PROBLEMAS

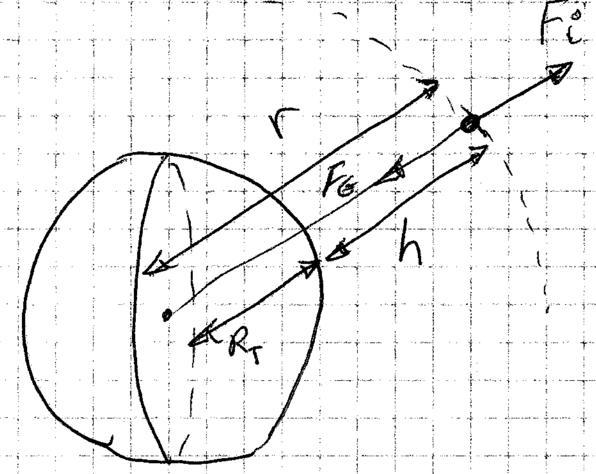
nº 1 (Páx 91) $T = 24h \cdot \frac{3600s}{1h} = 86.400s$

Datos: $R_T = 6'38 \cdot 10^6 m$ ; $M_s = 8 \cdot 10^2 Kg$

$M_T = 5'98 \cdot 10^{24} Kg$

$G = 6'67 \cdot 10^{-11} N \cdot m^2 \cdot Kg^{-2}$

a)




$F_G = F_i$

$$G \frac{M_T m}{r^2} = m \frac{v^2}{r}$$

$$\Downarrow$$

$$G \frac{M_T}{r^2} = \frac{v^2}{r}$$

$$G \frac{M_T}{r^2} = \frac{\omega^2 r^2}{r} \Rightarrow G \frac{M_T}{r^2} = \frac{4\pi^2}{T^2} r$$

$$r^3 = \frac{G M_T T^2}{4\pi^2} ; \quad r = \left( \frac{6'67 \cdot 10^{-11} N \cdot m^2 \cdot Kg^{-2} \cdot 5'98 \cdot 10^{24} Kg \cdot (86400 s)^2}{4\pi^2} \right)^{1/3}$$

$r = 4'23 \cdot 10^7 m$ ; $h = r - R_T = 3'59 \cdot 10^7 m$

b) $$F = G \frac{M \cdot m}{r^2} = m \frac{v^2}{r}$$

$$F = G \frac{M \cdot m}{r^2} = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{5'98 \cdot 10^{24} Kg \cdot 800 Kg}{(4'23 \cdot 10^7 m)^2} =$$

$= 178'3 N$

c) $E_m = E_c + E_p$

Non foi pulla calcular $E_c$ e $E_p$, sempre que se trate dun obxecto que xire nunha órbita podemos facer o seguinte

$$F_G = F_i \quad ; \quad G\frac{M\cdot m}{r^{\cancel{2}}} = m\frac{v^2}{\cancel{r}}$$

$$G\frac{Mm}{r} = m\cdot v^2$$

$$E_c = \frac{1}{2}mv^2 = \frac{1}{2}\,\frac{GM\cdot m}{r}$$

$$E_p = -G\frac{M\cdot m}{r}$$

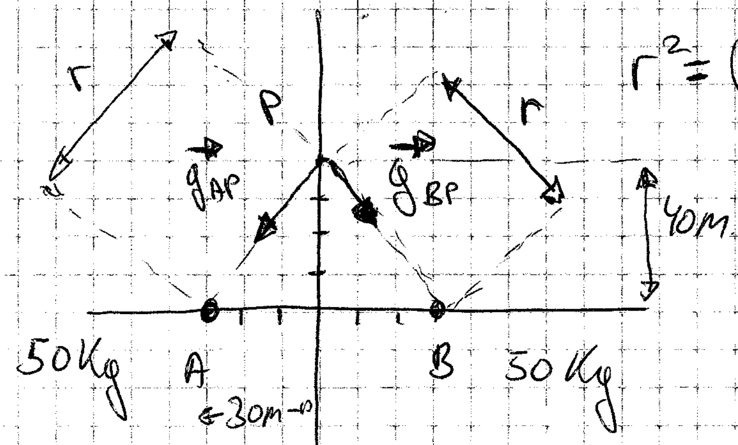
$$E_M = \frac{1}{2}\,\frac{GM\cdot m}{r} - \frac{GMm}{r} = -\frac{1}{2}\,\frac{GMm}{r}$$

$$E_m = -\frac{1}{2}\,6'67\cdot 10^{-11}\frac{N\cdot m^2}{kg^2}\cdot\frac{5'98\,10^{24}\,kg\cdot 800\,kg}{4'23\,10^{7}\,m}$$

$$E_m = -3'77\cdot 10^{9}\ J$$

PROBLEMA Nº 2 (Páx 91)

2.-

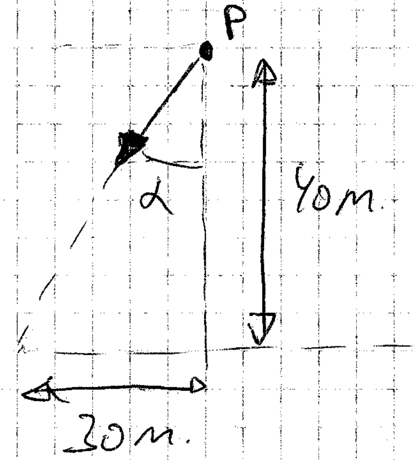



$r^2 = (30m)^2 + (40m)^2$

$r^2 = 2500 m^2$

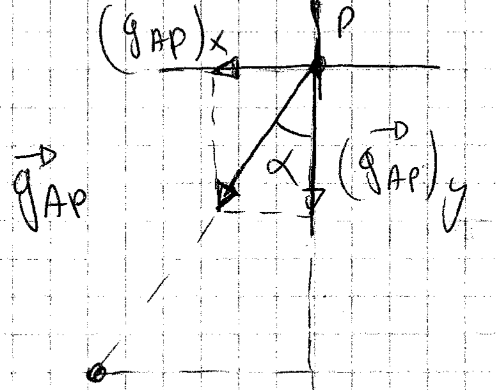
$r = 50m.$

$$|\vec{g}_{AP}| = |\vec{g}_{BP}| = G\frac{M}{r^2} = 6'67\,10^{-11}\frac{N.m^2}{Kg^2}\frac{50Kg}{2500m^2} = 1'33\,10^{-12}\frac{N}{Kg}$$




$$\operatorname{sen}\alpha = \frac{30m}{50m} = \frac{3}{5} = 0'6$$

$$\cos\alpha = \frac{40m}{50m} = \frac{4}{5} = 0'8$$

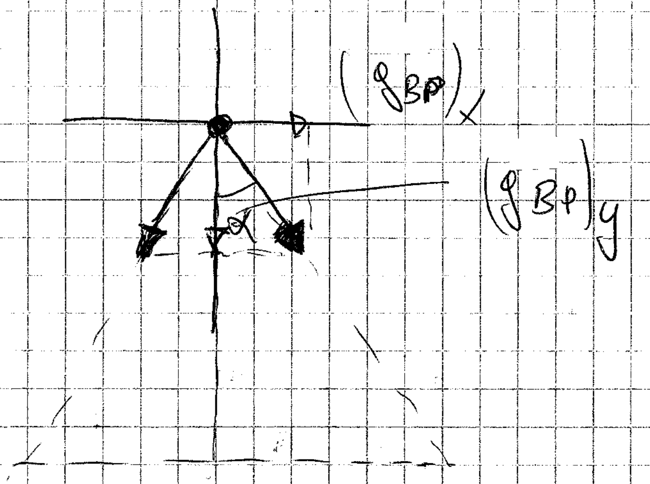



$$\operatorname{sen}\alpha = \frac{(g_{AP})_x}{|\vec{g}_{AP}|}$$

$$\cos\alpha = \frac{(g_{AP})_y}{|\vec{g}_{AP}|}$$

$$\vec{g}_{AP} = -1'33\,10^{-12} \cdot 0'6\,\vec{i} - 1'33\,10^{-12} \cdot 0'8\,\vec{j}$$

$$\vec{g}_{AP} = -7'98 \cdot 10^{-13}\,\vec{i} - 1'06\,10^{-12}\,\vec{j}\ (N/Kg)$$




$$\vec{g}_{BP} = |\vec{g}_{BP}| \cdot \operatorname{sen}\alpha \, \vec{i} - |\vec{g}_{BP}| \cdot \cos\alpha \, \vec{j}$$

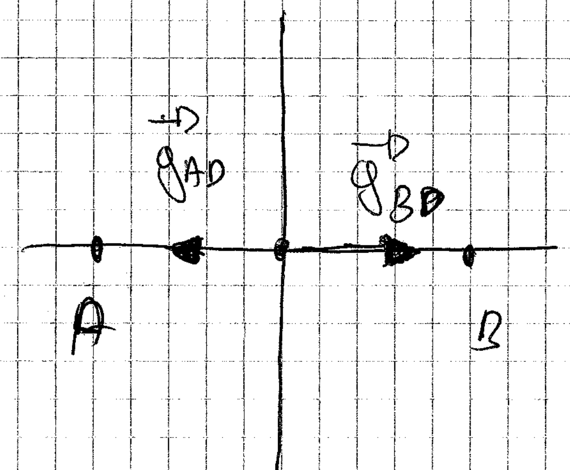
$$\vec{g}_{BP} = 1'33 \cdot 10^{-12} \cdot 0'6 \, \vec{i} - 1'33 \cdot 10^{-12} \cdot 0'8 \, \vec{j} \; (N/Kg)$$

$$\vec{g}_{BP} = 7'98 \cdot 10^{-13} \vec{i} - 1'06 \cdot 10^{-12} \vec{j} \; (N/Kg)$$

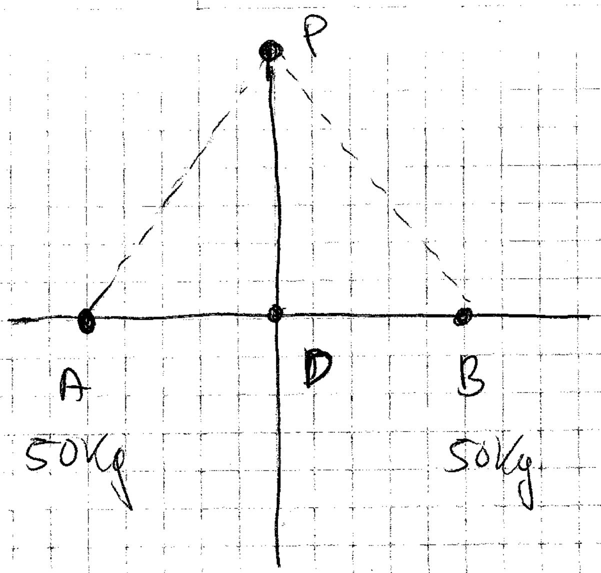
$$\vec{g}_P = \vec{g}_{AP} + \vec{g}_{AP} = 2\left(-1'06 \cdot 10^{-12} \vec{j}\right) (N/Kg)$$

$$\vec{g}_P = -2'12 \cdot 10^{-12} \; (N/Kg)$$

No punto $O(0,0)$



c)




$$V = -G\frac{m}{r}$$

$V_p = V_{AP} + V_{BP}$ => Distancia a mesma r, a masa tamen e igual

$$V_{PA} = V_{PB} = -6'67 10^{-11}\frac{N.m^2}{Kg^2}\frac{50Kg}{50m} = -6'67.10^{-11}\frac{N.m}{Kg}$$

$$V_p = 2\left(-6'67 10^{-11}\frac{N.m}{Kg}\right) = -1'33.10^{-10}\frac{J}{Kg}$$

$$V_D = V_{AD} + V_{BD}$$

$$V_D = 2\left(-6'67.10^{-11}\frac{N.m^2}{Kg^2}\frac{50Kg}{30m}\right) = -2'22 10^{-10} J/Kg$$

Problema nº3

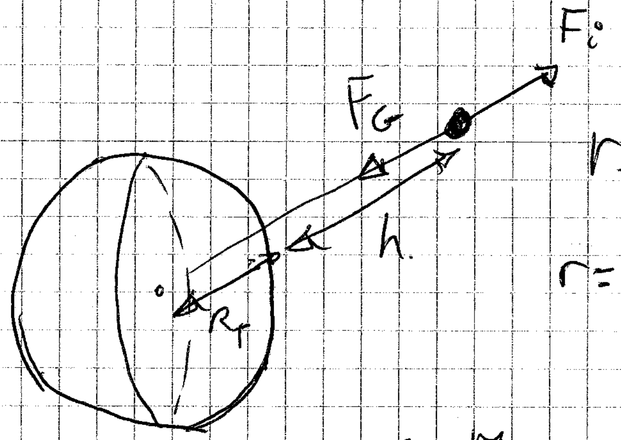
$m = 100\ Kg$

$h = 6000\ Km = 6\,10^6\ m.$

$g_0 = 9'80\ m/s^2$

$R_T = 6.400\ Km = 6'4\,10^6\ m.$

a) T? b) Peso?




$r = R_T + h.$

$r = 12'4\,10^6\ m.$

$$G\frac{M_T m}{r^2} = m\frac{v^2}{r}$$

$$G\frac{M_T}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}}\ ;\quad G\frac{M_T}{r} = w^2 r^2\ ;\quad G\frac{M_T}{r} = \frac{4\pi^2}{T^2}r^2$$

$$T^2 = \frac{4\pi^2 r^3}{G M_T}$$

$$g_0 = \frac{G M_T}{r_T^2} \Rightarrow G M_T = g_0\, r_T^2$$

$$T^2 = \frac{4\pi^2 r^3}{g_0\, r_T^2}$$

$$T = \left[\frac{4\pi^2 (12'4\,10^6 m.)^3}{9'80\frac{m}{s^2}\,(6'4\,10^6 m)^2}\right]^{1/2} = 1'37\,10^4 s$$

b) $g = \frac{GM_T}{r^2}$

poñendo esta expresión en función de $g_0$

$g_0 = \frac{GM_T}{r_T^2} \Rightarrow g_0 r_T^2 = GM_T$

$g = \frac{g_0 r_T^2}{r^2}$ ; $g = \frac{9'80 \, m/s^2 \cdot (6'4 \cdot 10^6 m)^2}{(12'4 \cdot 10^6 m)^2} = 2'61 \, m/s^2$

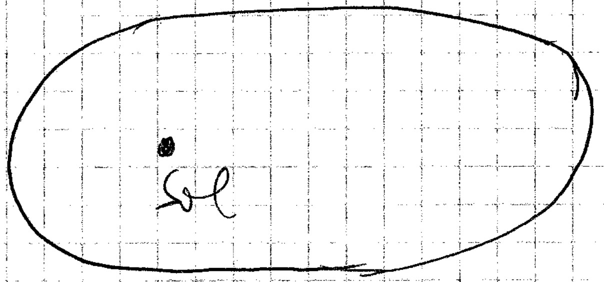
$P = mg = 100 \, Kg \cdot 2'61 \, m/s^2 = 261 \, N$

Problema nº 4

$T_T = 1$ ano $\quad R_T = 1'5 \cdot 10^{11} m$

$T_X = 12$ anos $\quad R_X = ?$

$T_N = ?$ $\quad R_N = 4'5 \cdot 10^{12} m.$



$\frac{T^2}{R^3} = cte$ $\quad$ 3ª lei de Kepler

$\frac{T_T^2}{R_T^3} = \frac{T_X^2}{R_X^3} \rightarrow R_X^3 = \frac{T_X^2}{T_T^2} \cdot R_T^3$

$$R_X^3 = \frac{T_X^2}{T_T^2} \cdot R_T^3$$

$$R_X = R_T \left(\frac{T_X}{T_T}\right)^{2/3}$$

$$R_X = 1'5\,10^{11}\,m \left(\frac{12\ \text{anos}}{1\ \text{ano}}\right)^{2/3} = 7'86 \cdot 10^{11}$$

b) $\frac{T_T^2}{R_T^3} = \frac{T_N^2}{R_N^3} \Rightarrow T_N^2 = \frac{R_N^3}{R_T^3} \cdot T_T^2$

$$T_N = T_T \left(\frac{R_N}{R_T}\right)^{3/2}$$

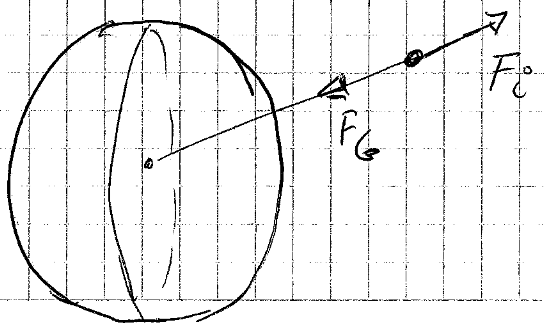
$$T_N = 1\ \text{año} \left(\frac{4'5\,10^{12}\,m}{1'5\,10^{11}\,m}\right)^{3/2}$$

$$T_N = 164'3\ \text{anos}$$

Problema nº 5.

$m = 64'5\ Kg$ | $R_T = 6370\ Km = 6'370\,10^6\,m.$

$R = 2'32\ R_T$ | $g_0 = 9'80\ m/s^2$



a)

$$G\frac{M_T \cancel{m}}{r^2} = \cancel{m}\frac{v^2}{r} \;;\quad G\frac{M_T}{r^2} = \frac{4\pi^2 r^2}{T^2 \cdot r}$$

$$T^2 = \frac{4\pi^2 r^3}{G M_T}$$

poñer $GM_T$ en función de $g_0$

$$g_0 = G\frac{M_T}{R_T^2}$$

$$T^2 = \frac{4\pi r^3}{g_0 R_T^2}$$

$$T^2 = \frac{4\pi^2 (2'32 R_T)^3}{g_0 R_T^2} \;;\quad T^2 = \frac{4\pi^2 (2'32)^3 R_T^{\cancel{3}}}{g_0 \cancel{R_T^2}}$$

$$T = \left(\frac{4\pi^2 (2'32)^3 \; 6'370 \cdot 10^6}{9'80 \, m/s^2}\right)^{1/2} = 17.900\,s$$

b) Peso $= m \cdot g$

$$g = G\frac{M_T}{r^2} \;;\quad g = \frac{g_0 R_T^2}{r^2}$$

$$g = g_0 \frac{\cancel{R_T^2}}{(2'32)^2 \cancel{R_T^2}} \Rightarrow g = 1'82\, m/s^2$$

$$\text{Peso} = m \cdot g = 64'5\,kg \cdot 1'82\,\frac{m}{s^2} = 117'4\,N$$

PROBLEMA Nº 6

$M_L = 0'011 M_T$

$R_L = R_T/4$

$(Peso)_T = 980N \quad (g_0 = 9'80\ m.s^{-2})$

a) a masa non depende do campo gravitatorio, é a mesma en todolos puntos do Universo

$$M_T = M_L = \frac{980N}{9'80 m.s^{-2}} = 100\ Kg$$

$$g_L = G\frac{M_L}{R_L^2} \quad ; \quad g_0 = G\frac{M_T}{R_T^2} \Rightarrow G = \frac{g_0 R_T^2}{M_T}$$

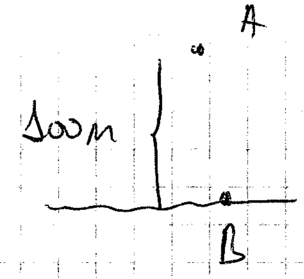
$$g_L = \frac{g_0 R_T^2}{M_T}\cdot\frac{M_L}{R_L^2} \quad ; \quad g_L = \frac{g_0 R_T^2\ 0'011 M_T}{M_T (R_T/4)^2}$$

$$g_L = \frac{g_0 \cancel{R_T^2}\ 0'011 \cancel{M_T}\ 16}{\cancel{M_T}\ \cancel{R_T^2}} \Rightarrow g_L = 16 \cdot 0'011\ g_0$$

$g_L = 1'72\ m/s^2$

$P_L = 100\ Kg \cdot 1'72\ m/s^2 = 172N$

b) Dado que a distancia é moi pequena sobre a superficie da Lua podemos utilizar a seguinte expresión para a enerxía potencial.



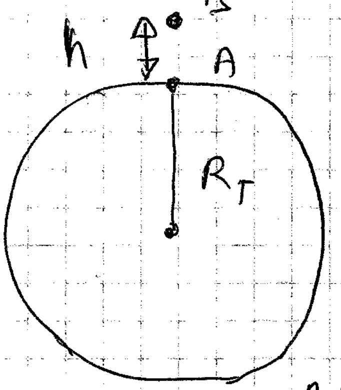

Enerxía A = Enerxía B.

$$(E_p)_A = (E_c)_B$$

$$m g_L h_A = \frac{1}{2} m v_B^{} \Rightarrow v_B = \sqrt{2 g_L h_A}$$

$$v_B = (2 \cdot 1'72\ m/s^2 \cdot 100 m)^{1/2} = 18'55\ m/s$$

Por que é posible facer esta aproximación




$$R_A = R_T$$
$$R_B = R_T + h.$$

$$(E_p)_B = -G \frac{M_T m}{(R_T + h)} \quad ; \quad (E_p)_A = -G \frac{M_T m}{R_T}$$

$$\Delta E_p = (E_p)_B - (E_p)_A$$

$$\Delta E_p = -G \frac{M_T m}{(R_T + h)} - \left(-G \frac{M_T m}{r_T}\right) = G M_T m \left(\frac{1}{R_T} - \frac{1}{R_T + h}\right)$$

$$= G M_T m \left(\frac{\cancel{R_T} + h - \cancel{R_T}}{R_T (R_T + h)}\right) = G \frac{M_T m h}{R_T (R_T + h)}$$

$$R_T (R_T + h) \simeq R_T^2 \quad \text{si } h << R_T$$

$$\Delta E_p = (E_p)_B - (E_p)_A = \frac{G M_T m \cdot h}{R_T^2}$$

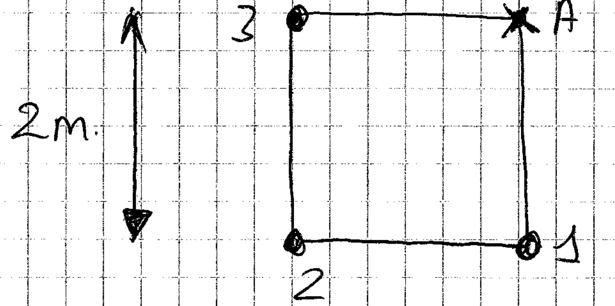
$$g = G \frac{M_T}{R_T^2} \quad ; \quad \Delta E_p = m g h$$

$$(R_T + h) R_T \simeq R_T^2$$

exemplo

$$(6'370 \, 10^6 m + 100 m) \cdot 6'370 \, 10^6 m = (6'370 \, 10^6 m)^2$$

7.- $m = 10 \, Kg$



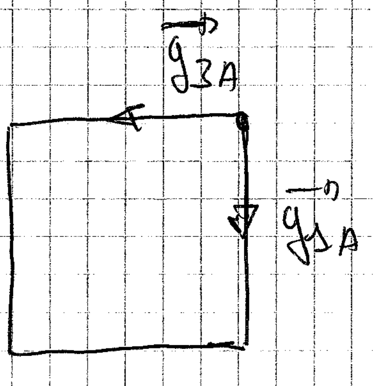

A vértice onde colocado $\vec{g}$ e V

$$\vec{g} = -G \frac{M}{r^2} \vec{u}_r$$

A distancia de 1, e 3 a A e a mesma.

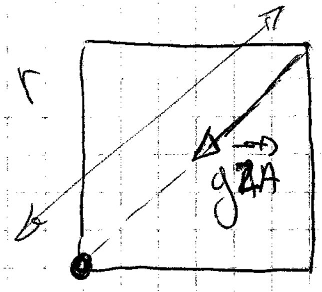
$$|\vec{g}_{3A}| = |\vec{g}_{1A}| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{10 Kg}{(2m)^2}$$

$$= 1'67 \cdot 10^{-10} \frac{N}{Kg}$$




$$\vec{g}_{1A} = -1'67 \cdot 10^{-10} \, \vec{j} \; N/Kg$$

$$\vec{g}_{3A} = -1'67 \cdot 10^{-10} \, \vec{i} \; N/Kg$$

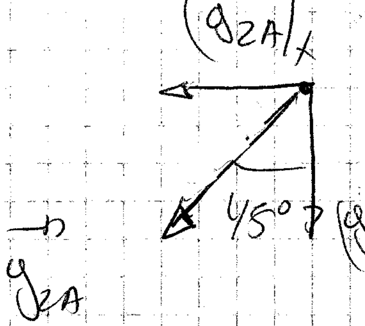



$m = 10 Kg$

$$r^2 = (2m)^2 + (2m)^2$$

$$r^2 = 8m^2 \Rightarrow r = 2'83m$$

$$|\vec{g}_{2A}| = 6'67\cdot 10^{-11}\,\frac{N\cdot m^2}{Kg^2}\;\frac{10Kg}{8m^2} = 8'34\cdot 10^{-11}\,\frac{N}{Kg}$$




$$(g_{2A})_x = (g_{2A})_y = |\vec{g}_{2A}| = 5'90\cdot 10^{-11}\, N/Kg$$

$$\text{sen } 45^\circ = \cos 45^\circ = 0'71$$

$$\vec{g}_{2A} = -5'90\cdot 10^{-11}\,\vec{i} - 5'90\cdot 10^{-11}\,\vec{j}\;(N/Kg)$$

$$\vec{g}_2 = \vec{g}_{1A} + \vec{g}_{2A} + \vec{g}_{3A}$$

$$\vec{g}_A = -2'26\cdot 10^{-10}\,\vec{i} - 2'26\cdot 10^{-10}\,\vec{j}\;(N/Kg)$$

potencial.

$$V_{3A} = V_{1A} = -G\,\frac{m}{r} = -6'67\cdot 10^{-11}\,\frac{N\cdot m^2}{Kg^2}\cdot\frac{10\,Kg}{2m} = -3'34\cdot 10^{-10}\, J/Kg$$

$$V_{2A} = -G\,\frac{m}{r'} = -6'67\cdot 10^{-11}\,\frac{N\cdot m^2}{Kg}\;\frac{10Kg}{2'83m} = -2'36\cdot 10^{-10}\, J/Kg$$

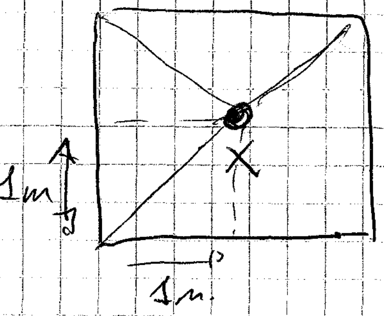
$$V_A = 2\times(-3'34\cdot 10^{-10}\, J/Kg) + (-2'36\cdot 10^{-10}\, J/Kg)$$

$$V_A = -9'04\cdot 10^{-10}\, J/Kg$$

b) $W_{\infty} \rightarrow$ CENTRO DO CUADRADO

$W_{\infty}^{\cancel{x}} = -(E_{p\cancel{x}} - E_{p\infty})$ $\qquad E_{p\infty} = 0$

$E_{px} = m' V_x$



$$V_x = 3\left(-G\frac{m}{\sqrt{2}\,m}\right)$$

$$V_x = -3 \cdot 6'67 \cdot 10^{-11}\frac{N \cdot m^2}{Kg^2} \cdot \frac{10\,Kg}{\sqrt{2}\,m} = -1'41\,10^{-9}\,J/Kg$$

$$W_{\infty}^{x} = -1\,Kg\left(-1'41\,10^{-9}\,J/Kg\right) = 1'41\,10^{-9}\,J$$

$$G\frac{M_T}{r^2}=\frac{v^2}{r}\ ;\ G\frac{M_T}{r}=v^2$$

$$g_0=\frac{G M_T}{r_T^2} \Rightarrow G M_T = g_0 r_T^2$$

$$v^2=\frac{g_0 r_T^2}{r}\ ;\ v=\left(\frac{9'80\,m/s^2\cdot(6'378\,10^6 m)^2}{3'64\,10^7 m}\right)^{1/2}$$

$$v=3309\ m/s$$

b)

$$E_m=-\frac{1}{2}G\frac{M_T m}{r}$$

$$E_m=-\frac{1}{2}\frac{g_0 r_T^2 m}{r}\ ,\ E_m=-\frac{1}{2}\frac{9'80\,m/s^2\cdot(6'378\,10^6 m)^2\cdot 300\,kg}{3'64\,10^7 m}$$

$$E_m=-1'64\,10^9\ J$$

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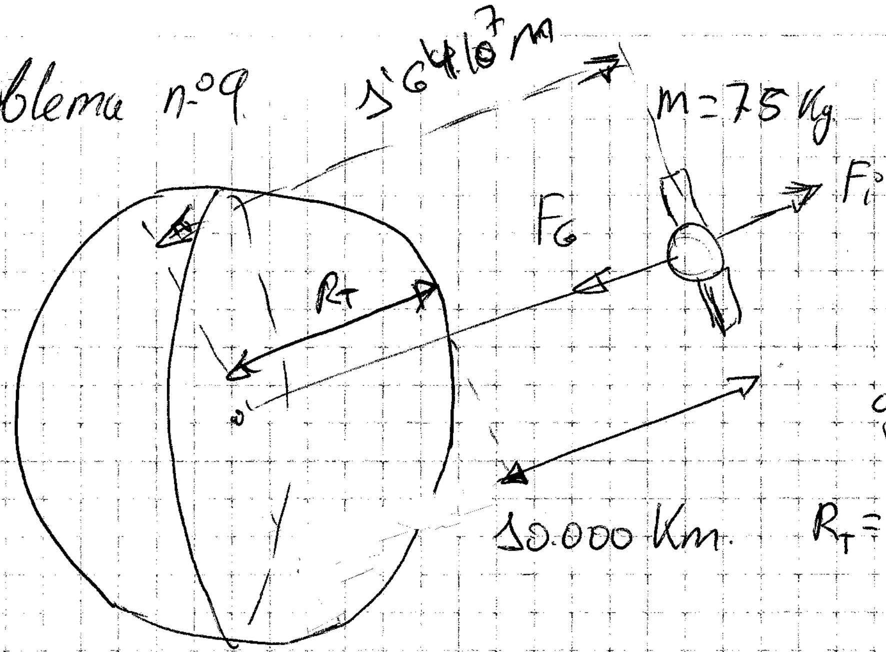
$$E_m=\frac{1}{2}mv^2-G\frac{M.m}{r}$$

$$G\frac{Mm}{r^2}=m\frac{v^2}{r}\Rightarrow m v^2=G\frac{Mm}{r}$$

$$E_m=\frac{1}{2}G\frac{Mm}{r}-G\frac{Mm}{r}=-\frac{1}{2}G\frac{Mm}{r}$$

Problema nº9




$g_T = 9'80\ m/s^2$

$R_T = 6.400\ Km = 6'4 \cdot 10^6\ m$

a) $v$? ; $T$? ; b) Peso

$$F_G = F_i$$

$$G\frac{M_T m}{r^2} = m\frac{v^2}{r} \; ; \; G\frac{M_T}{r^2} = \frac{v^2}{r}$$

$$v^2 = \frac{G M_T}{r}$$

$$g_0 = G\frac{M_T}{r_T^2} \Rightarrow g_0 r_T^2 = G M_T$$

$$v^2 = \frac{g_0 r_T^2}{r} \; ; \; v = \left(\frac{9'80\,m/s^2 \cdot (6'4 \cdot 10^6 m)^2}{1'64 \cdot 10^7}\right)^{1/2}$$

$$v = 4947\ m/s$$

$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T} \cdot r \Rightarrow T = \frac{2\pi \cdot r}{v}$$

$$T = \frac{2 \cdot \pi \cdot 1'64 \cdot 10^7 m}{4947\ m/s} = 20.829\ m$$

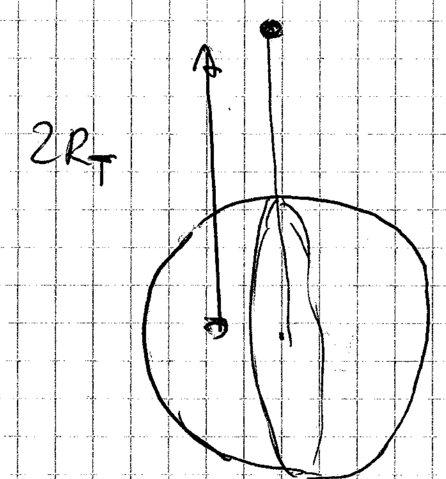
b)

$$Peso = m \cdot g$$

$$Peso = m \, G \frac{M_T}{r^2} \; ; \; Peso = m \cdot \frac{g_0 R_T^2}{r_T^2}$$

$$Peso = 75 Kg \cdot 9'80 m/s^2 \left( \frac{6'4 \cdot 10^6 m}{1'64 \cdot 10^7 m} \right)^2 = 112 N$$

Problema nº 10




$$F_G = F_i$$

$$G \frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{r}}$$

$$v^2 = G \frac{M_T}{r}$$

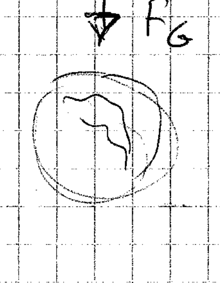
$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_T = 6400 Km.$

$g_0 = 9'80 m/s^2$

$$v^2 = \frac{g_0 R_T^2}{r} \; ; \; v^2 = \frac{9'80 m/s^2 \cdot (6'4 \cdot 10^6 m)^{\cancel{2}}}{2 \cdot \cancel{(6'4 \cdot 10^6 m)}}$$

$$v = 5.600 m/s$$



b) $Peso = m g$

$g = G \frac{M_T}{(2R_T)^2}$ ; $m = \frac{P_T}{g_T}$

$g_T = G \frac{M_T}{R_T^2}$

$Peso = \frac{P_T}{\cancel{G \frac{M_T}{R_T^2}}} \cdot \cancel{G} \frac{\cancel{M_T}}{4\cancel{R_T^2}} = \frac{5000\,N}{4} = 1250\,N$

Problema nº 11

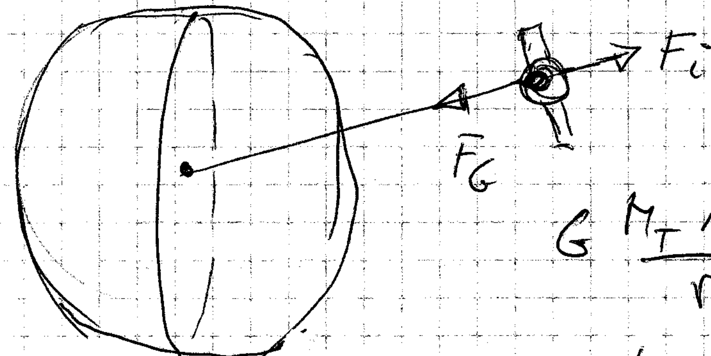
$M = 200\,Kg.$

$v = 10.800 \frac{Km}{h} \frac{1000\,m}{1\,Km} \frac{1\,h}{3600\,s} =$

$= 3.000\,m/s$

$g_0 = 9'80\,m/s^2$

$R_T = 6370\,Km.$




$G \frac{M_T m}{r^{\cancel{2}}} = m \frac{v^2}{\cancel{r}}$

$g_0 = G \frac{M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$

$r = \frac{G M_T}{v^2}$ ; $r = \frac{g_0 R_T^2}{v^2}$ ;

$r = \frac{9'80\,m/s^2 \cdot (6'37 \cdot 10^6\,m)^2}{(3000\,m/s)^2} = 4'42 \cdot 10^7\,m.$

$r = 4'42 \cdot 10^7 m$

$r = R_T + h \Rightarrow h = r - R_T = 4'42 \cdot 10^7 m - 6'370 \cdot 10^6 m$

$h = 3'78 \cdot 10^6 m$

b)

$E_m = -\frac{1}{2} G \frac{M_T m}{r}$ ; $E_m = E_c + E_p$

$E_m = \frac{1}{2} m v^2 - G \frac{M_T m}{r}$

$F_G = F_c$ ; $G \frac{M_T m}{r^2} = m \frac{v^2}{r}$

$G \frac{M_T}{r} = m v^2$ ;

$E_m = \frac{1}{2} G \frac{M_T}{r} - G \frac{M_T m}{r} = -\frac{1}{2} G \frac{Mm}{r}$

$E_m = E_{TOTAL} = -\frac{1}{2} G \frac{Mm}{r} = -\frac{1}{2} \frac{g_0 R_T^2 \cdot m}{r}$

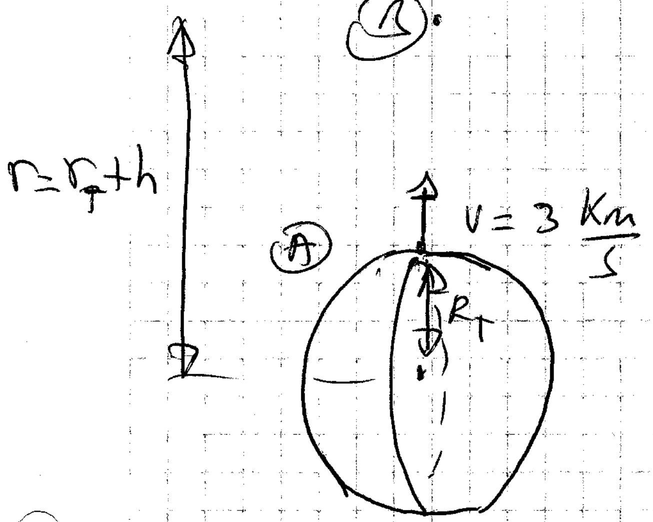
$E_m = -\frac{1}{2} \frac{9'80 m/s^2 \cdot (6'370 \cdot 10^6 m)^2 \cdot 200 Kg}{4'42 \cdot 10^7 m}$

$E_m = -9'0 \cdot 10^8 J$

UNIDADES $\frac{\frac{m}{s^2} \cdot m^2 \; Kg}{\cancel{m}} = Kg \frac{m}{s^2} \cdot m$

$N$

PROBLEMA Nº 12




$G = 6'67 \cdot 10^{-11}\ \frac{N \cdot m^2}{Kg^2}$

$R_T = 6'378 \cdot 10^6 m$

$M_T = 5'98 \cdot 10^{24}\ Kg$

a) $h$?   b) $v$?

Enerxía (A) = Enerxía (B)

$$(E_p)_A + (E_c)_A = (E_p)_B$$

$$-G\frac{M_T \cancel{m}}{R_T} + \frac{1}{2}\cancel{m}v^2 = -G\frac{M_T \cancel{m}}{r}$$

$$r = \frac{-G\ M_T}{-\frac{G M_T}{R_T} + \frac{1}{2}v^2}$$

$$r = \frac{-6'67 \cdot 10^{-11} N \cdot m^2/Kg^2 \cdot 5'98\ 10^{24} Kg}{\frac{-6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98 \cdot 10^{24} Kg}{6'378\ 10^6 m} + \frac{1}{2}\left(3\ 10^3\ m/s\right)^2}$$

$r = 6'87 \cdot 10^6\ m$ // $h = 4'94\ 10^5\ m$

para facelo na calculadora, probar a utilizar as memorias

$$r = \frac{A}{(B+C)}$$

$$A = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98 \, 10^{24} \, Kg$$

$$B = \frac{-6'67 \, 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98 \cdot 10^{24} \, Kg}{6'378 \cdot 10^6 \, m}$$

$$C = \frac{1}{3} \left( 3 \, 10^3 \, m/s \right)^2$$

b)

$$F_G = F_c$$

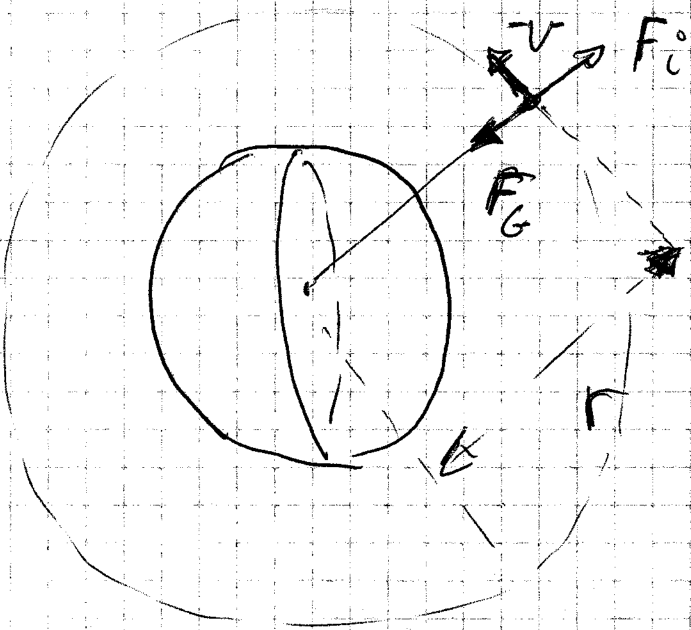
$$G \frac{M_T m}{r^2} = m \frac{v^2}{r}$$

$$v = \left( \frac{2 \, G \, M_T}{r} \right)^{1/2}$$

$$v = \left( \frac{2 \cdot 6'67 \cdot 10^{-11} \, N \cdot m^2/Kg^2 \cdot 5'98 \cdot 10^{24} \, Kg}{6'87 \, 10^6 \, m} \right)^{1/2}$$

$$v = 10.775 \, m/s$$

Problema n°13.-




$m = 25\,Kg$

$T = 24h = 86400s$

$G = 6'67\,10^{-11}\,\frac{N.m^2}{Kg^2}$

$M_T = 5'98.\,10^{24}\,Kg$

a) r?  b) $E_c$, $E_p$ ; $E_m$

a)

$$G\frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}} \; ; \; \frac{G M_T}{r} = v^2$$

$$G\frac{M_T}{r} = w^2 r^2 \Rightarrow G\frac{M_T}{r} = \frac{4\pi^2}{T^2} r^2$$

$$r^3 = \frac{G\,M_T\,T^2}{4\pi^2} \; ; \; r = \left(\frac{G\,M_T T^2}{4\pi^2}\right)^{1/3}$$

$$r = \left(\frac{6'67.10^{-11} N.m^2/Kg^2 .\; 5'98.10^{24} Kg \cdot (86400s)^2}{4\pi^2}\right)^{1/3}$$

$r = 4'225\,10^{7}\,m.$

b) $E_c = \frac{1}{2} m v^2$

para non calcular a velocidade

$$E_c = \frac{1}{2} m v^2 = \frac{1}{2} G \frac{M_T m}{r}$$

$$E_m = -\frac{1}{2} G \frac{M_T m}{r}$$

$$E_p = -\frac{G M_T m}{r}$$

$$E_p = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{5'98 \cdot 10^{24} Kg \cdot 25 Kg}{4'225 \cdot 10^6 m} =$$

$$E_p = -2'36 \cdot 10^8 J$$

$$E_m = \frac{1}{2} E_p =$$

$$E_c = -\left(\frac{1}{2} E_p\right) =$$

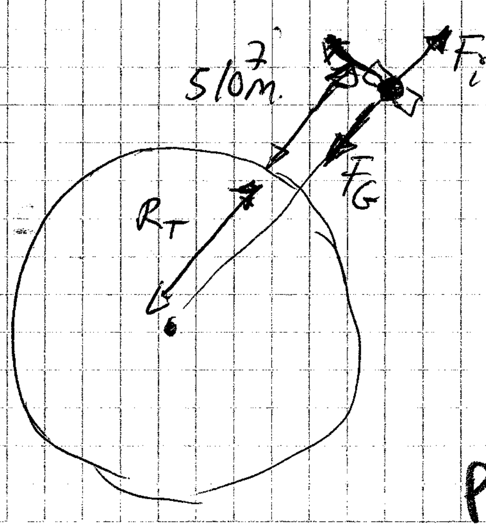
Problema nº14-

$m = 200 Kg$ | $g_0 = 9'81 m/s^2$

$r = 5 \cdot 10^7 m$ | $R_T = 6370 Km$

$F_G = ?$

$T ?$



$F_G = F_i$

$G\dfrac{M_T m}{r^2} = m\dfrac{v^2}{r}$ ; $g_0 = G\dfrac{M_T}{R_T^2}$

$\dfrac{g_0 R_T^2}{r^2} = \dfrac{v^2}{r} \Rightarrow$

$F_G = \dfrac{g_0 R_T^2 m}{r^2}$

$F_G = 9'81\,m/s^2 \cdot 200\,kg \left(\dfrac{6'37\cdot10^6 m}{(6'37\cdot10^6 + 5\cdot10^7)m}\right)^2$

$F_G = 9'81\,\dfrac{m}{s^2} \cdot 200\,kg \left(\dfrac{6'37\cdot10^6 m}{5'64\cdot10^7 m}\right)^2 = 25'1\,N$

b/

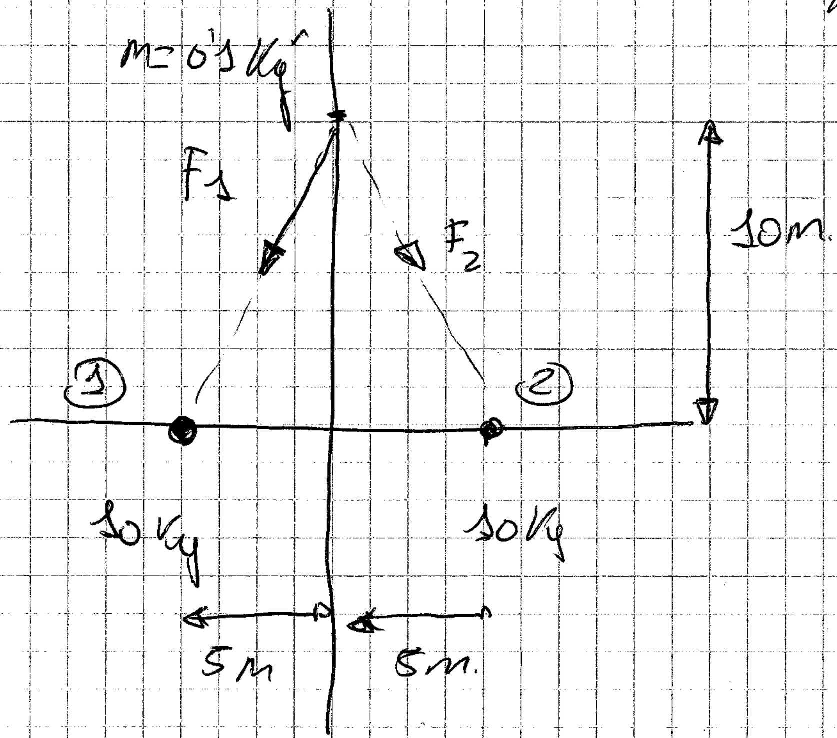
$\dfrac{g_0 R_T^2}{r^{\cancel{2}}} = \dfrac{v^2}{\cancel{r}}$ ; $\dfrac{g_0 R_T^2}{r} = \omega^2 \cdot r^2$

$\dfrac{g_0 R_T^2}{r^3} = \dfrac{4\pi^2}{T^2} \Rightarrow T^2 = \dfrac{4\pi^2}{\dfrac{g_0 R_T^2}{r^3}}$

$T^2 = \dfrac{4\pi^2 \cdot r^3}{g_0 R_T^2}$ ; $T = \left(\dfrac{4\pi^2 \cdot (5'64\cdot10^7 m)^3}{9'81\,\frac{m}{s^2} \cdot (6'37\cdot10^6 m)^2}\right)^{1/2}$

$T = 1'33\cdot10^5\,s$

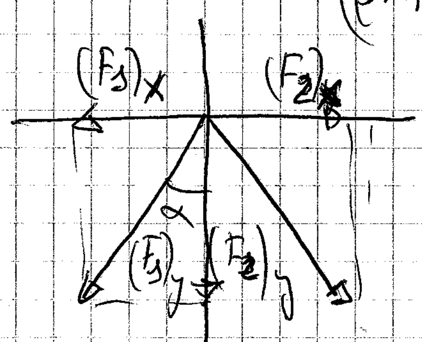
Problema nº 15 $\qquad G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$




$a$? $(0,10)$ // $a$? $(0,0)$

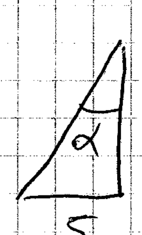
$$F = m\,a \Rightarrow a = \frac{F}{m}$$

$$|\vec{F_1}| = |\vec{F_2}| = G\,\frac{10\,Kg \cdot 0'1\,Kg}{(5m)^2 + (10m)^2} = 5'34 \cdot 10^{-13}\,N$$




$$(F_1)_x = (F_2)_x$$

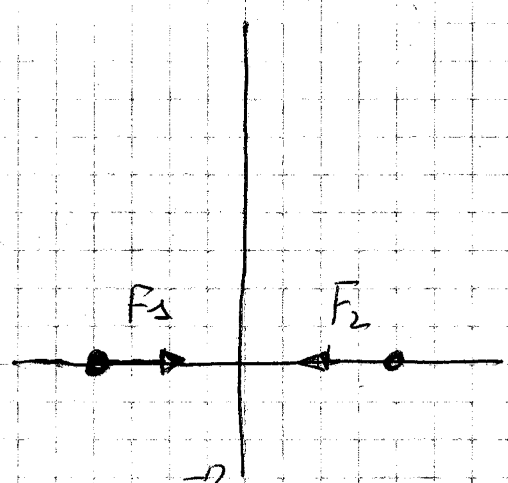
$$(F_1)_y = |F_1| \cos\alpha$$




$$\cos\alpha = \frac{10\,m}{\sqrt{125\,m^2}} = 0'89$$

$$\vec{F}_{(0,10)} = -2 \times 0'89 \cdot 5'34 \cdot 10^{-13}\,\vec{j}\ (N)$$

$$\vec{F}_{(0,10)} = -9'5 \cdot 10^{-13}\,\vec{j}\ N$$

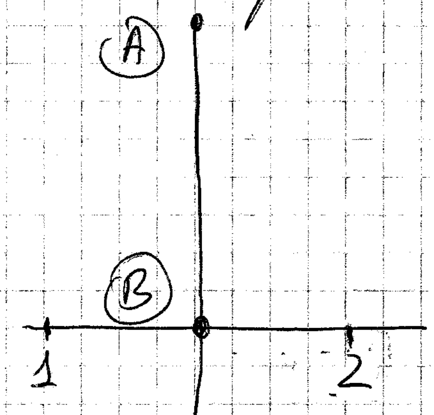



$\vec{F_1}(0,0) = 0$

$$\vec{a}(0,10) = \frac{\vec{F}(0,10)}{m} = \frac{-9'5\,10^{-13}\,\vec{j}\,(N)}{1\,kg} = -9'5\,10^{-13}\,\vec{j}\ m/s^2$$

$$\vec{a}(0,0) = \frac{\vec{F}(0,0)}{m} = 0$$

b) A aceleración non é constante na traxectoria dende o punto (0,10) o punto (0,0) polo que é necesario facelo mediante enerxías




Enerxía A = Enerxía B

$$(E_p)_A = (E_p)_B + (E_c)_B$$

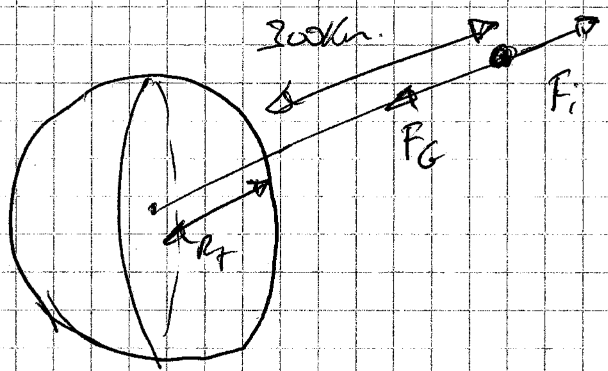
$$(E_p)_A = (E_p)_{A1} + (E_p)_{A2} = -2\,\frac{G\,M.m.}{\sqrt{125 m^2}}$$

$$(E_p)_B = (E_p)_{B1} + (E_p)_{B2} = -2\,\frac{G\,Mm.}{5m}$$

$$-2\,G\,\frac{M\cancel{m}}{\sqrt{125m^2}} = \frac{1}{2}\cancel{m}v_B^2 - 2\,G\,\frac{M.\cancel{m}}{5m}$$

$$v = \left(4.G.M.\left(\frac{1}{5m} - \frac{1}{\sqrt{125m^2}}\right)\right)^{1/2} = 1'7\,10^{-5}\ m/s$$

Problema nº16.-




$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_T = 6378\ Km$

$M_T = 5'98 \cdot 10^{24}\ Kg$

a) $v$ velocidade orbital ?

b) $T$ ?

$$F_G = F_i$$

$$G \frac{M_T \cancel{m}}{r^2} = \cancel{m} \frac{v^2}{r} \Rightarrow v = \left( \frac{G M_T}{r} \right)^{1/2}$$

$$v = \left( \frac{6'67 \cdot 10^{-11} N m^2/Kg^2 \cdot 5'98 \cdot 10^{24} Kg}{6'678 \cdot 10^6 m} \right)^{1/2} = 7728\ m/s$$

b)

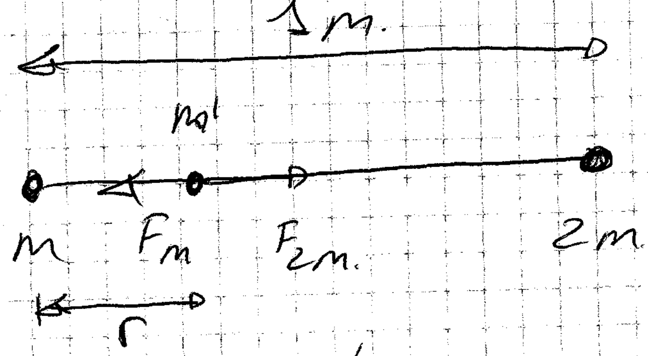
$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T} \cdot r$$

$$T = \frac{2\pi \cdot r}{v} ; \quad T = \frac{2\pi\ 6'678 \cdot 10^6 m}{7728\ m/s}$$

$$T = 5429 s$$

Problema nº 17

a)




$$|\vec{F}_m| = |\vec{F}_{2m}|$$

$$F_m = G\frac{mm'}{r^2}$$

$$F_{2m} = G\frac{2mm'}{(1-r)^2}$$

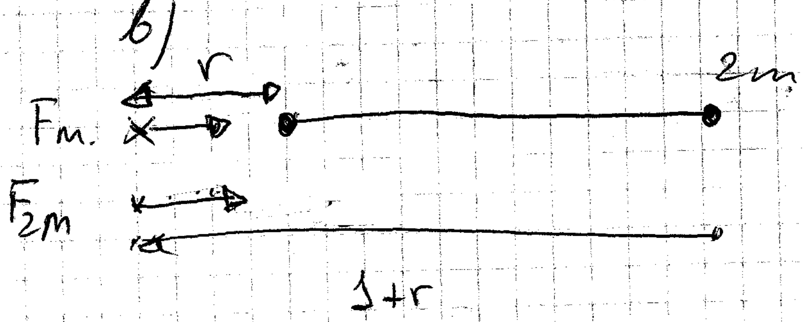
$$F_m = F_{2m} \; ; \quad G\frac{mm'}{r^2} = G\frac{2mm'}{(1-r)^2}$$

$$(1-r)^2 = 2r^2 \; ; \quad 1-2r+r^2 = 2r^2$$

$$-r^2-2r+1=0$$

$$\frac{2 \pm \sqrt{4+4}}{-2} \begin{cases} \to \underline{0'41} \\ \to - \quad \underline{\text{non}} \end{cases}$$

b)




$$F_m = F_{2m} \; ; \quad \cancel{G}\frac{m\cancel{m'}}{r^2} = \cancel{G}\frac{m2\cancel{m}}{(1+r)^2}$$

$$(1+r)^2 = r^2 \Rightarrow 1+2r+r^2 = 2r^2$$

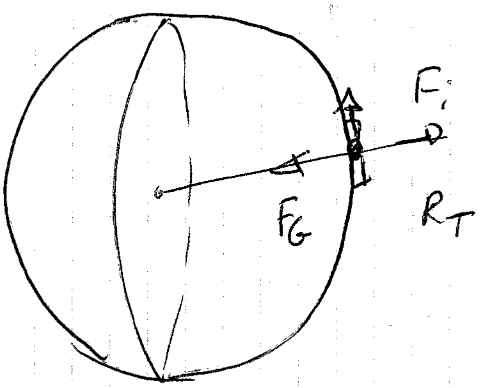
$(1+r)^2 = 2r^2 \Rightarrow 1+2r+r^2 = 2r^2$

$1+2r-r^2 = 0 \Rightarrow r = \dfrac{-2 \pm \sqrt{4+2}}{-2}$ → $2'22\,m.$ / $\ominus$ NON

Probleme nº 18

$G = 6'67 \cdot 10^{-11} \dfrac{N \cdot m^2}{kg^2}$ ; $R_T = 6'38 \cdot 10^6\,m.$

$M_T = 5'98 \cdot 10^{24}\,kg$




$$G \frac{M_T m}{r^2} = m \frac{v^2}{r}$$

a)

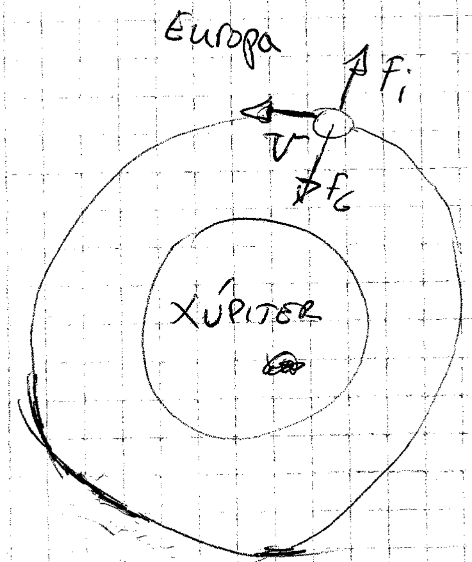
$$v = G \frac{M}{R_T} \; ; \; v = \left( \frac{6'67 \cdot 10^{-11} N \cdot m^2/kg \cdot 5'98\, 10^{24} kg}{6'38\, 10^6 m} \right)^{1/2}$$

$v = 7.907\,m/s$

b) $v = \omega \cdot r \Rightarrow v = \dfrac{2\pi}{T} \cdot r$

$$T = \frac{2\pi \cdot r}{v} = \frac{2\pi \cdot 6'38\, 10^6 m}{7.907 m/s} = 5.070\,s$$

Problema nº 19.-




$r = 6'7 \cdot 10^5$ km

$T = 3$ días, 13 horas e 13 minutos

a) $v$?

b) $M_x$?

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$3 \text{ días} \cdot \frac{24h}{1 \text{ día}} \cdot \frac{3600 s}{1h} = 2'592 \cdot 10^5 s$

$13h \cdot \frac{3600}{1h} = 4'68 \cdot 10^4 s$

$13 \text{ minutos} \cdot \frac{60 s}{1 min} = 780 s$

$T = 3'068 \cdot 10^5 s$

a) $v = \omega \cdot r = \frac{2\pi}{T} \cdot r$

$v = \frac{2\pi}{3'068 \cdot 10^5 s} \cdot 6'7 \cdot 10^8 m = 13.721 \text{ m/s}$

b) $G \frac{M_x \cancel{m}}{r^2} = \cancel{m} \frac{v^2}{r}$; $M_x = \frac{v^2 \cdot r^2}{G r}$;

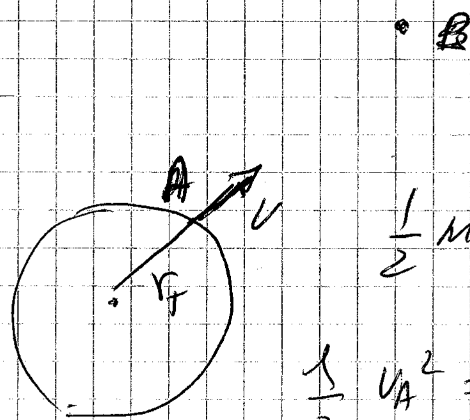
$M_x = \frac{(13.721 \text{ m/s})^2 \cdot 6'7 \cdot 10^8 m}{6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}} = 1'89 \cdot 10^{27}$ Kg

20.-

$G = 6'67 \cdot 10^{-11} N \cdot m^2/Kg^2$

$R_T = 6'38 \cdot 10^6 m$

$M_T = 5'98 \cdot 10^{24} Kg$

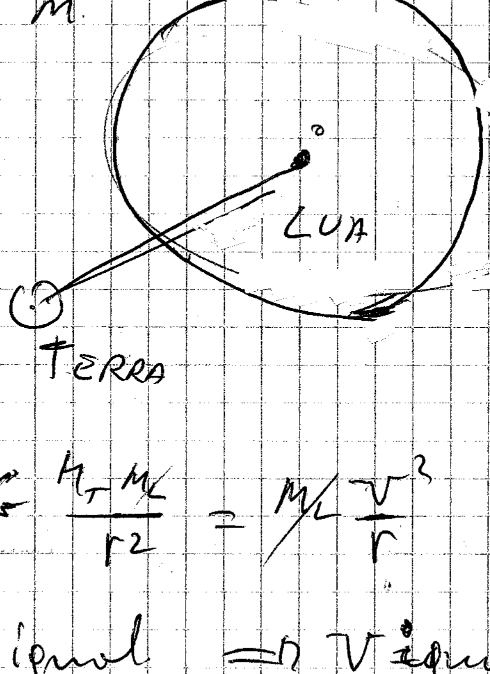



$$\frac{1}{2} m v_A^2 - G \frac{M_T m}{r} = 0$$

$$\frac{1}{2} v_A^2 = \frac{G M_T}{r}$$

$$r = \frac{2 G M_T}{v_A^2} \; ; \; r = \frac{2 \cdot 6'67 \cdot 10^{-11} N m^2/Kg^2 \cdot 5'98 \cdot 10^{24} Kg}{(3 \cdot 10^8 m/s)^2}$$

$r = 8'9 \cdot 10^{-3} m$




$$G \frac{M_T M_L}{r^2} = \cancel{M_L} \frac{v^2}{r}$$

r igual $\Rightarrow$ v igual

Pàx31

Problema nº21

$d_{T-L} = 60 R_T$

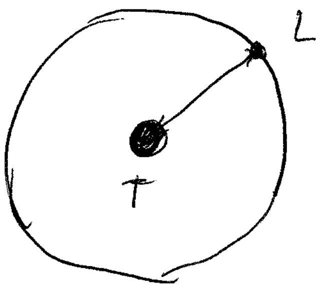
$R_T = 6400$ km.

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$

$M_T = 5'98 \cdot 10^{24}$ kg

a) A Lúa describe unha órbita circular arredor da Terra.

$$G \frac{M_T m_L}{d_{T-L}^2} = m_L \frac{v_L^2}{d_{T-L}}$$




$$v_L = \left( \frac{G M_T}{d_{T-L}} \right)^{1/2}$$

$$v_L = \left( \frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 5'98 \cdot 10^{24} kg}{60 \cdot 6'4 \cdot 10^6 m} \right)^{1/2} = 1'02 \cdot 10^3 m/s$$

b)

$$v = \omega \cdot r \Rightarrow v_L = \omega r$$

$$v_L = \frac{2\pi}{T} r \Rightarrow T = \frac{2\pi}{v_L} \cdot r$$

$$T = \frac{2\pi}{1'02 \cdot 10^3 m/s} \cdot 60 \cdot 6'4 \cdot 10^6 m = 2'37 \cdot 10^6 s$$

$$2'37 \cdot 10^6 s \ \frac{1 h}{3600 s} \ \frac{1 \text{ día}}{24 h} = 27'4 \text{ días}$$

Problema nº 22

DATOS

$r_P = \frac{r_T}{2}$

$g_P = 5 m/s^2$

$g_T = 10 m/s^2$

a) $M_T/M_P$ ?

b) $\underline{h}$ ? $h_P$ $h_T = 100 m$ a mesma velocidade

$$g_P = G\frac{M_P}{r_P^2} \; ; \; g_T = G\frac{M_T}{r_T^2}$$

$$G = \frac{g_P r_P^2}{M_P} \; ; \; G = \frac{g_T r_T^2}{M_T}$$

$$\frac{g_P r_P^2}{M_P} = \frac{g_T r_T^2}{M_T} \Rightarrow \frac{M_P}{M_T} = \frac{g_P r_P^2}{g_T r_T^2} = \frac{5 m/s^2}{10 m/s^2}\frac{\frac{r_T^2}{4}}{r_T^2}$$

$$\frac{M_P}{M_T} = \frac{1}{2 \times 4} = \frac{1}{8}$$

b)

$$\cancel{m} g_T h_T = \frac{1}{2}\cancel{m} v$$

$$g_T h_T = g_P h_P$$

So nas proximidades da superficie para que podamos utilizar $E_p = mgh$.

Problema nº23

$M_L = 0'012\, M_T$

$R_L = 0'27\, R_T$

DATOS

$g_{0T} = 9'8\ m/s^2$

$R_L = 1'7\ 10^6 m.$

a) $g_L$

b) v escape na lúa.

c) T? pendulo na Terra $\underline{\underline{2s}}$

a)

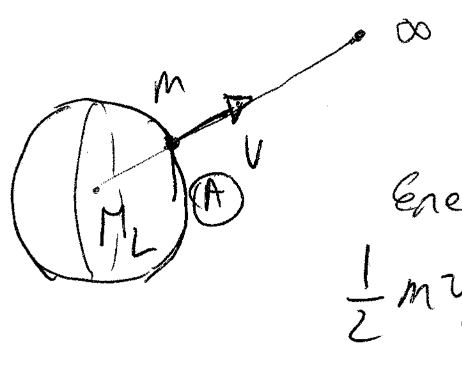
$$g_T = G\frac{M_T}{R_T^2} \quad ; \quad g_L = G\frac{M_L}{R_L^2}$$

$$G = \frac{g_T R_T^2}{M_T} \quad ; \quad G = \frac{g_L R_L^2}{M_L}$$

$$\frac{g_T R_T^2}{M_T} = \frac{g_L R_L^2}{M_L} \Rightarrow g_L = g_T\frac{R_T^2}{M_T}\frac{M_L}{R_L^2}$$

$$g_L = 9'8\frac{m}{s^2}\cdot\frac{\cancel{R_T^2}}{\cancel{M_T}}\frac{0'012\cancel{M_T}}{(0'27)^2\cancel{R_T^2}} = 1'61\ m/s^2$$

b)




Punto A na superficie da Lúa

Enerxia A = Enerxia ∞

$$\frac{1}{2}mv_A^2 - G\frac{M_L m}{r_L} = 0$$

$$\frac{1}{2}v_A^2 = \frac{GM_L}{r_L} \Rightarrow v_A^2 = \frac{2GM_L}{r_L};$$

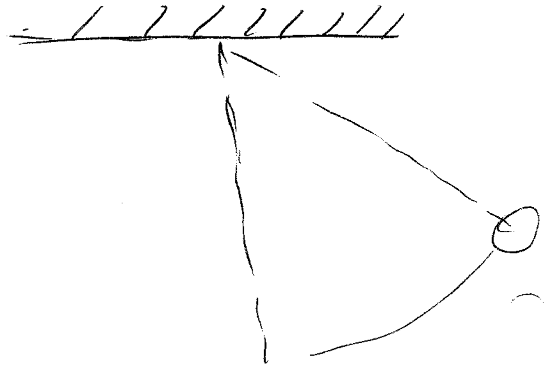
non coñecemos G

$$g_L = \frac{G M_L}{r_L^2} \Rightarrow G M_L = g_L r_L^2$$

$$V_A^2 = \frac{2 \cdot g_L r_L^2}{r_L} \; ; \; V_A^2 = 2 g_L r_L$$

$$V_A = \left(2 \cdot 1'61\, m/s^2 \cdot 1'7\, 10^6\, m\right)^{1/2}$$

c)



$$T = 2\pi \sqrt{\frac{l}{g}}$$

igual lonxitude

$$T^2 = 4\pi^2 \frac{l}{g} \Rightarrow$$

$$T_T^2 = 4\pi^2 \frac{l}{g_T} \; ; \; T_L^2 = 4\pi^2 \frac{l}{g_L}$$

$$T_T^2 g_T = T_L^2 g_L$$

$$T_L^2 = T_T^2 \frac{g_T}{g_L} \; ;$$

$$T_L = \left[(2s)^2 \frac{9'81\, m/s^2}{1'61\, m/s^2}\right]^{1/2} = \underline{\underline{4'9\, s}}$$

Problema nº 24

$M_T / M_L = 79'63$ | DATOS

$R_T / R_L = 3'66$ | $g_0 = 9'8\ m/s^2$

| $R_L = 1'7 \cdot 10^6 m.$

a) $g_L$? ; b) V satelite $r = 2300$ Km. centro da Lúa

c) T dun péndulo onde é maior?

a) $g_T = G \dfrac{M_T}{R_T^2}$ ; $g_L = G \dfrac{M_L}{R_L^2}$

$G = g_T \dfrac{R_T^2}{M_T}$ ; $G = g_L \dfrac{R_L^2}{M_L}$

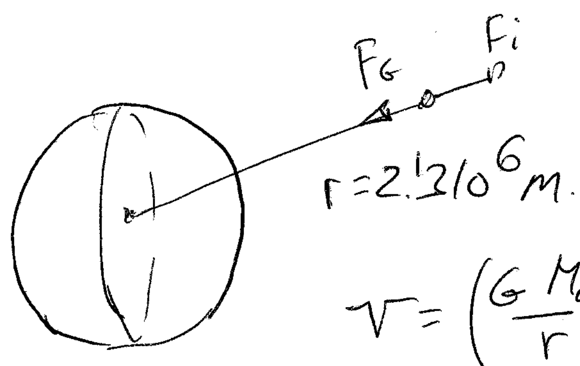
$g_T \dfrac{R_T^2}{M_T} = g_L \dfrac{R_L^2}{M_L}$

$g_L = g_T \dfrac{M_L}{M_T} \dfrac{R_T^2}{R_L^2}$

$g_L = 9'8 \dfrac{m}{s^2} \dfrac{M_T}{M_T\, 79'63} \cdot \dfrac{R_T^2}{\dfrac{R_T^2}{(3'66)^2}}$

$g_L = 9'8 \dfrac{m}{s^2} \dfrac{(3'66)^2}{79'63} = 1'65\ m/s^2$

b)




$F_G = F_i$

$G \dfrac{M_L \cancel{m}}{r^{\cancel{2}}} = \cancel{m} \dfrac{v^2}{\cancel{r}}$

$v = \left( \dfrac{G M_L}{r} \right)^{1/2}$ ; $M_L$

$g_L = G \dfrac{M_L}{R_L^2} \Rightarrow M_L = \dfrac{g_L R_L^2}{G}$

$$v = \left( \frac{G\, g_L R_L^2}{G\, r} \right)^{1/2}$$

$$v = \left( \frac{g_L R_L^2}{r} \right)^{1/2} ; \quad v = \left( \frac{1'65\, m/s^2 \cdot (1'7 \cdot 10^6 m)^2}{2'3\, 10^6 m} \right)^{1/2}$$

$$v = 1'44\, 10^3\, m/s$$

Pág 37

nº25

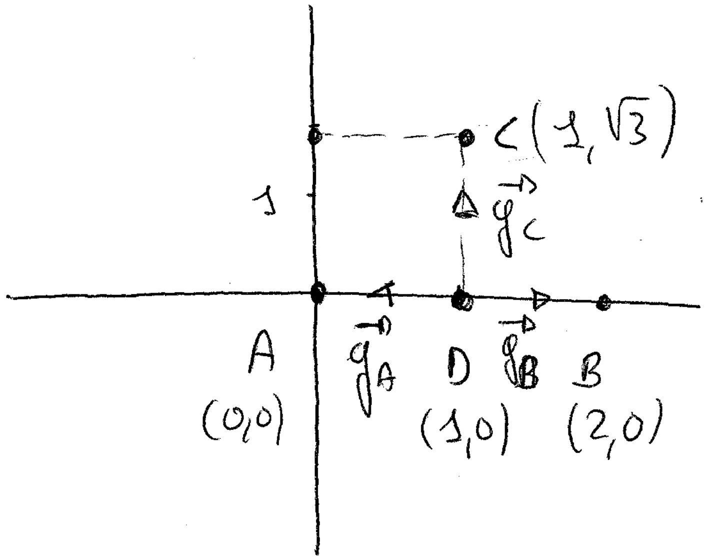
$m = 100\,kg$

$A(0,0)$ ; $B(2,0)$, $C(1,\sqrt{3})\,m.$

a) g? en D(1,0)

b) Ep unha masa de 5Kg en D.

a)




$r_A = r_B = 1m$ ; $r_C = \sqrt{3}m$ $r_C^2 = 1m.$

$|\vec{g}_A| = |\vec{g}_B|$

$$\vec{g}_C = G\,\frac{100\,kg}{3m}\,\vec{j} = 2'22\cdot10^{-4}\,\vec{j}\ (N/kg)$$

b) $V_A = V_B = -G\,\frac{100\,kg}{1m}$ ; $V_C = -G\,\frac{100\,kg}{\sqrt{3}m}$

$$V_T = -G\,100\,kg\left(\frac{2}{1m} + \frac{1}{\sqrt{3}m}\right) = -1'72\cdot10^{-8}\ J/kg$$

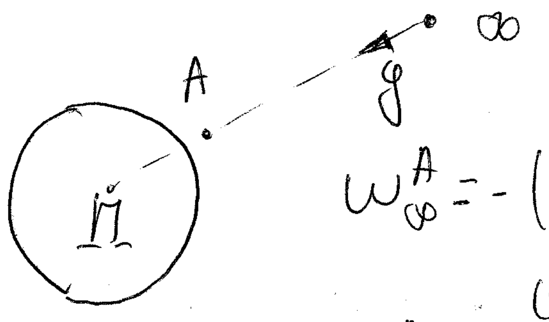
$$E_p = 5\,kg\,V_T = -8'59\cdot10^{-8}\ J$$

c) $W = -\Delta E_p \Rightarrow W_D^{\infty} = -(E_{p\infty} - E_D) \Rightarrow W_D^{\infty} = -[0-(-8'59\cdot10^{-8}\,J)]$

$W_D^{\infty} = -8'59\cdot10^{-8}\ J$ $\quad W_D^{\infty} = -$

Para saber si é feito polo campo ou por forzas exteriores o campo gravitatorio

Si movemos unha masa dende o infinito ata un punto A



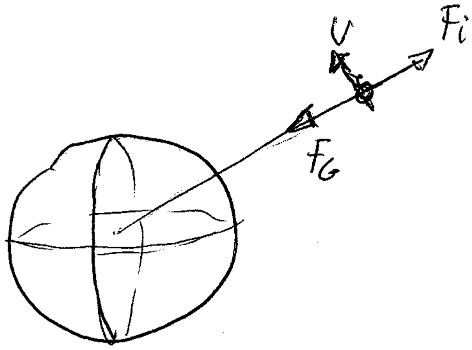

$$W_{\infty}^{A} = -(E_{PA} - \cancel{E_{P\infty}})$$

$(-)$ $\quad$ $0$

$$W_{\infty}^{A} = +$$

Si o traballo o fai o campo gravitatorio é (+). Si o traballo é negativo fano forzas externas

Problema nº 26




$m = 1800\ Kg$

$12'5$ voltas/día

$G = 6'67 \cdot 10^{-11}\ \frac{N \cdot m^2}{Kg^2}$

$R_T = 6378\ Km.$

$M_T = 5'98 \cdot 10^{24}\ Kg$

a) $12'5\ \frac{voltas}{dia}$

12'5 voltas cada día é a frecuencia

a) $T = \frac{1}{f} = \frac{1}{25} \frac{dia}{volta} \frac{24h}{1\ dia} \frac{3600s}{1h} = 6'91 \cdot 10^3 \frac{s}{volta}$

o período é o tempo que tarda en dar unha volta.

b) $G \frac{M_T \cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r}}$ ; $G \frac{M_T}{r} = v^2$

$G \frac{M_T}{r} = \omega^2 \cdot r^2 \Rightarrow G \frac{M_T}{r} = \frac{4\pi^2}{T^2} r^2$

$r^3 = \frac{GMT^2}{4\pi^2}$

$r = \left( \frac{6'67 \cdot 10^{-11} N \cdot m^2/Kg^2 \cdot 5'98 \cdot 10^{24} Kg \ (6'91 \cdot 10^3 s)^2}{4\pi^2} \right)^{1/3}$

$r = 7'84 \cdot 10^6 m$

$h = 7'84 \cdot 10^6 m - R_T = 1'46 \cdot 10^6 m$

c) $E_c = \frac{1}{2} m v^2$ ; teño que calcular a velocidade?

$G \frac{Mm}{r^{\cancel{2}}} = m \frac{v^2}{\cancel{r}}$

$E_c = \frac{1}{2} \frac{GMm}{r} = \frac{1}{2} 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{5'98 \cdot 10^{24} Kg \cdot 1800 Kg}{7'84 \cdot 10^6 m}$

$E_c = 4'58 \cdot 10^{10} J$

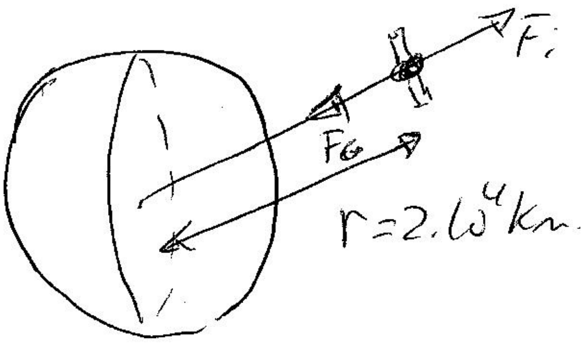
27.-

$M = 500 Kg$

$r = 2.10^4 Km$

a) $v$? e $T$? b) $E_p$ e $E_m$

DATOS: $g_0 = 9'8\ m.s^{-2}$ $R_T = 6370\ Km$




$$F_i = m\frac{v^2}{r} \; ; \; F_G = G\frac{M_T m}{r^2}$$

$$\cancel{m}\frac{v^2}{\cancel{r}} = \frac{G M_T \cancel{m}}{r^{\cancel{2}}} \Rightarrow v^2 = \frac{G M_T}{r}$$

$$g_0 = G\frac{M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

$$v^2 = \frac{g_0 R_T^2}{r} \; ; \; v^2 = \frac{9'8\,m/s^2 \cdot (6'37\,10^6\,m)^2}{2\,10^7\,m}$$

$$v = 4'46\,10^3\,m/s$$

$$v = \omega . r \Rightarrow v = \frac{2\pi}{T} . r \Rightarrow T = \frac{2\pi r}{v}$$

$$T = \frac{2.\pi\, 2.10^7\,m}{4'46.10^3\,m/s} = 2'82\,10^4\,s.$$

b) $E_m = -\frac{1}{2} \frac{G M_T m}{r}$

$E_p = -G \frac{M_T m}{r} \Rightarrow E_p = -\frac{g_0 R_T^2 m}{r}$

$E_p = -\frac{9'8\, m/s^2 \cdot (6'37 \cdot 10^6 m)^2 \cdot 500 kg}{2 \cdot 10^7 m} = -9'94 \cdot 10^9 J$

$\Rightarrow$ $E_m$ a metade $-4'97 \cdot 10^9 J$

c) $E_m = -\frac{1}{2} G \frac{M_T m}{r}$

Se perde enerxía non varía G, $M_T$ e m polo tanto o radio diminue

PERDA DE ENERXÍA SIGNIFICA SIGNO NEGATIVO.

$\Delta E_m = -$

$\Delta E_m = -\frac{1}{2} G M_T m \left( \frac{1}{r_f} - \frac{1}{r_i} \right)$

$r_f < r_i$ O radio diminue.

$v^2 = \frac{G M_T}{r}$ si diminue o radio v aumenta

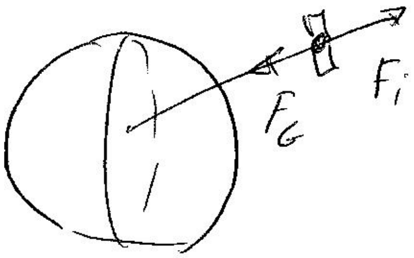
28.-

$m = 200\,Kg$

$h = 600\,Km.$

a) $v$? b) $T$? c) $E_m$?

DATOS: $R_T = 6.400\,Km$; $g_0 = 9'8\,m/s^2$




$F_G = F_i$

$$G\frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{G M_T}{r} = \frac{G M_T}{(R_T + h)}$$

$$g_0 = \frac{G M_T}{R_T^2} \Rightarrow g_0 R_T^2 = G M_T$$

$$v^2 = \frac{g_0 R_T^2}{(R_T + h)}$$

b) $T$?

$$\omega^2 r^2 = \frac{g \cdot R_T^2}{\underbrace{(R_T + h)}_{r}} \;;\; \frac{4\pi^2}{T^2} = \frac{g R_T^2}{(R_T + h)^3}$$

$$T^2 = \frac{4\pi^2 (R_T + h)^3}{g \cdot R_T^2} \;;\; T^2 = \frac{4\pi^2 (7\cdot 10^6 m)^3}{9'8\frac{m}{s^2}(6'4\cdot 10^6 m)^2}$$

$T = 5'81\,10^3\,s$

c)

$$E_m = -\frac{1}{2}\frac{G M_T m}{r} = -\frac{1}{2}\frac{g_0 R_T^2 \cdot m}{(R_T+h)}$$

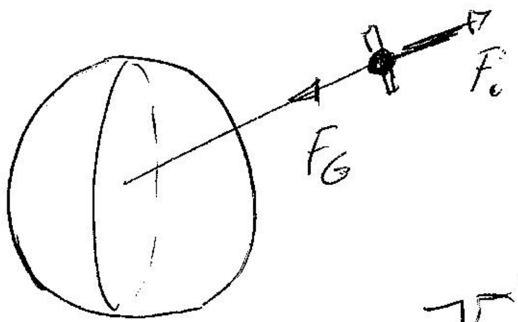
$$E_m = -\frac{1}{2}\frac{9'8\,m/s \cdot (6'4\,10^6\,m)^2 \cdot 200\,Kg}{(7\cdot 10^6\,m)} = -5'73\,10^9\,J$$

29.-

$m = 200\,Kg$
$h = 650\,Km$

a) $v$? $T$? b) $E_m$? c) $g_T / g_{SATELITE}$

$M_T = 5'98\,10^{24}\,Kg$; $R_T = 6'37\,10^6\,m$; $G = 6'67\,10^{-11}\ \frac{N\cdot m^2}{Kg^2}$




$$G\frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{G M_T}{r}$$

$$v^2 = \frac{6'67\cdot 10^{-11}\,N\cdot m^2/Kg^2 \cdot 5'98\cdot 10^{24}\,Kg}{(6'370'650)\cdot 10^6\,m} = 5'68\cdot 10^7\,m^2/s^2$$

$$v = 7'54\,10^3\,m/s$$

$$v = w\cdot r \Rightarrow v = \frac{2\pi}{T}\cdot r \Rightarrow T = \frac{2\pi\cdot r}{v}$$

$$T = \frac{2\cdot\pi\ 7'02\cdot 10^6\,m}{7'54\,10^3\,m/s} = 5'85\,10^3\,s$$

b)

$$E_M = -\frac{1}{2} G \frac{M_T m}{r}$$

$$E_m = -\frac{1}{2}\, 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{5'98\, 10^{24} Kg \cdot 200 Kg}{7'02 \cdot 10^6 m} = -5'68\, 10^9 J$$

c)

$$g_0 = G \frac{M_T}{R_T^2}$$

$$g_{SATE} = G \frac{M_T}{R_{SATELITE}^2}$$

$$\frac{g_{SATELITE}}{g_0} = \frac{\cancel{G}\frac{M_T}{R_S^2}}{\cancel{G}\frac{M_T}{R_T^2}}$$

$$\frac{g_S}{g_0} = \frac{R_T^2}{R_S^2}; \quad \frac{g_S}{g_0} = \left(\frac{R_T}{R_S}\right)^2$$

$$\frac{g_S}{g_0} = \left(\frac{6'37}{7'02}\right)^2; \quad \frac{g_S}{g_0} = 0'823$$

<u>24 BIS.-</u>

$M_L = 0'012\, M_T$ | a) $g_L$

$R_L = 0'27\, R_T$ | b) velocidade de escape na Lua

c) $T = 2s$ $\quad T_L = ?$ $\quad T = 2\pi\sqrt{\frac{l}{g}}$

DATOS $g_{0T} = 9'8\ m \cdot s^{-2}$; $R_L = 1'7 \cdot 10^6 m$

$$g_T = G \frac{M_T}{R_T^2}$$

$$g_L = G \frac{M_L}{R_L^2}$$

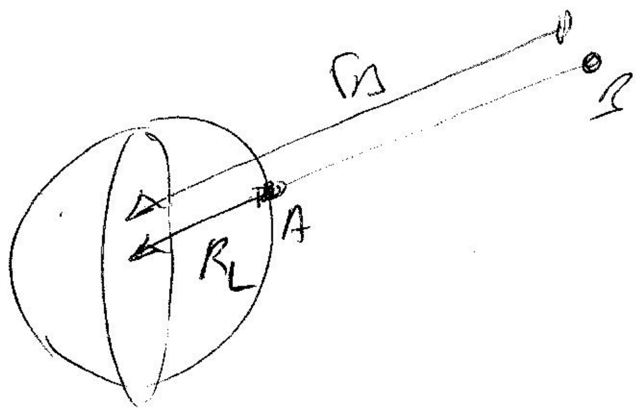
$$\frac{g_L}{g_T} = \frac{R_L^2}{R_T^2}$$

$$\frac{g_L}{g_T} = \frac{G\frac{M_L}{R_L^2}}{G\frac{M_T}{R_T^2}} \Rightarrow \frac{g_L}{g_T} = \frac{M_L \cdot R_T^2}{M_T R_L^2}$$

$$g_L = \frac{M_L R_T^2}{M_T \cdot R_L^2} g_T ; \quad g_L = \frac{0'012 \cancel{M_T} \cdot \cancel{R_T^2}}{\cancel{M_T} (0'27)^2 \cancel{R_T^2}} \, 9'8 \, m/s^2$$

$$g_L = 1'65 \, 10^{-1} \cdot 9'8 \, m/s^2 = 1'61 \, m/s^2$$

b)




O PUNTO B, É UN PUNTO TAN ALONXADO QUE UN CORPO NON SOFRE NINGUNHA ATRACCIÓN POR PARTE DA LUA.

ENERXÍA (A) = ENERXÍA (B)

$$\frac{1}{2} m v_A^2 + \left(-G\frac{M_L \cdot m}{r_L}\right) = \frac{1}{2} m v_B^2 + \left(-G\frac{M_L m}{r_B}\right)$$

Velocidade de escape

$v_B = 0$ non vai máis lonxe do punto B

$r_B \simeq \infty$ para que non sofra atracción por parte da Lua.

$$\frac{1}{2}\cancel{m}v_A^2 - G\frac{M_L \cancel{m}}{r_L} = 0$$

$$v_A^2 = 2\frac{GM_L}{R_L};$$

$$g_{0T} = \frac{G M_T}{r_T^2} \Rightarrow G = \frac{g_{0T}\, r_T^2}{M_T}$$

$$G = \frac{g_{0T}\left(\frac{R_L}{0'27}\right)^2}{\frac{M_L}{0'012}} = \frac{g_{0T}\, R_L^2 \cdot 0'012}{(0'27)^2 \cdot M_L}$$

$$v_A^2 = 2\,\frac{g_{0T}\, R_L^{\cancel{2}} \cdot 0'012}{(0'27)^2 \cdot \cancel{M_L}} \cdot \frac{\cancel{M_L}}{\cancel{R_L}}$$

$$v_A^2 = \frac{2 \cdot 0'012}{(0'27)^2} \cdot g_{0T} \cdot R_L$$

$$v_A^2 = \frac{2\cdot 0'012 \cdot 9'8\,m/s^2 \cdot 1'7 \cdot 10^6 m}{(0'27)^2} = 5'48 \cdot 10^6 m/s$$

$$v_A = 2'34 \cdot 10^3 m/s$$

c) $T = 2\pi\sqrt{\frac{l}{g}}$ ; $T_T = 2\pi\sqrt{\frac{l}{g_T}}$ ; $T_L = 2\pi\sqrt{\frac{l}{g_L}}$

$$\frac{T_L}{T_T} = \frac{2\pi\sqrt{\frac{l}{g_L}}}{2\pi\sqrt{\frac{l}{g_T}}} ; \frac{T_L}{T_T} = \sqrt{\frac{g_T}{g_L}}$$

$$T_L = T_T \sqrt{\frac{g_T}{g_L}} \; ; \; T_L = 2s \sqrt{\frac{9'8 \, m/s^2}{1'61 \, m/s^2}}$$

$$T_L = 4'93 \, s$$

30.-

$m = 1 \cdot 10^3 \, Kg$

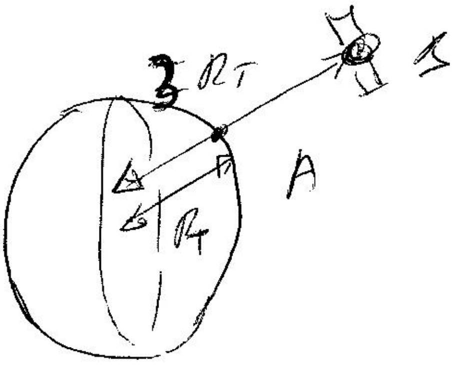
$r = 2 R_T$

a) Enerxia $\Rightarrow E_c$ ?

b) $F_c$ (forza centripeta

$T$ ?

DATOS: $g_0 = 9'8 \, m/s^2$ ; $R_T = 6.370 \, Km$.




$h = 2 R_T$

Enerxia (A) = Enerxia B.

$$\frac{1}{2} m v_A^2 + \left(-G \frac{Mm}{r_T}\right) = \frac{1}{2} m \cancel{v_B^2} + \left(-G \frac{Mm}{3 r_T}\right)$$

$v_B = 0$ non se desprazamáis lonxe.

$$(E_c)_A = G \frac{Mm}{r_T} - \frac{GMm}{3 r_T}$$

$$T_L = T_T \sqrt{\frac{g_T}{g_L}} \; ; \; T_L = 2\,s \sqrt{\frac{9'81\,m/s^2}{1'61\,m/s^2}}$$

$$T_L = 4'93\ s$$

30.-

$m = 10^3\ Kg$

$h = 2\ R_T$

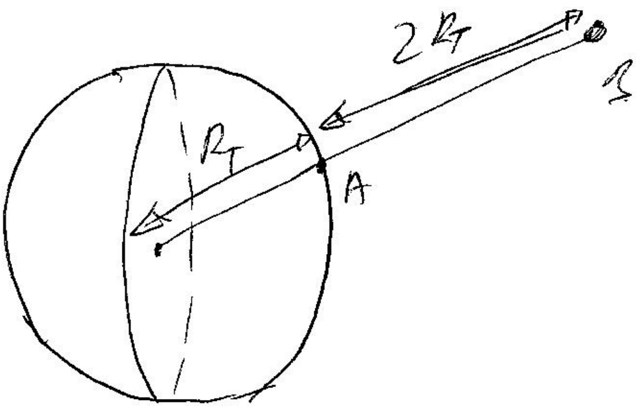
$r = 3\ R_T$

a) Enerxia $\Rightarrow E_c$

b) $F_i \Rightarrow$ Forza centripeta

c) $T$ ?

DATOS: $g_0 = 9'8\ m.s^{-2}$ ; $R_T = 6.370\ Km.$




Enerxia A = Enerxia B

$$\frac{1}{2} m v_A^2 - G\frac{M_T\,m}{r_T} = \frac{1}{2} m \cancel{v_B}^2 - \frac{G M_T m}{3 r_T}$$

$v_B = 0$ non se desplaza máis lonxe

$$\frac{1}{2} m v_A^2 = \frac{G M_T m}{r_T} - \frac{G M_T m}{3 R_T}$$

$$(E_c)_A = \frac{G M_T m}{R_T}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\,\frac{G M_T m}{r_T}$$

$$g_0 = \frac{G M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

$$(E_c)_A = \frac{2}{3}\,\frac{g_0 R_T^{\cancel{2}} m}{\cancel{R_T}} = \frac{2 \cdot 9'8\,m/s^2 \; 6'37\cdot10^6\,m \cdot 10^3\,Kg}{3}$$

$$(E_c)_A = 4'16 \cdot 10^{10}\ J$$

UNIDADES

$$Kg\,\frac{m^2}{s^2} \Rightarrow Kg\,\frac{m}{s^2}\cdot m = N \cdot m = J$$

b)

$$F_c = m\,\frac{v^2}{r}$$

$$F_G = F_c\ ;\quad m\,\frac{v^2}{r} = \frac{G\,M_T \cdot m}{r^2}$$

$$F_c = \frac{G\,M_T\,m}{r^2}\ ;\quad F_c = \frac{g_0\,R_T^2\,m}{(3R_T)^2}\ ;$$

$$F_c = \frac{g_0\,\cancel{R_T^2}\,m}{9\,\cancel{R_T^2}} = \frac{g_0\,m}{9} = \frac{9'8\,m/s^2 \cdot 10^3\,Kg}{9}$$

$$F_c = 1'09 \cdot 10^3\,N$$

c)

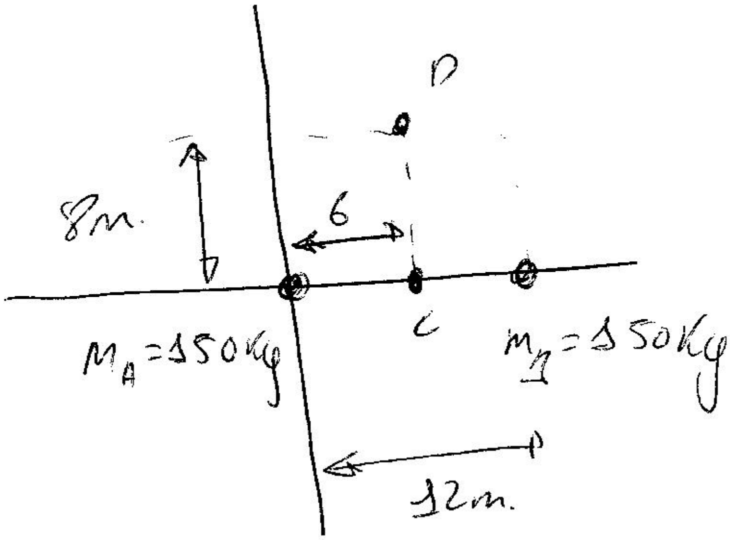
$$\cancel{m}\,\frac{v^2}{\cancel{r}} = \frac{G\,M_T\,\cancel{m}}{r^{\cancel{2}}}\ ;\quad v^2 = \frac{G\,M_T}{r}$$

$$\frac{4\pi^2}{T^2}\cdot r^2 = \frac{G\,M_T}{r} \Rightarrow T^2 = \frac{4\pi^2 r^3}{G\,M_T}$$

$$T^2 = \frac{4\pi^2\,(3R_T)^3}{g_0\,R_T^2}\ ;\quad T^2 = \frac{4\pi^2 \cdot 27\,R_T^{\cancel{3}}}{g_0\,\cancel{R_T^2}}$$

$$T^2 = \frac{4\pi^2 \cdot 27 \cdot 6'37\cdot10^6\,m}{9'8\,\frac{m}{s^2}}\ ;\quad T^2 = 6'93\cdot10^8\,s^2\ ;\quad \underline{\underline{T = 2'63\cdot10^4\,s}}$$

31 -

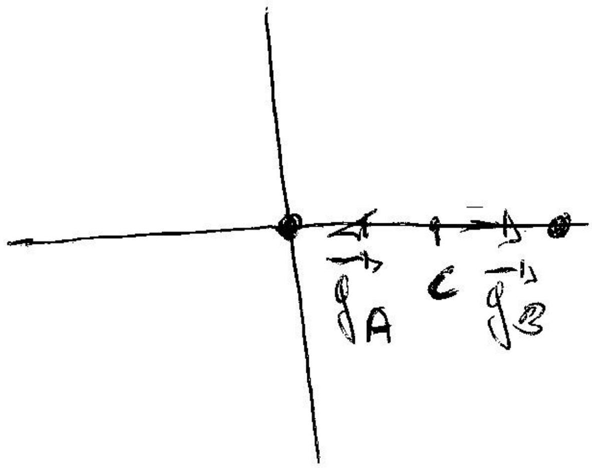



a) $\vec{g}_C$ $\vec{g}_D$

b) $V_D = -10^{-4}\ \vec{j}\ m/s$ $V_C$?

c) movemento entre $C \to D$ M.RUA ou do outro tipo

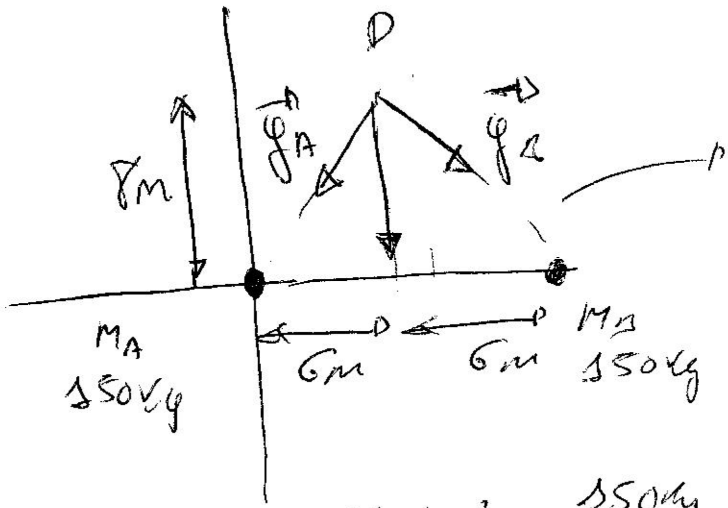
a)




$|\vec{g}_A| = |\vec{g}_B| \Rightarrow$ sentidos opostos

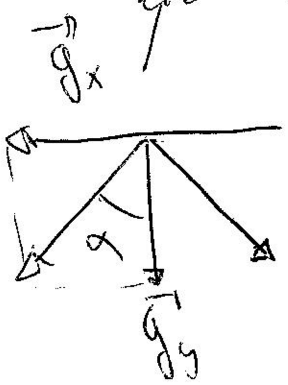
campo gravitatorio en C $\underline{\underline{0}}$

→ feces a partir do 27




$$d^2 = (6\,m)^2 + 8\,m^2$$
$$d^2 = 100\,m^2$$

$$|\vec{g}_A| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2} \, \frac{150\,kg}{100\,m^2} = 1 \cdot 10^{-10} \frac{N}{kg}$$




$$\operatorname{sen}\alpha = \frac{\vec{g}_x}{|\vec{g}|}$$

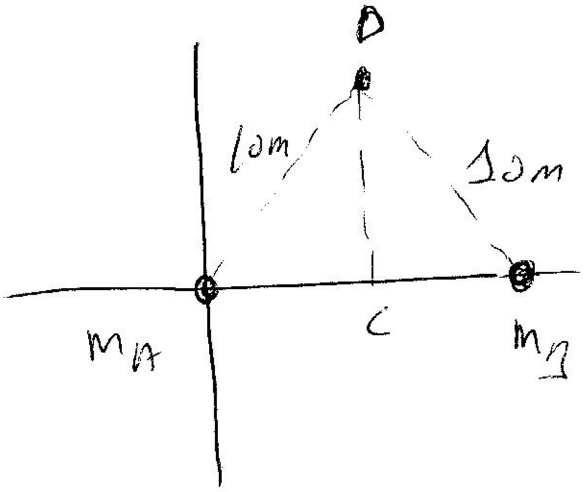
$$\cos\alpha = \frac{\vec{g}_y}{|\vec{g}|}$$

$$\vec{g}_x = |\vec{g}| \cdot 0'6 = 6 \cdot 10^{-11}\, N/kg$$
$$\vec{g}_y = |\vec{g}| \cdot 0'8 = 8 \cdot 10^{-11} \frac{N}{kg}$$

$$\vec{g}_A = -6 \cdot 10^{-11}\,\vec{i} - 8 \cdot 10^{-11}\,\vec{j} \; (N/kg)$$
$$\vec{g}_A = 6 \cdot 10^{-11}\,\vec{i} - 8 \cdot 10^{-11}\,\vec{j} \; (N/kg)$$

$$\vec{g}_T = -1'6 \cdot 10^{-10}\,\vec{j} \; (N/kg)$$

$V_D$




$V = -\frac{GM}{r}$ en D crean potencial gravitatorio

A e B

$$V_D = -\frac{GM_A}{r} + \left(-\frac{GM_B}{r}\right) = -2\frac{GM}{r}$$

$$V_D = -\frac{2\cdot 6'67\cdot 10^{-11}\, N\cdot m^2/kg^2 \cdot 150kg}{10m} = -2\cdot 10^{-9}\, J/kg$$

$$V_C = \frac{-2GM_A}{r'} + \left(-\frac{GM_B}{r'}\right) = -2\frac{GM}{r'}$$

$$V_C = -\frac{2\cdot 6'67\cdot 10^{-11}\, N\cdot m^2/kg^2 \cdot 150kg}{6m} = -3'34\cdot 10^{-9}\, J/kg$$

b) Enerxia D = Enerxia C

$$\frac{1}{2} m v_D^2 + (E_p)_D = \frac{1}{2} m v_C^2 + (E_p)_C$$

$$E_p = V\cdot m$$

$$\frac{1}{2} m v_D^2 + V_D m = \frac{1}{2} m v_C^2 + V_C\cdot m \Rightarrow \frac{1}{2} v_D^2 + V_D = \frac{1}{2} v_C^2 + V_C$$

velocidade → potencial

$$V_c^2 = V_D^2 + 2(V_D - V_c)$$

$$V_c^2 = \left(1 \cdot 10^{-4} m/s\right)^2 + 2\left(-2 \cdot 10^{-9} J/kg - \left(-3'34 \cdot 10^{-9} J/kg\right)\right)$$

$$V_c^2 = 1 \cdot 10^{-8} \frac{m^2}{s^2} + 2'68 \cdot 10^{-9} \frac{m^2}{s^2}$$

$$V_c^2 = 1'27 \cdot 10^{-8} \frac{m^2}{s^2} \Rightarrow V_c = 1'13 \cdot 10^{-4} m/s$$

c) Rectilineo si, uniformemente acelerado non porque a aceleración varia

32.-

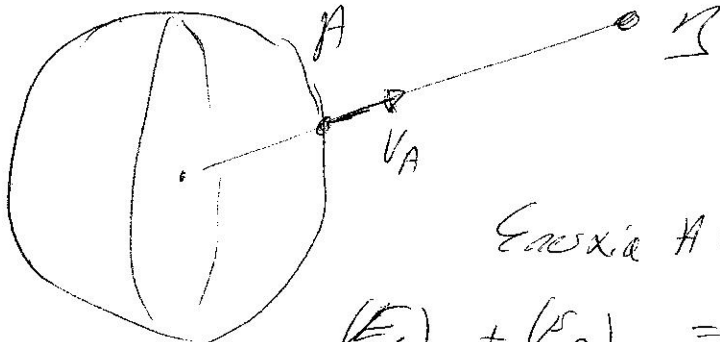
a)


$$g = G\frac{M_c}{R_c^2}$$

$$g = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{9'43 \cdot 10^{20} Kg}{(4'77 \cdot 10^5 m)^2}$$

$$g = 2'76 \cdot 10^{-1} \frac{N}{Kg}$$

b)




Enerxía A = Enerxía B

$$(E_c)_A + (E_p)_A = (E_c)_B + (E_p)_B$$

$v_B = 0 \qquad r_B = \infty$

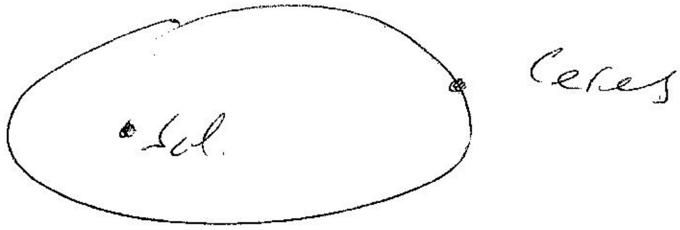
$$(E_c)_A + (E_p)_A = 0$$

$$(E_c)_A = -(E_p)_A \Rightarrow (E_c)_A = \frac{G M_c m}{R_c}$$

$$(E_c)_A = \frac{6'67 \cdot 10^{-11} N \cdot m^2/Kg^2 \cdot 9'43 \cdot 10^{20} Kg \cdot 10^3 Kg}{4'77 \cdot 10^5 m}$$

$$(E_c)_A = 1'32 \cdot 10^8 J$$

c)




lei de Kepler.

$$\frac{T^2}{r^3} = cte \qquad \frac{T_T^2}{r_T^3} = \frac{T_C^2}{r_C^3}$$

$$r_C^3 = r_T^3 \frac{T_C^2}{T_T^2} \Rightarrow r_C^3 = r_T^3 \frac{T_C^2}{T_T^2}$$

$$r_C = r_T \left(\frac{T_C}{T_T}\right)^{2/3} = 1'5\,10^{11}\,m \left(\frac{4'6\,anos}{1\,ano}\right)^{2/3}$$

$$r_C = 4'5\,10^{11}\,m.$$

34.-

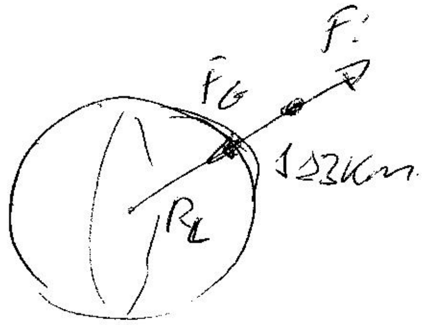
$v = 7'62\ Km.s^{-1}$ | a) h? b) T

c) voltas cada 24h.

DATOS: $G = 6'67\,10^{-11}\ N.m^2/Kg^2$ ; $R_T = 6370\ Km$

$M_T = 5'98.10^{24}\ Kg$

33.-




a) T?

b) v? ω?

c) velocidade de escapa

DATOS: $G : 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$ ; $R_L = 1740\,Km$ ; $M_L = 7'36 \cdot 10^{22}\,Kg$

$$G \frac{M_L \cancel{m}}{r^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{r}} ; \quad r = R_L + 113\,Km.$$

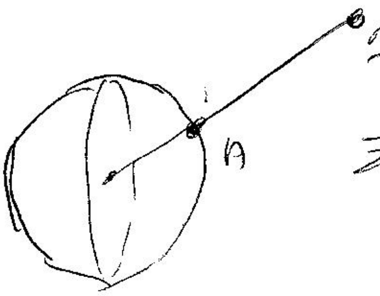
$$v^2 = \frac{G M_L}{r} = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{7'36 \cdot 10^{22}\,Kg}{(1740+113) \cdot 10^3\,m}$$

$$v^2 = 2'64 \cdot 10^6\,m^2/s^2 \; ; \quad v = 1628\,m/s$$

$$v = \omega \cdot r \Rightarrow \omega = \frac{v}{r} = \frac{1628\,m/s}{1'853 \cdot 10^6\,m} = 8'78 \cdot 10^{-4}\,\frac{rad}{s}$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi\,rad}{8'78 \cdot 10^{-4}\,\frac{rad}{s}} = 7153\,s$$

c)




Enerxia A = Enerxia 1

$$\frac{1}{2} m v_A^2 - G \frac{M m}{r} = \frac{1}{2} m v_1^2 - G \frac{M m}{r_1}$$

$$r_1 = \infty$$

$$v_1 = 0$$

$$r = r_L + h$$

$$v_A^2 = \frac{2GM}{r} = \frac{2 \cdot 6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 7'36 \cdot 10^{22} kg}{1'853 \cdot 10^6 m}$$

$$v_A^2 = 5'30 \cdot 10^6 m^2/s^2 \; ; \; v_A = 2301 m/s$$

34.-

$v = 7'62 \; km \cdot s^{-1}$

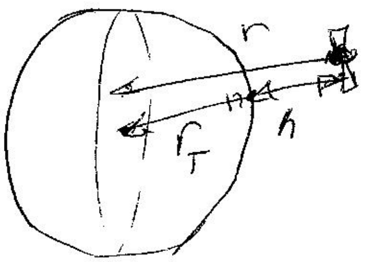
a) $h$?

b) $T$?

c) Vueltas cada 24h.

DATOS: $G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$ ; $R_T = 6370 km$ ; $M_T = 5'98 \cdot 10^{24} kg$

a)




$$F_G = F_c$$

$$G \frac{M_T m}{r^{\cancel{2}}} = m \frac{v^2}{\cancel{r}}$$

$$r = \frac{G M_T}{(v)^2}$$

$$r = \frac{6'67 \cdot 10^{-11} N m^2/kg^2 \cdot 5'98 \cdot 10^{24} kg}{(7620 \; m/s)^2}$$

$$r = 6'87 \cdot 10^6 m$$

$$h = r - r_T = 4'99 \cdot 10^5 m.$$

b) T?

$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T} \cdot r$$

$$T = \frac{2\pi r}{v} = \frac{2\pi \cdot 6'86 \cdot 10^6 m}{7620 \frac{m}{s}} = 5656 s$$

$$5656 s \quad \frac{1h}{3600 s} = \underline{\underline{1'57 h}}$$

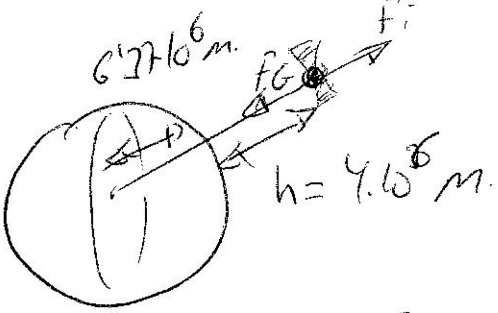
c)

$$\frac{24 h}{1'57 \frac{h}{volta}} = 15'3$$

DATOS

35.- $m = 10^2\ Kg$ | $g_0 = 9'81\ m/s^2$ | a) $v$, $a$ e $T$

$h = 4 \cdot 10^3\ Km$ | $R_T = 6'37 \cdot 10^6\ m$ | b) $|L|$




$$G\frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{G M_T}{r}$$

$$g_0 = G\frac{M_T}{R_T^2}\ ;\ g_0 R_T^2 = G M_T$$

$$v^2 = \frac{g_0 R_T^2}{r} \Rightarrow v^2 = \frac{9'81\ m/s^2 \cdot (6'37 \cdot 10^6\ m)^2}{10'370 \cdot 10^6\ m}$$

$$v^2 = 3'84 \cdot 10^7\ m^2/s^2\ ;\ v = 6'20 \cdot 10^3\ m/s$$

$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T} \cdot r \Rightarrow T = \frac{2\pi \cdot r}{v}$$

$$T = \frac{2\pi \cdot 10'370 \cdot 10^6\ m}{6'20 \cdot 10^3\ m/s} = 1'05 \cdot 10^4\ s$$

$$a_n = \frac{v^2}{r} = \frac{(6'20 \cdot 10^3\ m/s)^2}{10'370 \cdot 10^6\ m} = 3'71\ m/s^2$$

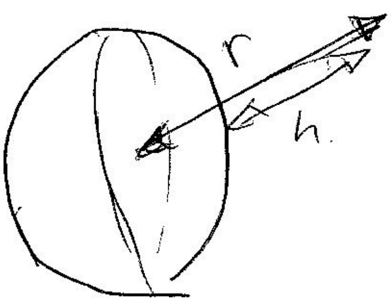
$$a_n = g = G\frac{M_T}{r^2} = \frac{g_0 R_T^2}{r} = \frac{9'81\ m/s^2 \cdot (6'37 \cdot 10^6\ m)^2}{(10'370 \cdot 10^6\ m)^2}$$

$$= 3'70\ m/s^2$$

$$|\vec{L}| = |\vec{r}| \cdot |\vec{p}| = |\vec{r}| \cdot m|\vec{v}| = 10'370 \cdot 10^6\ m \cdot 10^2\ Kg \cdot 6'20 \cdot 10^3\ m/s$$

$$|\vec{L}| = 6'43 \cdot 10^{12}\ Kg \cdot m^2 \cdot s^{-1}$$

36.-




$\frac{2 \text{ voltas}}{24 \text{ horas}}$

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$

$M_T = 5'98 \cdot 10^{24}\ kg$

$R_T = 6'37 \cdot 10^{6}\ m.$

a) h? ; b) $E_m$ ; c) T cando $r = 2r$

$$G \frac{M_T \cancel{m}}{r^2} = \cancel{m} \frac{v^2}{r} \; ; \; G \frac{M_T}{r} = v^2$$

$$G \frac{M_T}{r} = \omega^2 r^2 \Rightarrow G M_T = \frac{4\pi^2}{T^2} \cdot r^3 \quad (1)$$

$$r^2 = \frac{G M_T T^2}{4\pi^2} = \frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 5'98 \cdot 10^{24} kg \cdot (12 \times 3600 s)^2}{4\pi^2}$$

$$r^3 = 1'89 \cdot 10^{22} m^3 \Rightarrow r = 2'66 \cdot 10^7 m.$$

$$h = r - R_T = 2'66 \cdot 10^7 - 6'37 \cdot 10^6 m = 2'02 \cdot 10^7 m$$

b) $$E_m = -\frac{1}{2} \frac{G M_T m}{r} = -\frac{1}{2} \frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 5'98 \cdot 10^{24} kg \cdot 150 kg}{2'66 \cdot 10^7 m}$$

$$E_m = -1'12 \cdot 10^9 J$$

c) $$\frac{G M_T}{4\pi^2} = \frac{r^3}{T^2} \; ; \; \frac{r_a^3}{T_a^2} = \frac{r_b^3}{T_b^2}$$

$T_b = ?$

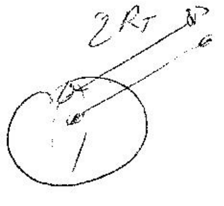
$r_b = R_T + 2h$

$r_a = 2'66 \cdot 10^7 m.$

$T_a = 12h$

$$T_b = T_a \left(\frac{r_b}{r_a}\right)^{3/2} \Rightarrow T_b = 12h \left(\frac{4'68 \cdot 10^7}{2'66 \cdot 10^7}\right)^{3/2} = 28h$$

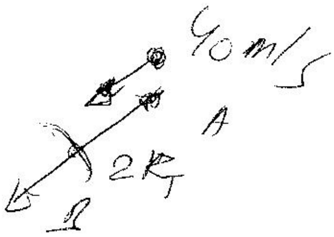
37.-




a) $v$?

b) $g_{2R_T}$

c)




$g_0 = 9'81\ m/s$

$R_T = 6370\ m.$

a)

$$G\frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}} \Rightarrow v^2 = \frac{G M_T}{r}$$

$$v^2 = \frac{G M_T}{2R_T} \Rightarrow g_0 = \frac{G M_T}{R_T^2}$$

$$v^2 = \frac{g_0 R_T^{\cancel{2}}}{2\cancel{R_T}} \Rightarrow v^2 = \frac{g_0 R_T}{2}$$

$$v^2 = \frac{9'81\ m.s^{-2} \cdot 6'370 \cdot 10^6\ m}{2} = 3'12 \cdot 10^7\ m^2/s^2$$

$$v = 5'59 \cdot 10^3\ m/s$$

b) $g = G\frac{M_T}{r^2}$; $g = G\frac{M_T}{(2R_T)^2}$; $g = \frac{g_0 \cancel{R_T^2}}{4\cancel{R_T^2}}$

$$g = \frac{9'81\ m/s^2}{4} = 2'45\ m/s^2$$

c)

$$(E_c)_A + (E_p)_A = (E_c)_B + (E_p)_B$$

$$\frac{1}{2} m v_B^2 = (E_p)_A - (E_p)_B + \frac{1}{2} m v_A^2$$

$$\frac{1}{2}\cancel{m} v_B^2 = -\frac{G M_T \cancel{m}}{r_A} + \frac{G M_T \cancel{m}}{r_B} + \frac{1}{2}\cancel{m} v_A^2$$

$$v_B^2 = 2 G M_T\left(\frac{1}{r_B} - \frac{1}{r_A}\right) + v_A^2$$

$$v_B^2 = 2 \cdot g_0 R_T^2\left(\frac{1}{r_B} - \frac{1}{r_A}\right) + v_A^2;$$

$$V_1^2 = 2 g_0 R_T^2 \left( \frac{1}{R_T} - \frac{1}{2R_T} \right) + V_A^2$$

$$V_1^2 = \cancel{2} g_0 \cancel{R_T^2} \left( \frac{2R_T - R_T}{\cancel{2} \cancel{R_T^2}} \right) + V_A^2 \; ; \; V_1^2 = g_0 R_T + V_A^2$$

$$V_1^2 = 9'81 \, m/s^2 \cdot 6'37 \, 10^6 \, m + (40 \, m/s)^2$$

$$V_1 = 7'91 \, 10^3 \, m/s$$