Indeterminaciones cero partido por cero y uno elevado a infinito

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INDETERMINACIÓN $\frac{0}{0}$

a) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-5x+4} = \frac{1^2-3+2}{1-5+4} = \frac{0}{0}$ INDET.

$x^2-3x^2+2=0 \rightarrow x = \frac{3\pm\sqrt{9-8}}{2} = \frac{3\pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$x^2-5x+4=0 \rightarrow x = \frac{5\pm\sqrt{25-16}}{2} = \frac{5\pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$\lim_{x\to 1} \frac{x^2-3x+2}{x^2-5x+4} = \lim_{x\to 1} \frac{(x-2)(x-1)}{(x-4)(x-1)} = \frac{-1}{-3} = \frac{1}{3}$

$-2x^2+6x-4=0 \begin{cases} 2 \\ 1 \end{cases} \Rightarrow -2x^2+6x-4 = -2(x-2)(x-1)$

b) $\lim_{x\to 2} \frac{\sqrt{3x-2}-2}{x-2} = \frac{\sqrt{6-2}-2}{2-2} = \frac{0}{0}$ INDET.

$\lim_{x\to 2} \frac{\sqrt{3x-2}-2}{x-2} = \lim_{x\to 2} \frac{(\sqrt{3x-2}-2)(\sqrt{3x-2}+2)}{(x-2)(\sqrt{3x-2}+2)} = \lim_{x\to 2} \frac{(\sqrt{3x-2})^2-2^2}{(x-2)(\sqrt{3x-2}+2)} = \lim_{x\to 2} \frac{3x-2-4}{(x-2)(\sqrt{3x-2}+2)} =$

$= \lim_{x\to 2} \frac{3x-6}{(x-2)(\sqrt{3x-2}+2)} = \lim_{x\to 2} \frac{3(x-2)}{(x-2)(\sqrt{3x-2}+2)} = \frac{3}{\sqrt{6-2}+2} = \frac{3}{4}$

INDETERMINACIÓN 1^∞

$\lim_{x\to\infty} \left(\frac{x^2+2x}{x^2-1}\right)^{3x+2} = 1^\infty$ INDETERMINADO

$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$

$\lim_{x\to\infty} \left(1+\frac{1}{f(x)}\right)^{f(x)} = e$ con $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to\infty} \left(\frac{x^2+2x}{x^2-1}\right)^{3x+2} = \lim_{x\to\infty} \left(1+\frac{x^2+2x}{x^2-1}-1\right)^{3x+2} = \lim_{x\to\infty} \left(1+\frac{x^2+2x-x^2+1}{x^2-1}\right)^{3x+2} = \lim_{x\to\infty} \left(1+\frac{2x+1}{x^2-1}\right)^{3x+2} =$

$= \lim_{x\to\infty} \left(1+\frac{\frac{2x+1}{2x+1}}{\frac{x^2-1}{2x+1}}\right)^{3x+2} = \lim_{x\to\infty} \left(1+\frac{1}{\frac{x^2-1}{2x+1}}\right)^{3x+2} = \lim_{x\to\infty} \left(1+\frac{1}{\frac{x^2-1}{2x+1}}\right)^{\frac{x^2-1}{2x+1}\cdot\frac{2x+1}{x^2-1}(3x+2)} =$

$= e^{\lim_{x\to\infty} \frac{2x+1}{x^2-1}(3x+2)} = e^{\lim_{x\to\infty} \frac{6x^2+4x+3x+2}{x^2-1}} = e^6$

$= e^{\lim_{x\to\infty} x^2-1} = e^{x\to\infty \quad x^2-1} = \boxed{e^6}$