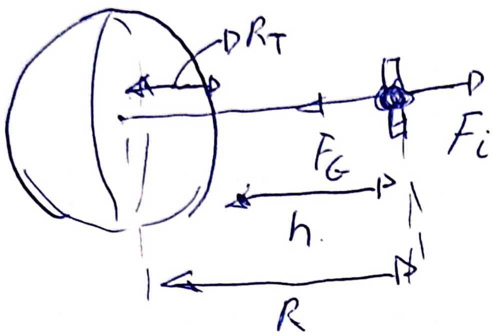
$h = 4\,10^3\,Km = 4.10^6\,m$

DATOS

$R_T = 6'37\,10^6\,m$

$g_0 = 9'81\,\frac{m}{s^2}$

a) v? ; T? ; g



$|F_G| = |F_i|$

$R = 6'37\,10^6\,m + 4.10^6\,m = 1'037.10^7\,m$

$G\frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{R}}$; $v^2 = \frac{G\,M_T}{R}$

$g_T = G\frac{M_T}{R_T^2} \Rightarrow G = \frac{g_T R_T^2}{M_T} \longrightarrow v^2 = \frac{g_T R_T^2}{\cancel{M_T}} \cdot \frac{\cancel{M_T}}{R}$

$v = R_T\left(\frac{g_T}{R}\right)^{1/2} \Rightarrow v = 6'37\,10^6\,m.\left(\frac{9'81\,m/s^2}{1'037.10^7\,m}\right)^{1/2} = 6196\,\frac{m}{s}$

$v = \omega.R \Rightarrow v = \frac{2\pi}{T}.R \Rightarrow T = \frac{2.\pi R}{v} = \frac{2.\pi.1'037\,10^7\,m}{6196\,m/s}$

$T = 10.525\,s$ (2 h, 55 min)

$a_c = g = G\frac{M_T}{R^2} = \frac{g_T R_T^2}{\cancel{M_T}}\cdot\frac{\cancel{M_T}}{R^2} = g_T\left(\frac{R_T}{R}\right)^2 = 9'81\,\frac{m}{s^2}\left(\frac{6'37\,10^6}{1'037\,10^7}\right)^2$

$a_c = g = 3'70\,m/s^2$

b) $\vec{L} = \vec{r} \wedge \vec{p}$; $|\vec{L}| = |\vec{r}|.|\vec{p}|.\,sen\,90^\circ$

$|\vec{L}| = |\vec{r}|\,|\vec{p}| = r.mv = R.m.v = 1'037.10^7\,m.\,10^2\,kg.\,6196\,\frac{m}{s}$

$|\vec{L}| = 6'425\,10^{12}\,kg\,\frac{m^2}{s}$

12.-

$v = 7'62 \frac{Km}{s}$

a) h?

b) T?

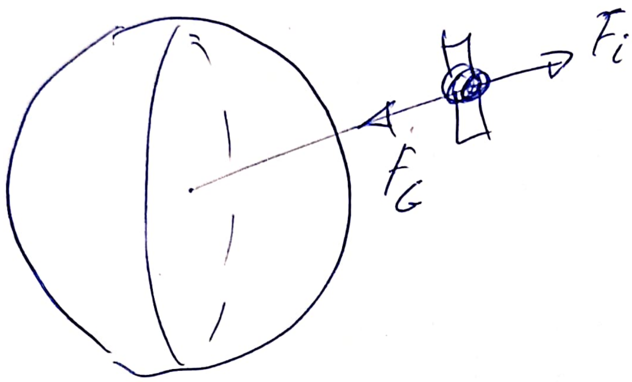
c) voltas en 24 horas

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_T = 6370\ Km$

$M_T = 5'93 \cdot 10^{24}\ Kg$

a)



$F_G = F_i$; $G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{R}}$

$$R = \frac{G M_T}{v^2} = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{5'93 \cdot 10^{24}\ Kg}{\left(7620 \frac{m}{s}\right)^2}$$

$R = 6'812 \cdot 10^6\ m$

$h = R - R_T = 6'812 \cdot 10^6 m - 6'370 \cdot 10^6 m = 4'42 \cdot 10^5 m$

b) $V = w \cdot R \Rightarrow V = \frac{2\pi}{T} \cdot R$

$$T = \frac{2\pi R}{V} = \frac{2 \cdot \pi \cdot 6'812 \cdot 10^6 m}{7620 \frac{m}{s}} = 5617\ s \ (1h,\ 33 minutos)$$

c) $f = \frac{1}{T} = 1'78 \cdot 10^{-5} \frac{1}{\cancel{s}} \cdot \frac{24 \times 3600 \cancel{s}}{} = 15'4$ voltas

13.- $m = 500\,Kg$

$h = 5000\,Km. \rightarrow 5 \cdot 10^6\,m$

DATOS

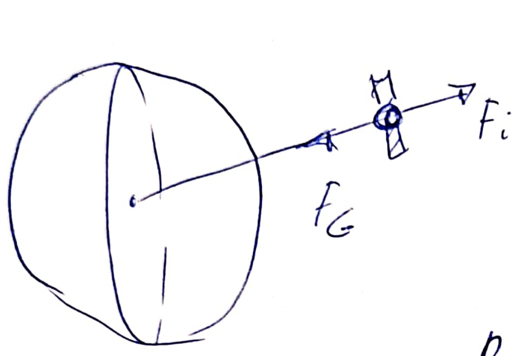
$R_T = 6370\,Km. = 6'37 \cdot 10^6\,m.$

$g_0 = 9'8\,ms^{-2}$

a) v?

b) E_m?

c) Enerxía para acadar o infinito



$$|\vec{F_G}| = |\vec{F_i}|$$

$$G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{R}}$$

$$R = 6'37 \cdot 10^6\,m + 5 \cdot 10^6\,m = 1'14 \cdot 10^7\,m.$$

$$v^2 = \frac{G M_T}{R} \quad ; \quad g_0 = \frac{G M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

$$v^2 = \frac{g_0 R_T^2}{R} \; ; \; v = \sqrt{\frac{g_0 R_T^2}{R}} = \left(\frac{9'8\,m/s^2 \cdot (6'37 \cdot 10^6\,m)^2}{1'14 \cdot 10^7\,m}\right)^{1/2}$$

$v = 5906\,m/s$

b)

$$E_m = E_c + U = \frac{1}{2} m v^2 - G \frac{M m}{R}$$

tendo en conta $G \frac{M_T m}{R^{\cancel{2}}} = \frac{m v^2}{\cancel{R}}$

$$m \cdot v^2 = G \frac{M_T m}{R}$$

$$\rightarrow E_m = \frac{1}{2} \frac{G M_T m}{R} - G \frac{M m}{R} = -\frac{1}{2} G \frac{M m}{R}$$

$$E_m = -\frac{1}{2}\frac{G M_T m}{R} \; ; \; g_0 = \frac{G M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

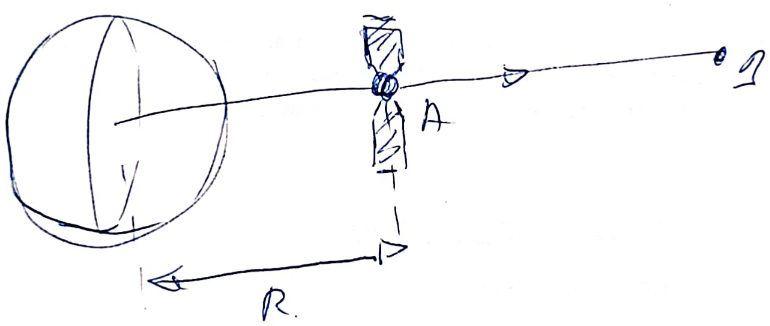
$$E_m = -\frac{1}{2}\frac{g_0 R_T^2 . m.}{R}$$

$$E_m = -\frac{1}{2}\frac{9'81\,m/s^2\,(6'37 \cdot 10^6 m)^2\,500\,Kg}{1'14\,10^7 m} = -8'73\,10^4\,J$$

unidades $= \dfrac{\frac{m}{s^2} \cdot m^2 . Kg}{\frac{m}{1}}$ $\Big\}$ $Kg\,\frac{m}{s^2} = N$

$$\frac{N . m^2}{\cancel{m}} = N . m = J$$

c)



Enerxia en A = Enerxia B

$$\frac{1}{2} m v_A^2 - \frac{G M_T m.}{R} = \frac{1}{2} m \underbrace{v_B^2}_{0} - \underbrace{\cancel{\frac{G M_T m}{\infty}}}_{0}$$

<u>igual o do apartado B</u>

a enerxia que hai que facer $8'73\,10^4\,J$

14.-

$113\ km = 1'13\ 10^{5}\ m$.

a) periodo da orbita

b) v e w

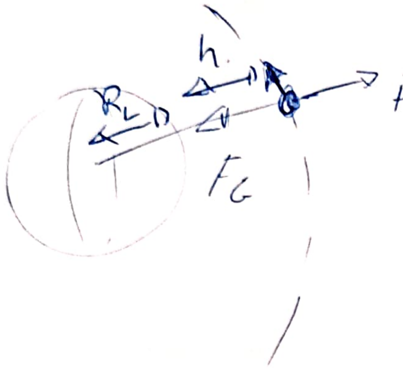
c) v_e

DATOS:

$G = 6'67 \cdot 10^{-11}\ \frac{N \cdot m^2}{Kg^2}$

$R_L = 1740\ km = 1'740\ 10^{6}\ m$.

$M_L = 7'36 \cdot 10^{22}\ Kg$



$$G\frac{M_L m}{R^2} = m\frac{v^2}{R}$$

$R = 1'740\ 10^{6}\ m + 1'13\ 10^{5}\ m =$

$$v^2 = \frac{G M_L}{R} = \frac{6'67 \cdot 10^{-11}\ N \cdot m^2/Kg^2 \cdot 7'36 \cdot 10^{22}\ Kg}{1'853 \cdot 10^{6}\ m}$$

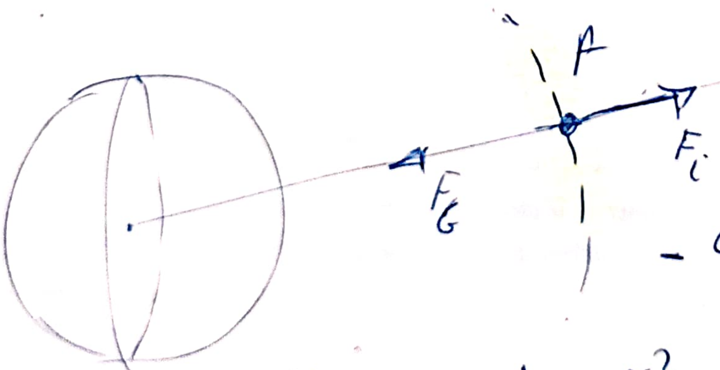
$v^2 = 2'64 \cdot 10^{8}\ \frac{m^2}{s^2}$; $v = 1624\ \frac{m}{s}$

$$v = w \cdot R \Rightarrow w = \frac{v}{R} = \frac{1624\ m/s}{1'853\ 10^{6}\ m} = 8'76 \cdot 10^{-4}\ \frac{rad}{s}$$

$$w = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{w} = \frac{2\pi\ rad}{8'76 \cdot 10^{-4}\ \frac{rad}{s}} = 7165\ s$$

$T = 2h$.

c)



B

Enerxia A = Enerxia B

$$-\frac{G M_L m}{R_A} + \frac{1}{2} m v_A^2 = -\frac{G M_L m}{\infty} + \frac{1}{2} m v_B^2$$

0

$$v_B = 0 \Rightarrow \frac{1}{2} m v_A^2 = +\frac{G M_L m}{R_A}$$

$$v_A = \sqrt{\frac{6'67 \cdot 10^{-11}\ N \cdot m^2/Kg \cdot 7'36 \cdot 10^{22}\ Kg \cdot 2}{1'853\ 10^{6}\ m}} = 2302\ \frac{m}{s}$$

15.-

$T_c = 4'60$ anos
$M_c = 9'43 \cdot 10^{20}$ Kg
$R_c = 477$ Km $= 4'77 \cdot 10^5$ m

DATOS
$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

a) g_c??

b) $m_{nave} = 1000$ Kg Enerxia de escape.

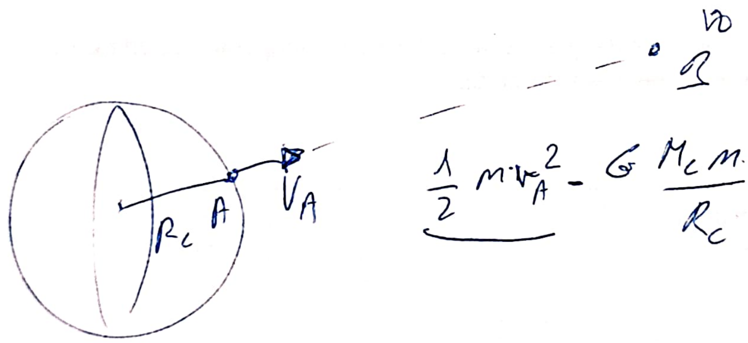
c) $d_{ceres-sol}$? DATO $d_{Terra-Sol} = 1'50 \cdot 10^{11}$ m; $T_{TERRA} = 1$ ano

a)

$$g_c = G \frac{M_c}{R_c^2} = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{9'43 \cdot 10^{20} Kg}{(4'77 \cdot 10^5 m)^2} =$$

$$g_c = 0'28 \frac{m}{s^2}$$

b)



$$\frac{1}{2} m v_A^2 - G \frac{M_c m}{R_c} = \frac{1}{2} m v_B^2 - G \frac{M_c m}{R}$$

$$E_c = G \frac{Mm}{r^2} = \frac{G M_c m}{R_c} = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{9'43 \cdot 10^{20} Kg \cdot 10^3 Kg}{4'77 \cdot 10^5 m} =$$

Enerxia $E = 1'32 \cdot 10^8$ J

c) $$\frac{T_c^2}{R_c^3} = \frac{T_T^2}{R_T^3} \Rightarrow R_c^3 = \frac{T_c^2}{T_T^2} \cdot R_T^3 \Rightarrow R_c = R_T \left(\frac{T_c}{T_T}\right)^{2/3}$$

$$R_c = 1'5 \cdot 10^{11} m \cdot \left(\frac{4'6}{1}\right)^{2/3} = 4'11 \cdot 10^5 \text{ anos}$$

mellor utilizar d. para o radio de xiro dos planetas

16.-

$m_A = m_B = 150\ Kg$ $\quad m_A\ A(0,0)$ $\quad m_B\ B(12,0)$

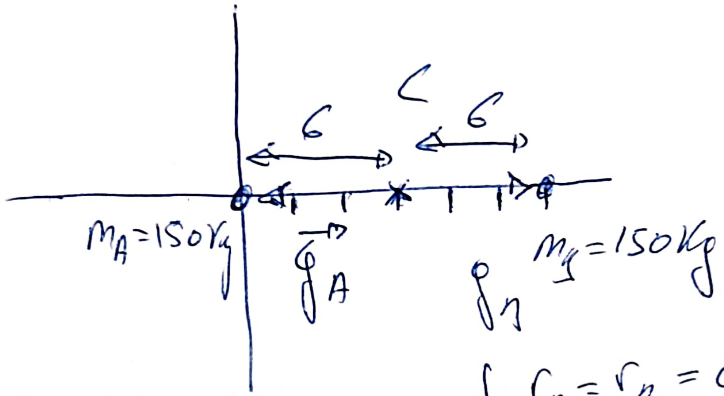
$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

a) g en $C(6,0)$ e D en $(6,8)$

b) $m = 2\ Kg$ $\quad V_D = -4 \cdot 10^{-4}\ \vec{j}\ (m/s)$; V_C?

c) $C - D$ mru? mru?

a)



$$|\vec{g}_A| = |\vec{g}_B| = G\frac{m}{r^2} \quad \left\{ \begin{array}{l} r_A = r_B = 6m \\ m_A = m_B = 150\ Kg \end{array} \right.$$

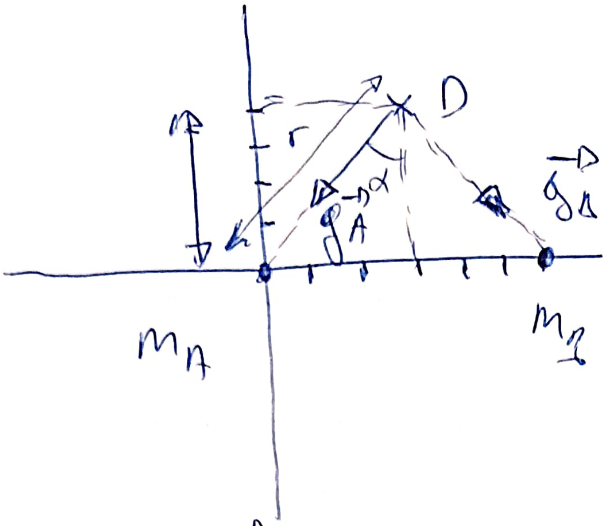
g en C cero

calculamos o seu valor

$$|\vec{g}_A| = |\vec{g}_B| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{150 Kg}{(6m)^2} = 2'78\ 10^{-10} \frac{N}{Kg}$$

$\vec{g}_A = -2'78 \cdot 10^{-10}\ \vec{i}\ N/Kg$

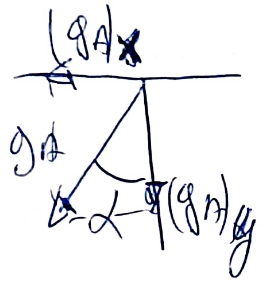
$\vec{g}_B = 2'78 \cdot 10^{-10}\ \vec{i}\ N/Kg$



$$r^2 = (6m)^2 + (8m)^2 = 100\,m^2$$

$$r = 10\,m.$$

$$|\vec{g}_A| = |\vec{g}_B| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2} \cdot \frac{150\,kg}{100\,m^2} = 1 \cdot 10^{-10} \frac{N}{kg}$$



$$\operatorname{sen}\alpha = \frac{6}{10} = 0'6$$

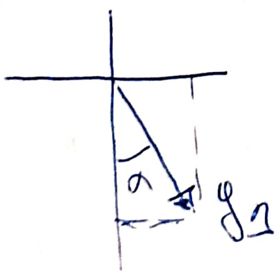
$$\cos\alpha = \frac{8}{10} = 0'8$$

$$\operatorname{sen}\alpha = \frac{(g_A)_x}{|g_A|} \Rightarrow (g_A)_x = |g_A| \cdot \operatorname{sen}\alpha$$

$$\cos\alpha = \frac{(g_A)_y}{|g_A|} \Rightarrow (g_A)_y = |g_A| \cdot \cos\alpha$$

$$(g_A)_x = 1 \cdot 10^{-10} \frac{N}{kg} \cdot 0'6 = 6 \cdot 10^{-11}\, N/kg$$

$$(g_A)_y = 1 \cdot 10^{-10} \frac{N}{kg} \cdot 0'8 = 8 \cdot 10^{-11}\, N/kg$$



$$\operatorname{sen}\alpha = \frac{(g_B)_x}{|g_B|} \Rightarrow (g_B)_x = |g_B| \cdot \operatorname{sen}\alpha$$

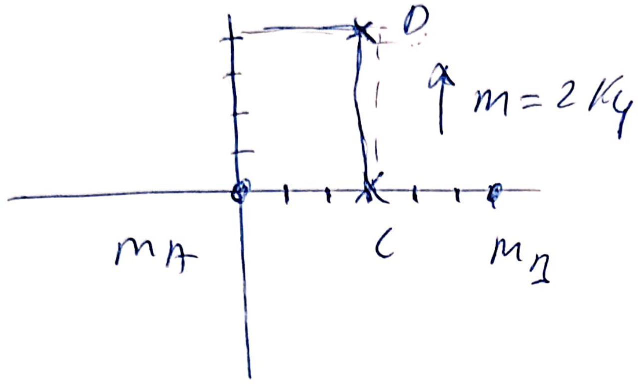
$$\cos\alpha = \frac{(g_B)_y}{|g_B|} \Rightarrow (g_B)_y = |g_B| \cdot \cos\alpha$$

$$\vec{g}_A = -(g_A)_x \vec{i} - (g_A)_y \vec{j}$$

$$\vec{g}_B = (g_B)_x \vec{i} - (g_B)_y \vec{j}$$

$$\vec{g}_A = -6\cdot10^{-11}\,\vec{i} - 8\cdot10^{-11}\,\vec{j}\ \frac{N}{Kg}$$
$$\vec{g}_B = 6\cdot10^{-11}\,\vec{i} - 8\cdot10^{-11}\,\vec{j}\ \frac{N}{Kg}$$
$$g_T = 16\cdot10^{-11}\,\vec{j}\ N/Kg$$

b) V_C?



enerxia C = Enerxia D

$$(E_p)_C + (E_c)_C = (E_p)_D + (E_c)_D$$

$$(E_p)_C = -G\frac{m_A m}{r_{AC}} - G\frac{m_B m}{r_{BC}}$$

enerxia potencial creados por la masa A e B en el punto C cuando esta la masa m.

$$(E_p)_D = -\frac{G m_A m}{r_{AD}} - G\frac{m_B m}{r_{BD}}$$

$$\frac{1}{2}v_C^2 - \left(\frac{G m_A}{r_{AC}} + \frac{G m_B}{r_{BC}}\right) = \frac{1}{2}v_D^2 - \left(G\frac{M_A}{r_{AD}} + G\frac{m_B}{r_{BD}}\right)$$

$$\frac{1}{2}v_C^2 - 2\frac{G m_A}{6m} = \frac{1}{2}v_D^2 - 2\frac{G M_A}{10m}$$

$$\frac{1}{2}V_C^2 = \frac{1}{2}V_D^2 + 2Gm_A\left(\frac{1}{6m} - \frac{1}{10m}\right)$$

$$V_C^2 = V_D^2 + 4\ 6'67\cdot10^{-11}\frac{N\cdot m^2}{Kg^2}\cdot 150Kg\cdot 6'66\cdot10^{-2}\,m^{-1}$$

$$V_C^2 = \left(1\cdot10^{-4}\frac{m}{s}\right)^2 + 2'67\cdot10^{-9}\frac{m^2}{s^2}\ ;\quad V_C^2 = 12'67\cdot10^{-9}\frac{m^2}{s^2}$$

$$V_C = 1'13\cdot10^{-4}\,m/s$$

28/35

19.- $\begin{cases} t = 5\,10^2 s \\ \text{sol-terra.} \end{cases}$

$t_{sol-xupiter} = 2'6\,10^3 s$

$T_T = 3'1\,10^7 s$

$c = 3 . 10^8 m/s$

$G = 6'67 . 10^{-11} \frac{N . m^2}{Kg^2}$

a) T_x ?

b) $(v_0)_x$?

c) M_s ?

$d = v . t \; (e = v . t)$

a) $d_{s-T} = 3 . 10^8 \frac{m}{s} . 5\,10^2 s = 1'5 . 10^{11} m.$

$d_{s-x} = 3 . 10^8 \frac{m}{s} . 2'6 . 10^3 s = 7'8 . 10^{11} m.$

$\frac{T^2}{R^3} = cte \; ; \; \frac{T_T^2}{d_T^3} = \frac{T_x^2}{d_x^3}$

$T_x^2 = \frac{d_x^3}{d_T^3} . T_T^2 \; ; \; T_x = T_T \left(\frac{d_x}{d_T}\right)^{1'5}$

$T_x = 3'1\,10^7 s \left(\frac{7'8 . 10^{11}}{1'5 . 10^{11}}\right)^{1'5} = 3'68 . 10^8 s$

b)

$v = \omega . r$
$\quad\quad\quad\downarrow$
$\quad\quad\quad d$

$v = \frac{2\pi}{T} . d . = \frac{2\pi}{3'68 . 10^8 s} . 7'8 . 10^{11} m . = 1'33\,10^4 \frac{m}{s}$

c)

$F_G = F_i$ suponiendo orbitas circulares

$$G \frac{M_S \cancel{m_x}}{d_x^2} = \cancel{m_x} \frac{v_x^2}{\cancel{d_x}}$$

$$M_S = \frac{v_x^2 \cdot d_x}{G} = \frac{(1'33 \cdot 10^4 \, m/s)^2 \cdot 7'8 \cdot 10^{11} \, m}{6'67 \cdot 10^{-11} \, \frac{N \cdot m^2}{Kg^2}}$$

$M_S = 2'07 \cdot 10^{30}$ Kg