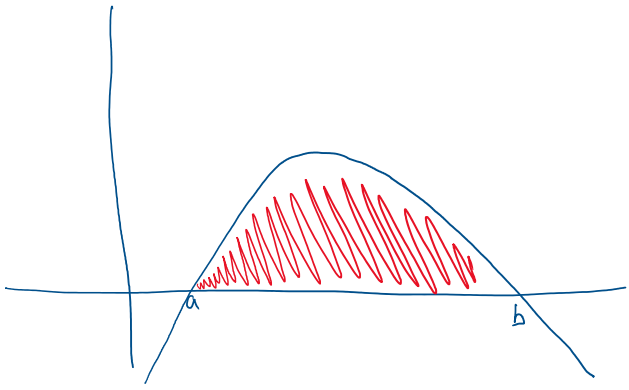
UD 3 Integrales -Vídeo 1 - Integral definida

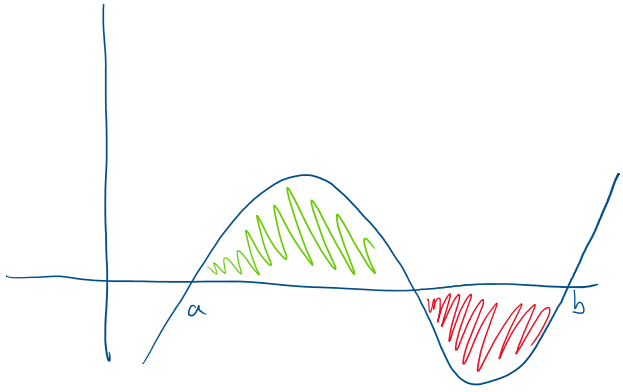
sábado, 13 de enero de 2024 23:58

1. INTEGRAL DEFINIDA

Si f es una función no negativa en $[a,b]$, llamamos integral definida desde a hasta b al área que encierra f con el eje x:

$$\int_a^b f(x)\,dx$$





Ejemplo

$f(x) = x^3 - x^2 - 9x + 9$ en $[-2,4]$

$$\int_{-2}^{4} f(x)\,dx = \left|\int_{-2}^{1} f(x)\,dx\right| + \left|\int_{1}^{3} f(x)\,dx\right| + \left|\int_{3}^{4} f(x)\,dx\right| =$$

$x^3 - x^2 - 9x + 9 = 0$

	1	-1	-9	9
1		1	0	-9
	1	0	-9	0

$\rightarrow x^2 - 9 = 0$

$x^2 = 9 \rightarrow x = \pm 3$

$$= \left|\int_{-2}^{1} (x^3 - x^2 - 9x + 9)\,dx\right| + \left|\int_{1}^{3} (x^3 - x^2 - 9x + 9)\,dx\right| + \left|\int_{3}^{4} (x^3 - x^2 - 9x + 9)\,dx\right| =$$

$$= \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x\right]_{-2}^{1}\right| + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x\right]_{1}^{3}\right| + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x\right]_{3}^{4}\right| =$$

$$= \left|\left(\frac{1}{4} - \frac{1}{3} - \frac{9}{2} + 9\right) - \left(4 + \frac{8}{3} - 18 + 18\right)\right| + \left|\left(\frac{81}{4} - 9 - \frac{81}{2} + 27\right) - \left(\frac{1}{4} - \frac{1}{3} - \frac{9}{2} + 9\right)\right| +$$

$$\left|\left(-\frac{64}{3} + \ldots - 36\right) - \left(\frac{81}{4} - 9 - \frac{81}{2} + 27\right)\right| = \boxed{148\ u^2}$$

$$+\left|\left(64-\frac{64}{3}-72+36\right)-\left(\frac{81}{4}-9-\frac{81}{2}+27\right)\right| = \frac{148}{3}\,u^2$$

Ejemplo

$f(x) = x+1 \qquad g(x) = 2x^2-3x+1$

Ptos de corte entre las funciones

$$x+1 = 2x^2-3x+1 \Rightarrow 2x^2-4x=0$$

$$x(2x-4)=0 \quad \begin{cases} x=0 \\ x=2 \end{cases}$$

$$\left|\int_0^2 \left[(x+1)-(2x^2-3x+1)\right]dx\right| = \left|\int_0^2 (-2x^2+4x)\,dx\right| = \left|\left[\frac{-2x^3}{3}+\frac{4x^2}{2}\right]_0^2\right| =$$

$$\left|\left(\frac{-16}{3}+8\right)-\left(0+0\right)\right| = \left|\frac{-16}{3}+8\right| = \left|\frac{-16+24}{3}\right| = \left|\frac{8}{3}\right| = \frac{8}{3}\,u^2$$