Unidade 3– Integración – Solucións Ex1 a Ex20

Ex 1 Obtén a integral indefinida en cada caso:

a) $\int 3\,dx \quad = 3x + C$

b) $\int x^2\,dx \quad = \frac{x^3}{3} + C$

c) $\int x^7\,dx \quad = \frac{x^8}{8} + C$

d) $\int 3x^5\,dx \quad \Rightarrow F(x) = \frac{x^6}{2} + C$

e) $\int \frac{1}{x^3}\,dx \quad \Rightarrow F(x) = \frac{1}{-2x^2} + C$

f) $\int \sqrt{x^3}\,dx \quad \Rightarrow F(x) = \frac{2}{5} x^{\frac{5}{2}} + C$

g) $\int \frac{1}{\sqrt[3]{x^2}}\,dx \quad \Rightarrow = 3\sqrt[3]{x} + C$

h) $\int \frac{5}{x^2}\,dx \quad \Rightarrow F(x) = \frac{-5}{x} + C$

i) $\int \frac{\sqrt{2x}}{\sqrt[3]{5x}}\,dx = \int \frac{\sqrt[6]{8x^3}}{\sqrt[6]{25x^2}}\,dx = \sqrt[6]{\frac{8}{25}} \int \sqrt[6]{x}\,dx = \frac{6}{7}\sqrt[6]{\frac{8}{25}}\sqrt[6]{x^7} + C$

j) $\int (3x^3 - 5x^2 + 2)\,dx \quad = \frac{3}{4}x^4 - \frac{5}{3}x^3 + 2x + C$

k) $\int \frac{1}{x}\,dx \quad = \ln|x| + C$

l) $\int \frac{x^4 - 5x^2 + 3x - 4}{x}\,dx \quad = \int \left(x^3 - 5x + 3 - \frac{4}{x}\right)dx = \quad = \frac{x^4}{4} - \frac{5}{2}x^2 + 3x - 4\ln|x| + C$

m) $\int \frac{1}{x-5}\,dx \quad = \ln|x-5| + C$

n) $\int \frac{x^3 - 3x^2 + x + 2}{x-2}\,dx$
$= \int (x^2 - x - 1)\,dx = \frac{x^3}{3} - \frac{x^2}{2} - x + C$

o) $\int x^3\,dx \quad = \frac{x^4}{4} + C$

p) $\int (\sqrt[5]{x^2} - 3x^2 + 7)\,dx \quad = \frac{5}{7}x^{\frac{7}{5}} - x^3 + 7x + C$

q) $\int \left(\frac{4}{x} + \frac{6}{x^3} + e^x\right)dx$
$= 4\ln|x| - \frac{3}{x^2} + e^x + C$

r) $\int \frac{x^2 + 3x - 1}{3}\,dx$
$= \frac{1}{3}\cdot\left(\frac{x^3}{3} + 3\frac{x^2}{2} - x + C_1\right) = \frac{x^3}{9} + \frac{x^2}{2} - \frac{x}{3} + C_2$

s) $\int (1 + \sqrt[3]{x^2})\,dx \quad = x + \frac{3}{5}x^{\frac{5}{3}} + C$

t) $\int \frac{\sqrt{x} - x^3 + 2x}{x^2}\,dx$
$= \int \left(x^{\frac{-3}{2}} - x + \frac{2}{x}\right)dx = \frac{-2}{\sqrt{x}} - \frac{x^2}{2} + 2$

u) $\int (2x-4)(3x+1)\,dx$
$= \int (6x^2 - 10x - 4)\,dx = 2x^3 - 5x^2 - 4x + C$

v) $\int (2^x + 3^x)\,dx \quad = \frac{2^x}{\ln 2} + \frac{2^3}{\ln 3} + C$

w) $\int \frac{2x}{x^2+1}\,dx \quad = \ln|x^2+1| + C$

x) $\int \frac{6x+6}{x^2+2x+7}\,dx$
$= 3\int \frac{2x+2}{x^2+2x+7}\,dx = \ln|x^2+2x+7| + C$

Ex 2 Obtén a integral indefinida en cada caso:

a) $\int \frac{(\ln x)^3}{x}\,dx \quad \Rightarrow F(x) = \frac{(\ln x)^4}{4} + C$

b) $\int \left(\frac{3-5x}{2}\right)^3 dx$
$\Rightarrow F(x) = \frac{-1}{10}\left(\frac{3-5x}{2}\right)^4 + C$

c) $\int \frac{x-1}{x^2-2x+1}\,dx$
$\Rightarrow F(x) = \frac{1}{2}\ln|x^2-2x+1| + C$

d) $\int \frac{e^x}{1+e^x}\,dx \quad \Rightarrow F(x) = \ln(e^x+1) + C$

e) $\int 7^{2x-5}\,dx \quad \Rightarrow F(x) = \frac{7^{2x-5}}{2\ln 7}$

f) $\int \sqrt{3}x + 1\,dx \quad \Rightarrow F(x) = \frac{\sqrt{3}}{2}\cdot x^2 + C$

g) $\int \frac{x^2+2}{x^3+6x+1}\,dx$
$\Rightarrow F(x) = \frac{1}{3}\ln|x^3+6x+1| + C$

h) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \quad \Rightarrow F(x) = -2e^{\sqrt{x}} + C$

i) $\int \frac{t+1}{\sqrt{t^2+2t+3}}\,dt \quad \Rightarrow F(x) = \sqrt{t^2+2t+3} + C$

j) $\int (x^2+1)^{20}\,dx$: É inhumana: mira a solución nesta ligazón

k) $\int \frac{3}{s+1}\,ds$
$\Rightarrow F(x) = 3\ln|s+1| + C$

l) $\int \frac{x}{(x^2+3)^5}\,dx \quad \Rightarrow F(x) = \frac{-1}{8}\cdot(x^2+3)^{-4} + c$

m) $\int 5^{x^3-6x+1}\cdot(x^2-2)\,dx$
$\Rightarrow F(x) = \frac{1}{3\ln 5}5^{x^3-6x+1} + C$

n) Función $f(x)$ tal que $f'(x) = \frac{1}{\sqrt{x}} - 2x + e^x + 1$ e $f(0) = 2$
$\Rightarrow f(x) = 2\sqrt{x} - x^2 + e^x + x + 1;$ pois $C = 1$