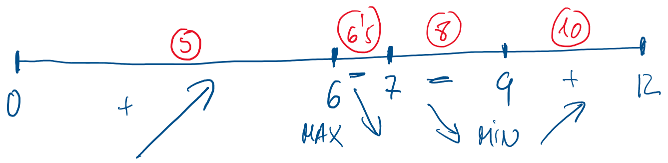
Soluciones simulacro segundo examen

martes, 21 de noviembre de 2023 21:12

(1) $V(t)=\begin{cases}12t-t^2 & \text{si} \quad 0\le t\le 7\\ t^2-18t+112 & \text{si} \quad 7<t\le 12\end{cases} \rightarrow V'(t)=\begin{cases}12-2t & \text{si} \quad 0\le t<7\\ 2t-18 & \text{si} \quad 7<t\le 12\end{cases}$

a) $12-2t=0$ $\qquad\qquad$ $2t-18=0$

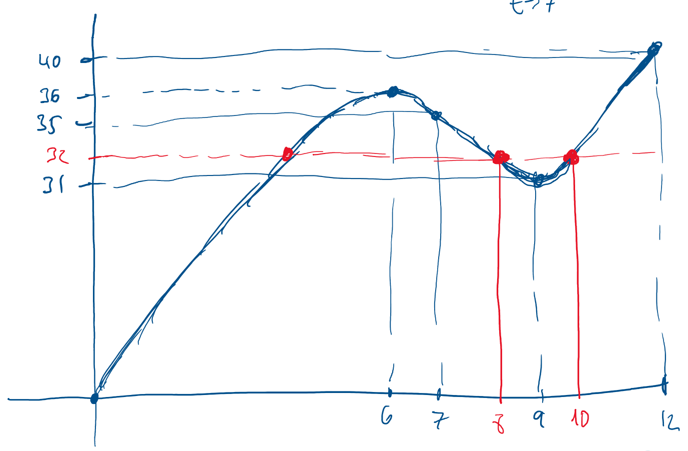
$t=6$ ← Posible máximo o mínimo ⟶ $t=9$



$\boxed{V(6)=12\cdot 6-6^2=36}$

En los 7 primeros meses el máximo se alcanza en el 6º mes y venden 36000 unidades.

$V(0)=0 \qquad V(7)=35 \qquad \lim_{t\to 7^+}(t^2-18t+112)=35 \qquad V(9)=31 \qquad V(12)=40$



El máximo de ventas en todo el año lo alcanza en el mes 12 y es de 40000 unidades.

b) $t^2-18t+112=32 \Rightarrow t^2-18t+80=0 \begin{cases} t=8\\ t=10\end{cases}$

El nº de ventas fue menor o igual a 32000 unidades desde el mes 8 al mes 10.

(2) $B(x)=ax^3-3x^2+bx \qquad 0\le x\le 7 \qquad\qquad B'(x)=3ax^2+6x+b$

$B(2)=8 \rightarrow \boxed{8a-12+2b=8} \qquad\qquad \boxed{12a+12+b=0}$

$B'(2)=0$

$b=-12a-12$

$8a-12+2(-12a-12)=8$

$-44 \quad -11$

$b(2)=0$

$$8a-12+2(-12a-12)=8$$

$$8a-12-24a-24=8 \rightarrow -16a=8+36 \Rightarrow \boxed{a=\frac{-44}{16}=\frac{-11}{4}}$$

$$b=-12\left(\frac{-11}{4}\right)-12=33-12 \Rightarrow \boxed{b=21}$$

3) $f(x)=-x^3+6x^2$ $\quad f'(x)=-3x^2+12x=0$

$x(-3x+12)=0$ $\quad x=0$, $x=4$

$-1 \quad 1 \quad 5$

0 MAX $\quad 4$ MIN

$x=0 \quad f(0)=0 \Rightarrow (0,0)$ MAX.

$x=4 \quad f(4)=-64+96=32 \Rightarrow (4,32)$ MIN

$$f'(x)=9=-3x^2+12x \Rightarrow 3x^2-12x+9=0 \Rightarrow x^2-4x+3=0 \quad x=1,\ x=3$$

4)
$$f(x)=\begin{cases}-x^2+6x+a & \text{si } 0\le x\le 6\\ 50 & \text{si } 6<x\le 8\\ 50+(x-8)(x-12) & \text{si } 8<x\le 12\end{cases}
\qquad
f'(x)=\begin{cases}-2x+6 & \text{si } 0\le x<6\\ 0 & \text{si } 6<x<8\\ 2x-20 & \text{si } 8<x\le 12\end{cases}$$

a) $f(0)=50 \Rightarrow 0^2+0+a=50 \Rightarrow \boxed{a=50}$

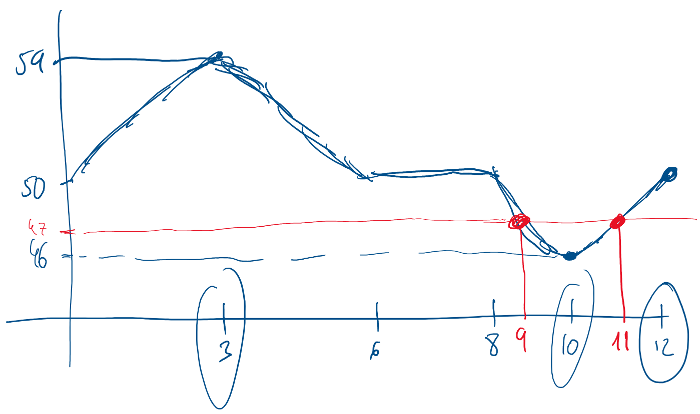
b) $-2x+6=0$ $\quad 2x-20=0$

$x=3$ $\quad x=10$

$2 \quad 4 \quad 7 \quad 9$

$0 \quad 3 \quad 6 \quad 8 \quad 10 \quad 12$

MAX $\quad$ MIN



$f(3)=59$

$f(10)=46$

$f(12)=50$

$$50+(x-8)(x-12)=47 \rightarrow (x-8)(x-12)=-3 \Rightarrow x^2-12x-8x+96=-3 \Rightarrow x^2-20x+99=0$$

$$x = \frac{20 \pm \sqrt{400 - 4\cdot 1\cdot 99}}{2} = \frac{20 \pm \sqrt{4}}{2} = \frac{20 \pm 2}{2} = \begin{cases} 11 \\ 9 \end{cases}$$

Tuvieron pérdidas entre los meses 9 y 11

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$$f(x) = \begin{cases} x^2 + ax + b & \text{si} \quad x < 1 \\ cx & \text{si} \quad x \geq 1 \end{cases} \qquad f'(x) = \begin{cases} 2x + a & \text{si} \quad x < 1 \\ c & \text{si} \quad x \geq 1 \end{cases}$$

$$\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2 + ax + b) = 1 + a + b \\ \lim_{x \to 1^+} cx = c \end{array}\right\} \boxed{1 + a + b = c}$$

$$\left.\begin{array}{l} \lim_{x \to 1^-} (2x + a) = 2 + a \\ \lim_{x \to 1^+} c = c \end{array}\right\} \boxed{2 + a = c}$$

$$f(0) = f(4) \qquad \left.\begin{array}{l} f(0) = b \\ f(4) = 4c \end{array}\right\} \boxed{b = 4c} \qquad \boxed{b = 1}$$

$$1 + a + 4c = c$$

$$a = -1 - 3c$$

$$\boxed{a = -1 - \frac{3}{4} = \frac{-7}{4}}$$

$$2 - 1 - 3c = c$$

$$1 = 4c \Rightarrow \boxed{c = 1/4}$$

6 a) $\lim_{x \to \infty} \sqrt{4x^2 - 3x + 7} - 2x = \infty - \infty$ Indet.

$$\lim_{x \to \infty} \frac{\left(\sqrt{4x^2 - 3x + 7} - 2x\right)\left(\sqrt{4x^2 - 3x + 7} + 2x\right)}{\sqrt{4x^2 - 3x + 7} + 2x} = \lim_{x \to \infty} \frac{\left(\sqrt{4x^2 - 3x + 7}\right)^2 - (2x)^2}{\sqrt{4x^2 - 3x + 7} + 2x} = \lim_{x \to \infty} \frac{4x^2 - 3x + 7 - 4x^2}{\sqrt{4x^2 - 3x + 7} + 2x} =$$

$$= \lim_{x \to \infty} \frac{-3x + 7}{\sqrt{4x^2 - 3x + 7} + 2x} = \frac{-3}{\sqrt{4} + 2} = \boxed{\frac{-3}{4}}$$

b) $\lim_{x \to \infty} \left(\frac{x}{x-1}\right)^{\frac{x}{2}} = 1^{\infty}$ $\qquad \lim_{x \to \infty} \left(1 + \frac{1}{f(x)}\right)^{f(x)}$

$$\lim_{x \to \infty} \left(1 + \frac{x}{x-1} - 1\right)^{\frac{x}{2}} = \lim_{x \to \infty} \left(1 + \frac{x - x + 1}{x - 1}\right)^{\frac{x}{2}} = \lim_{x \to \infty} \left(1 + \frac{1}{x-1}\right)^{\frac{x}{2}} =$$

$$\left(1 + \frac{1}{\ }\right)^{x-1 \cdot \frac{1}{x-1} \cdot \frac{x}{2}} \qquad \lim_{x \to \infty} \frac{x}{2x - 2} \qquad \frac{1}{2}$$

$$= \lim_{x\to\infty} \left(1 + \frac{1}{x-1}\right)^{x-1 \cdot \frac{1}{x-1} \cdot \frac{x}{2}} = e^{\lim_{x\to\infty} \frac{x}{2x-2}} = e^{1/2}$$