

UD 5 Determinantes - Video 2 - Menor complementario, adjunto e inversa.

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3. MENOR COMPLEMENTARIO

$$A = \begin{pmatrix} -1 & 3 & 2 \\ 1 & 0 & 4 \\ -1 & 1 & 1 \end{pmatrix}$$

$a_{ij} \rightarrow \alpha_{ij}$ (Menor complementario)

$$\alpha_{22} = \begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = -1 - (-2) = 1$$

$$\alpha_{13} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1 - 0 = 1$$

4. ADJUNTO DE UN ELEMENTO

$$A = \begin{pmatrix} -1 & 3 & 2 \\ 1 & 0 & 4 \\ -1 & 1 & 1 \end{pmatrix}$$

$$a_{ij} \rightarrow A_{ij} = (-1)^{i+j} \alpha_{ij}$$

$$A_{22} = (-1)^{2+2} \alpha_{22} = 1 \cdot 1 = 1$$

$$A_{13} = (-1)^{1+3} \alpha_{13} = 1 \cdot 1 = 1$$

$$A_{12} = (-1)^{1+2} \alpha_{12} = (-1) \begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} = (-1)(1+4) = -5$$

5. INVERSA DE UNA MATRIZ

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{(\text{Adj } A)^t}{\det A}$$

$$\text{Adj } A = \begin{pmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \\ - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \\ + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$(\text{Adj } A)^t = \begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\det A = (0+0+0) - (0+1+1) = -2$$

$$A^{-1} = \frac{\begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}}{-2} = \frac{-1}{2} \begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \end{pmatrix}$$

Ejemplo

$A = \begin{pmatrix} 1 & 0 & x \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ calcula x para que A tenga inverso.

$$\det A = (1-x+0) - (0+0+0) = 1-x=0 \Rightarrow \textcircled{x=1} \Rightarrow \text{Si } x=1 \text{ la matriz no tiene inversa.}$$

Para que A tenga inversa $x \neq 1$