

Boletín de rectas tangentes (soluciones)

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1 a) $y = \frac{1-3x^2}{2}$ en $x=1$ $f(1) = \frac{1-3}{2} = -1$ \Rightarrow $y+1 = -3(x-1)$
 $f'(x) = -3x$ $f'(1) = -3$

b) $f(x) = \sqrt{x+12}$ en $x=-3$ $f(-3) = 3$ \Rightarrow $y-3 = \frac{1}{6}(x+3)$
 $f'(x) = \frac{1}{2\sqrt{x+12}}$ $f'(-3) = \frac{1}{6}$

c) $f(x) = \frac{1}{x}$ en $x=2$ $f(2) = \frac{1}{2}$ \Rightarrow $y - \frac{1}{2} = -\frac{1}{4}(x-2)$
 $f'(x) = -\frac{1}{x^2} \Rightarrow f'(2) = -\frac{1}{4}$

d) $y = x \ln x$ en $x=e$ $f(e) = e$ \Rightarrow $y-e = 2(x-e)$
 $f'(x) = \ln x + 1 \Rightarrow f'(e) = 2$

e) $y = \frac{x+5}{x-5}$ en $x=3$ $f(3) = \frac{8}{-2} = -4$ \Rightarrow $y+4 = -\frac{5}{2}(x-3)$
 $f'(x) = \frac{-10}{(x-5)^2}$ $f'(3) = \frac{-10}{4} = -\frac{5}{2}$

f) $y = \ln(x+1)$ en $x=0$ $f(0) = 0$ \Rightarrow $y-0 = 1(x-0)$
 $y' = \frac{1}{x+1} \Rightarrow f'(0) = 1$

2) $y = x^2 + 4x + 1$ paralela a $4x - 2y + 5 = 0 \Rightarrow y = 2x + \frac{5}{2} \Rightarrow$ la pendiente es 2 $\Rightarrow f'(a) = 2$
 $f'(x) = 2x + 4 \Rightarrow 2x + 4 = 2 \Rightarrow x = -1 \Rightarrow a = -1$
 $f(-1) = 1 - 4 + 1 = -2 \Rightarrow y + 2 = 2(x + 1)$

3) $y = \frac{2x}{x-1}$ paralela a $2x + y = 0 \Rightarrow y = -2x \Rightarrow$ la pendiente es -2 $\Rightarrow f'(a) = -2$
 $f'(x) = \frac{-2}{(x-1)^2} \Rightarrow \frac{-2}{(x-1)^2} = -2 \Rightarrow \frac{-2}{-2} = (x-1)^2 \Rightarrow (x-1)^2 = 1 \Rightarrow$
 $\left. \begin{array}{l} x-1 = -1 \Rightarrow x=0 \Rightarrow a=0 \\ x-1 = 1 \Rightarrow x=2 \Rightarrow a=2 \end{array} \right\}$

$\left. \begin{array}{l} a=0 \\ f'(a) = -2 \\ f(a) = 0 \end{array} \right\} \Rightarrow y - 0 = -2(x - 0)$
 $\left. \begin{array}{l} a=2 \\ f'(a) = -2 \\ f(a) = 4 \end{array} \right\} \Rightarrow y - 4 = -2(x - 2)$

$$\left. \begin{array}{l} f'(a) = -2 \\ f(a) = 0 \end{array} \right\} \Rightarrow |y - 0 = -2(x - 0)|$$

$$\left. \begin{array}{l} f(a) = -2 \\ f(a) = 4 \end{array} \right\} | \dots \dots \dots |$$

④ $y = 4x - x^2 \quad y' = 4 - 2x$

Pts corte ejes

$$\begin{array}{l} 0 = 4x - x^2 \\ 0 = x(4 - x) \end{array} \left\{ \begin{array}{l} x = 0 \\ x = 4 \end{array} \right.$$

$$\left. \begin{array}{l} a = 0 \\ f(a) = 0 \\ f'(a) = 4 \end{array} \right\} \Rightarrow |y - 0 = 4(x - 0)|$$

$$\left. \begin{array}{l} a = 4 \\ f(a) = 0 \\ f'(a) = -4 \end{array} \right\} \Rightarrow |y - 0 = -4(x - 4)|$$

⑤ Si la tg es horizontal $\Rightarrow f'(a) = 0$

$$f(x) = \frac{6x}{x^2 + 1} \quad f'(x) = \frac{-6x^2 + 6}{(x^2 + 1)^2}$$

$$\frac{-6x^2 + 6}{(x^2 + 1)^2} = 0 \Rightarrow -6x^2 + 6 = 0 \Rightarrow x^2 = 1 \Rightarrow |x = \pm 1|$$

$$\left. \begin{array}{l} a = 1 \\ f(a) = 3 \\ f'(a) = 0 \end{array} \right\} \left\{ \begin{array}{l} y - 3 = 0(x - 1) \\ y = 3 \end{array} \right.$$

$$\left. \begin{array}{l} a = -1 \\ f(a) = -3 \\ f'(a) = 0 \end{array} \right\} \left\{ \begin{array}{l} y + 3 = 0(x + 1) \\ y = -3 \end{array} \right.$$