BOLETÍN DE REPASO SEMEJANZA Y TRIGONOMETRÍA

1) Escala 1:150

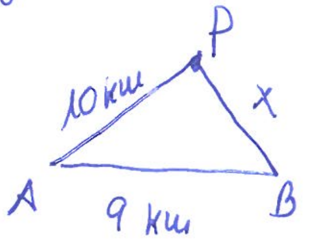
a) 25 cm x 52 cm

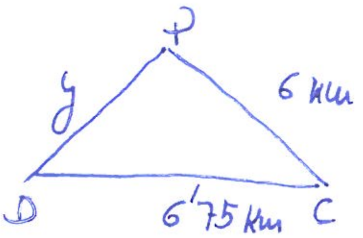
$25 \times 150 = 3750 \text{ cm} = 3'75 \text{ m}$
$52 \times 150 = 7800 \text{ cm} = 7'8 \text{ m}$

b) $405 \text{ m}^3 = 405000 \text{ dm}^3 = 405000 \text{ l}$.

$405.000 : 150^3 = 0'12 \text{ l} = 0'12 \text{ dm}^3 = 120 \text{ cm}^3$

2) Los triángulos son semejantes por tener un ángulo opuesto por el vértice y por alternos internos entre paralelas

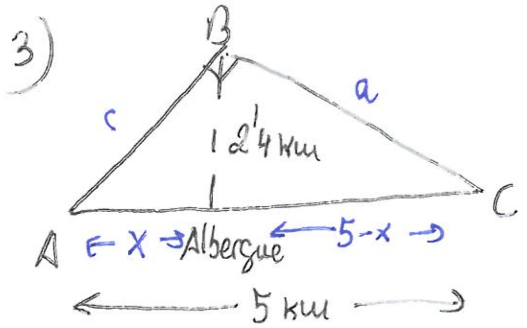




$\frac{10}{9} = \frac{y}{6'75} \rightarrow y = 7'5 \text{ km}$

$\frac{x}{9} = \frac{6}{6'75} \rightarrow x = 8 \text{ km}$

3)



Teorema de la altura $h^2 = m \cdot n$

$2'4^2 = x \cdot (5-x)$

$5'76 = 5x - x^2 \rightarrow x^2 - 5x + 5'76 = 0$

$x = \frac{5 \pm \sqrt{25 - 4 \cdot 5'76}}{2} = \frac{5 \pm 1'4}{2} = \begin{cases} 3'2 \\ 1'8 \end{cases}$

Podemos tomar como x cualquiera de los dos valores, el otro será 5-x

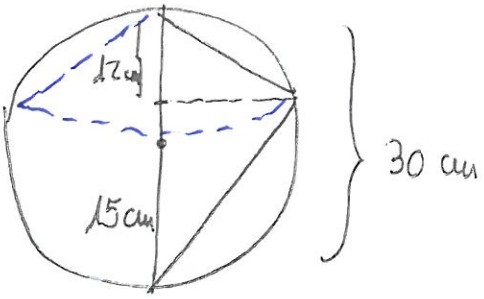
Por ejemplo $x = 1'8$, $5 - x = 3'2$

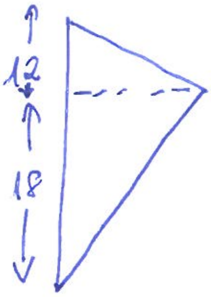
Por el teorema del cateto $c^2 = 5 \cdot 1'8 \rightarrow c = 3$

Para hallar a podemos repetir el teorema del cateto o hacer Pitágoras (3, 4, 5 es una terna pitagórica) $a^2 = 5 \cdot 3'2 \rightarrow a = 4$

LONGITUD CIRCUITO $3 + 4 + 5 = 12 \text{ km}$

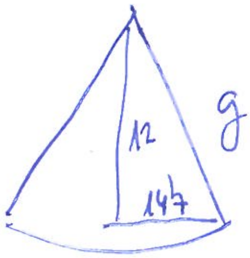
4)





Calculamos la altura del triángulo que es el radio del cono

$R^2 = 12 \cdot 18 \to R \simeq 14'7$



$g = \sqrt{12^2 + 14'7^2} \simeq 19$ cm $\Rightarrow A_{LAT} = \pi R g =$
$= 279'3\pi$ cm^3

5)

Distancia mapa 7'5 cm

Distancia real 153 Km = 15.300.000 cm

MAPA	REALIDAD
7'5	15.300.000
1	x

$x = \dfrac{15300000}{7'5} = 2.040.000$

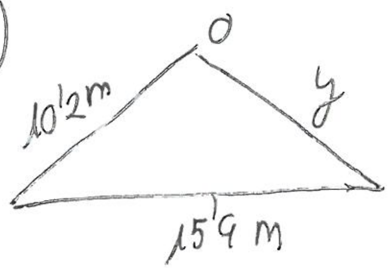
$\dfrac{\text{Distancia real}}{\text{Distancia mapa}}$

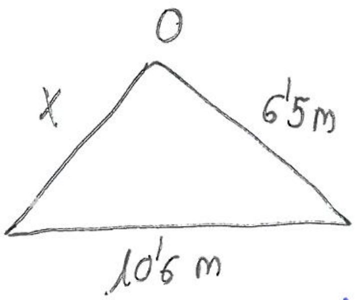
1: 2.040.000

Superficie mapa 12 cm^2

Superficie real : $12 \times 2.040.000^2 = 5 \times 10^{13}$ cm^2 = 5000 km^2

6)





Son semejantes

$\dfrac{10'2}{x} = \dfrac{15'9}{10'6}$

$x = 6'8$ m

$\dfrac{y}{6'5} = \dfrac{15'9}{10'6}$

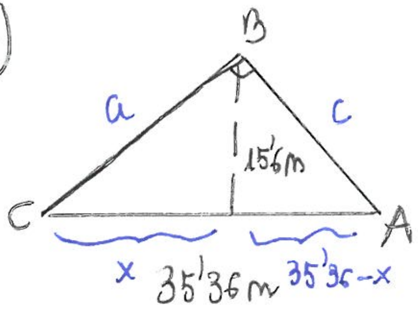
$y = 9'75$ m

Longitud puentes

$10'2 + x = 17$ m

$6'5 + y = 16'25$ m

7)



Precio cable : 0'3 €/m

Teorema de la Altura $\quad 15'6^2 = x\cdot(35'36 - x)$

$$243'36 = 35'36x - x^2 \rightarrow x^2 - 35'36x + 243'36 = 0$$

$$x = \frac{35'36 \pm \sqrt{35'36^2 - 4\cdot 243'36}}{2} = \frac{35'36 \pm 16'64}{2} = \begin{cases} 26 \\ 9'36 \end{cases}$$

Por ejemplo:

$$\begin{cases} x = 9'36 \\ 35'36 - x = 26 \end{cases}$$

Por el Teorema del cateto:

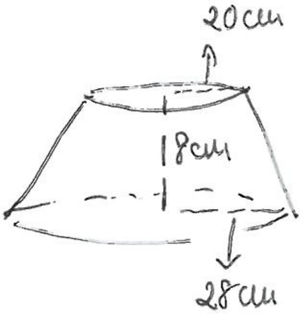
$$a^2 = 35'36 \cdot 9'36 \rightarrow a \simeq 18'2 \text{ m}$$

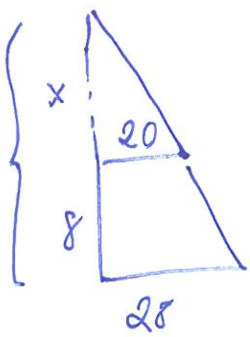
$$c^2 = 35'36 \cdot 26 \rightarrow c \simeq 30'3 \text{ m}$$

Longitud cable: $35'36 + 18'2 + 30'3 = 83'86$ m

Coste: $0'3 \times 83'86 = 25'16$ €

8)





$$\frac{x}{20} = \frac{x+8}{28}$$

$$28x = 20x + 160$$

$$8x = 160 \rightarrow x = 20 \text{ cm}$$

$$V_{\text{TRONCO}} = V_{\substack{\text{CONO}\\\text{GRANDE}}} - V_{\substack{\text{CONO}\\\text{PEQUEÑO}}} = \frac{\pi\cdot 28^2\cdot 28}{3} - \frac{\pi\cdot 20^2\cdot 20}{3} = \frac{\pi\cdot 28^3 - \pi\cdot 20^3}{3} = \frac{\pi(28^3 - 20^3)}{3} =$$

$$= \frac{13952\pi}{3} \text{ cm}^3$$

VOLUMEN CONO $= \frac{\pi r^2 h}{3}$

TRIGONOMETRÍA

1)

sen α	0'8	0'8
cos α	0'6	0'6
tg α	0'75	4/3

$$\begin{cases} sen^2\alpha + cos^2\alpha = 1 \\ tg\alpha = \dfrac{sen\alpha}{cos\alpha} \end{cases}$$

$$0'8^2 + cos^2\alpha = 1 \rightarrow cos^2\alpha = 0'36 \rightarrow cos\alpha = 0'6$$

$$tg\alpha = \frac{0'8}{0'6} = \frac{8}{6} = \frac{4}{3}$$

$tg\alpha = 0'75$

$$\frac{sen\alpha}{cos\alpha} = 0'75 \rightarrow sen\alpha = 0'75\, cos\alpha \rightarrow 0'75^2 (cos\alpha)^2 + cos^2\alpha = 1$$

$$0'56 (cos\alpha)^2 + (cos\alpha)^2 = 1$$

$$1'56\,(cos\alpha)^2 = 1$$

$$(cos\alpha)^2 = \frac{1}{1'56} = 0'64 \rightarrow cos\alpha = 0'8$$

$$sen\alpha = 0'75 \cdot 0'8 = 0'6$$

2) $cos\,\alpha = -\dfrac{\sqrt{5}}{5}$, $\alpha \in \text{III}$

$$sen^2\alpha + \left(-\frac{\sqrt{5}}{5}\right)^2 = 1 \rightarrow sen^2\alpha + \overset{1/5}{\frac{5}{25}} = 1 \rightarrow sen^2\alpha = 1 - \frac{1}{5}$$

$$sen^2\alpha = \frac{4}{5}$$

$$sen\alpha = -\sqrt{\frac{4}{5}} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

$$tg\alpha = \left(\frac{\frac{-2\sqrt{5}}{5}}{\frac{-\sqrt{5}}{5}}\right) = \frac{-\cancel{5} \cdot 2\cancel{\sqrt{5}}}{-\cancel{5}\cancel{\sqrt{5}}} = 2$$

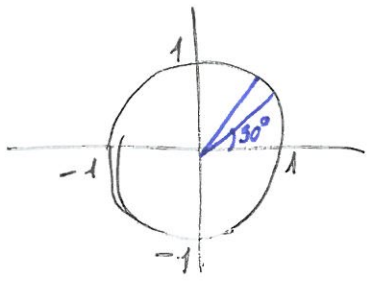
3) $cos\,\alpha = \dfrac{\sqrt{2}}{3}$, $\alpha \in \text{IV}$

$$sen^2\alpha + \left(\frac{\sqrt{2}}{3}\right)^2 = 1 \rightarrow sen^2\alpha + \frac{2}{9} = 1 \rightarrow sen^2\alpha = \frac{7}{9} \rightarrow sen\alpha = \frac{-\sqrt{7}}{3}$$

$$tg\alpha = \frac{-\sqrt{7}/\cancel{3}}{\sqrt{2}/\cancel{3}} = -\sqrt{7/2}$$

4)

α	0°	30°	45°	90°
sen α	0	1/2	√2/2	1
cos α	1	√3/2	√2/2	0
tg α	0	√3/3	1	∄



5) $\left.\begin{array}{l} tg\alpha = 4/3 \\ \alpha < 90^\circ \end{array}\right\}$

$\frac{sen\alpha}{cos\alpha} = \frac{4}{3} \rightarrow sen\alpha = \frac{4}{3} cos\alpha$

$\left(\frac{4}{3} cos\alpha\right)^2 + cos^2\alpha = 1$

$\frac{16}{9}(cos\alpha)^2 + cos^2\alpha = 1 \rightarrow \frac{25}{9}(cos\alpha)^2 = 1$

$cos^2\alpha = \frac{9}{25} \rightarrow cos\alpha = 3/5$ //

$sen\alpha = \frac{4}{3} \cdot \frac{3}{5} = \frac{4}{5}$ //

6) $\left.\begin{array}{l} tg\alpha = \frac{3}{4} \\ \alpha \in III \end{array}\right\}$

$\frac{sen\alpha}{cos\alpha} = \frac{3}{4} \rightarrow sen\alpha = \frac{3}{4} cos\alpha$

$\left(\frac{3}{4} cos\alpha\right)^2 + cos^2\alpha = 1 \rightarrow \frac{9}{16}(cos\alpha)^2 + (cos\alpha)^2 = 1$

$\frac{25}{16}(cos\alpha)^2 = 1 \rightarrow cos^2\alpha = \frac{16}{25}$

$cos\alpha = -4/5$ //

$sen\alpha = \frac{3}{4} \cdot \left(-\frac{4}{5}\right) = -3/5$ //

7) $\left.\begin{array}{l} tg\alpha = -\sqrt{5} \\ \alpha \in II \end{array}\right\}$

$\frac{sen\alpha}{cos\alpha} = -\sqrt{5} \rightarrow sen\alpha = -\sqrt{5}\, cos\alpha$

$(-\sqrt{5}\, cos\alpha)^2 + (cos\alpha)^2 = 1 \rightarrow 5(cos\alpha)^2 + (cos\alpha)^2 = 1$

$6(cos\alpha)^2 = 1 \rightarrow cos\alpha = -\sqrt{\frac{1}{6}} = \frac{-\sqrt{6}}{6}$ //

$sen\alpha = -\sqrt{5} \cdot \left(\frac{-\sqrt{6}}{6}\right) = \frac{\sqrt{30}}{6}$ //

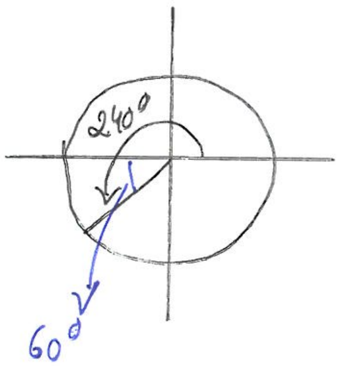
8) $\left.\begin{array}{l} sen\alpha = \frac{\sqrt{5}}{3} \\ \alpha \in II \end{array}\right\}$

$\left(\frac{\sqrt{5}}{3}\right)^2 + cos^2\alpha = 1 \rightarrow \frac{5}{9} + cos^2\alpha = 1 \rightarrow cos^2\alpha = \frac{4}{9}$

$cos\alpha = -\frac{2}{3}$ //

$tg\alpha = \frac{\sqrt{5}/\cancel{3}}{-2/\cancel{3}} = -\frac{\sqrt{5}}{2}$ //

9)

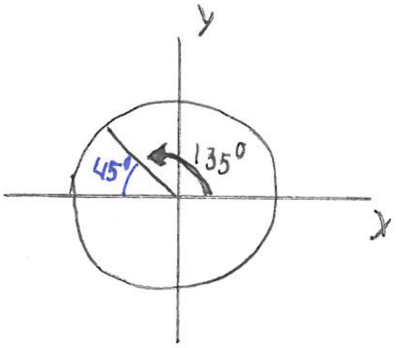


$sen 240^\circ = -sen 60^\circ = -\sqrt{3}/2$

$cos 240^\circ = -cos 60^\circ = -1/2$

$tg 240^\circ = tg 60^\circ = \sqrt{3}$

10)



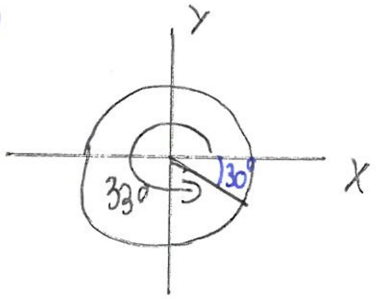
$\text{sen}\,135^\circ = \text{sen}\,45^\circ = \sqrt{2}/2$

$\cos 135^\circ = -\cos 45^\circ = -\sqrt{2}/2$

$\text{tg}\,135^\circ = -\text{tg}\,45^\circ = -1$

11) $2130^\circ = 330^\circ = -30^\circ$

$$\begin{array}{l|l} 2130 & 360 \\ \hline 330 & 5 \end{array}$$

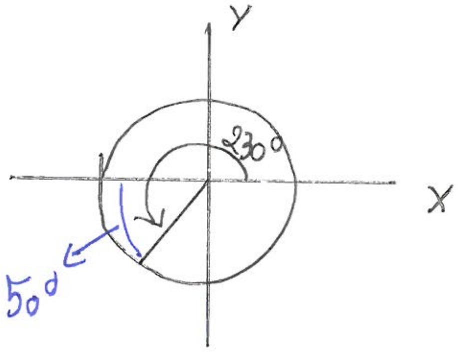


$\cos 2130^\circ = \cos 30^\circ = \sqrt{3}/2$

$\text{sen}\,2130^\circ = -\text{sen}\,30^\circ = -1/2$

$\text{tg}\,2130^\circ = -\text{tg}\,30^\circ = -\sqrt{3}/3$

12)

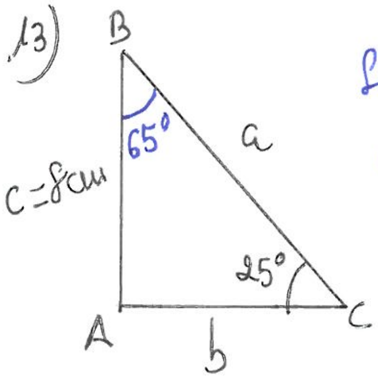


$\text{sen}\,230^\circ = -\text{sen}\,50^\circ = -0'57$

$\cos 230^\circ = -\cos 50^\circ = -0'64$

$\text{tg}\,230^\circ = \text{tg}\,50^\circ = 0'84$

13)



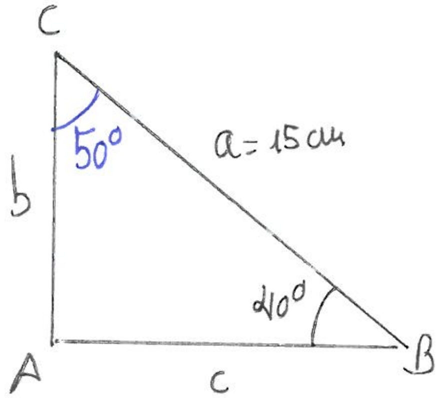
$\text{sen}\,25^\circ = \dfrac{8}{a}$

$a = \dfrac{8}{\text{sen}\,25^\circ}$

$a = 18'9$ cm.

$\text{tg}\,25^\circ = \dfrac{8}{b} \rightarrow b = \dfrac{8}{\text{tg}\,25^\circ} \simeq 17'2$ cm

$b \simeq 17'2$ cm



$\text{sen}\,40^\circ = \dfrac{b}{15} \rightarrow b = 15\,\text{sen}\,40^\circ \simeq 9'6$ cm

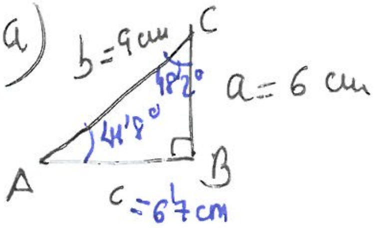
$b \simeq 9'6$ cm

$\cos 40^\circ = \dfrac{c}{15} \rightarrow c = 15 \cos 40^\circ \simeq 11'5$ cm

$c \simeq 11'5$ cm

14)

a)

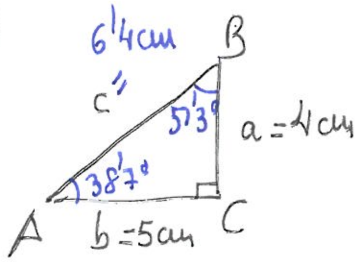


$c=\sqrt{9^2-6^2}=6'7$ cm

$\text{sen}\,\hat{A}=\frac{6}{9}\rightarrow \hat{A}=\text{arcsen}\,\frac{2}{3}=41'8^\circ$

$\hat{C}=90^\circ-41'8^\circ=48'2^\circ$

b)



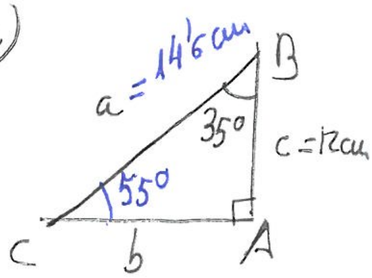
$c=\sqrt{4^2+5^2}=\sqrt{41}=6'4$ cm

$\text{tg}\,\hat{A}=\frac{4}{5}=0'8$

$\hat{A}=\text{arctg}\,0'8=38'7^\circ$

$\hat{B}=90^\circ-38'7^\circ=51'3^\circ$

c)



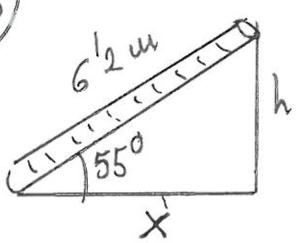
$\text{sen}\,55^\circ=\frac{12}{a}$

$a=\frac{12}{\text{sen}\,55^\circ}=14'6$ cm

$\text{tg}\,55^\circ=\frac{12}{b}$

$b=\frac{12}{\text{tg}\,55^\circ}=8'4$ cm

15)



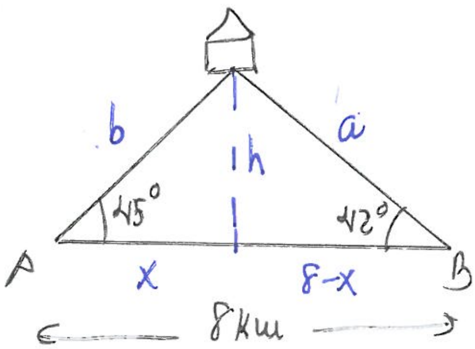
a) $\text{sen}\,55^\circ=\frac{h}{6'2}$

$h=6'2\cdot\text{sen}\,55^\circ\simeq 5'1$ m

b) $\cos 55^\circ=\frac{x}{6'2}$

$x=6'2\cdot\cos 55^\circ\simeq 3'6$ m

16)



$\text{tg}\,45^\circ=\frac{h}{x}\rightarrow h=x\cdot\text{tg}\,45^\circ$

$\text{tg}\,42^\circ=\frac{h}{8-x}\rightarrow h=(8-x)\,\text{tg}\,42^\circ$

(Se pueden sustituir las tangentes por su valor)

$x\,\text{tg}\,45^\circ=(8-x)\,\text{tg}\,42^\circ\rightarrow x\,\text{tg}\,45^\circ=8\cdot\text{tg}\,42^\circ-x\,\text{tg}\,42^\circ$

$x\,\text{tg}\,45^\circ+x\,\text{tg}\,42^\circ=8\,\text{tg}\,42^\circ$

$x(\text{tg}\,45^\circ+\text{tg}\,42^\circ)=8\cdot\text{tg}\,42^\circ$

$x=\frac{8\cdot\text{tg}\,42^\circ}{\text{tg}\,45^\circ+\text{tg}\,42^\circ}\simeq 3'8$ km

$h=x\,\text{tg}\,45^\circ=x=3'8$ km

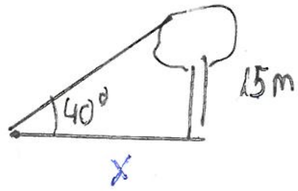
$\text{sen}\,45^\circ=\frac{h}{b}=\frac{3'8}{b}$

$b=\frac{3'8}{\text{sen}\,45^\circ}=\frac{3'8}{\frac{\sqrt{2}}{2}}=\frac{2\cdot 3'8}{\sqrt{2}}=3'8\sqrt{2}\simeq 5'4$ km

$\text{sen}\,42^\circ=\frac{h}{a}=\frac{3'8}{a}\rightarrow a=\frac{3'8}{\text{sen}\,42^\circ}\simeq 5'7$ km

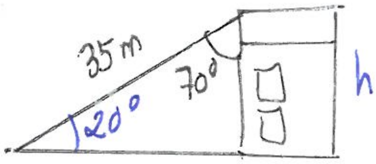
$b\simeq 5'4$ km ; $a=5'7$ km

17)



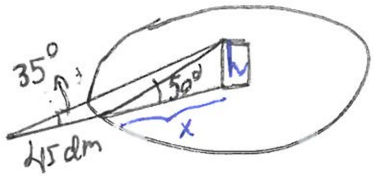
$tg\,40^\circ = \frac{15}{x} \rightarrow x = \frac{15}{tg\,40^\circ} = 17'9$ m.

18)



$sen\,20^\circ = \frac{h}{35} \rightarrow h = 35\,sen\,20^\circ \simeq 12$ m

19)



$$\left.\begin{array}{l} tg\,50^\circ = \frac{h}{x} \\ tg\,35^\circ = \frac{h}{x+45} \end{array}\right\} \begin{array}{l} h = x\,tg\,50^\circ \\ h = tg\,35^\circ(x+45) \end{array}$$

$x \cdot tg\,50^\circ = (x+45)\,tg\,35^\circ$ (Se puede sustituir por los valores de las tangentes)

$x\,tg\,50^\circ = x\,tg\,35^\circ + 45\,tg\,35^\circ$

$x\,tg\,50^\circ - x\,tg\,35^\circ = 45 \cdot tg\,35^\circ$

$x(tg\,50^\circ - tg\,35^\circ) = 45\,tg\,35^\circ$

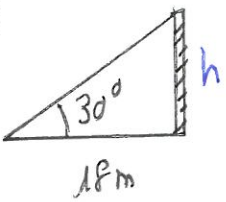
$x = \frac{45 \cdot tg\,35^\circ}{tg\,50^\circ - tg\,35^\circ} = 64'1$ dm $=$ Radio lago

$h = 64'1 \cdot tg\,50^\circ = 76'4$ dm

Superficie lago $= \pi r^2$

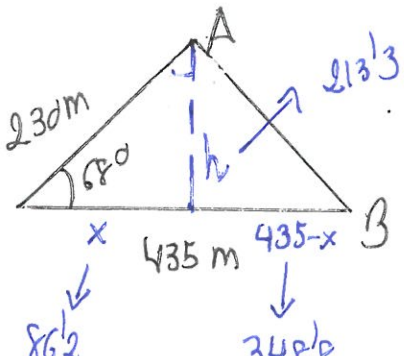
$A = \pi \cdot 64'1^2 = 4108'81\pi$ dm^2

20)



$h = 18 \cdot tg\,30^\circ = 18 \cdot \frac{\sqrt{3}}{3} = 6\sqrt{3}$ m

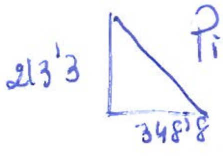
21)



$sen\,68^\circ = \frac{h}{230} \rightarrow h = 230 \cdot sen\,68^\circ = 213'3$ m

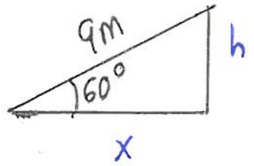
$cos\,68^\circ = \frac{x}{230} \rightarrow x = 230 \cdot cos\,68^\circ = 86'2$ m

$435 - x = 435 - 86'2 = 348'8$ m



Pitágoras $\rightarrow$ 408'9 m

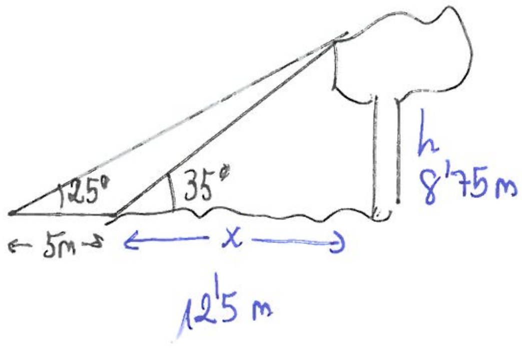
22)



$h = 9 \cdot \operatorname{sen} 60° = 9 \cdot \frac{\sqrt{3}}{2} = 4'5 \cdot \sqrt{3}$ m

$x = 9 \cdot \cos 60° = 9 \cdot \frac{1}{2} = 4'5$ m. . .

23)



$\left.\begin{array}{l} tg\,35° = \dfrac{h}{x} \\ tg\,25° = \dfrac{h}{x+5} \end{array}\right\}$ $\begin{array}{l} h = x\,tg\,35° \\ h = (x+5)\,tg\,25° \end{array}$

$x\,tg\,35° = (x+5) \cdot tg\,25°$

$0'7x = (x+5) \cdot 0'5$

$0'7x = 0'5x + 2'5$

$0'2x = 2'5 \rightarrow x = \frac{2'5}{0'2} = \frac{25}{2} = 12'5$ m

$h = x \cdot tg\,35° = 12'5 \cdot 0'7 = 8'75$ m

24) a) $240°$

$180° = \pi$ rad

$\left.\begin{array}{l} 180° — \pi \text{ rad} \\ 240° — x \end{array}\right\}$ $x = \frac{24\cancel{0}\,\pi}{18\cancel{0}} = \frac{4\pi}{3}$ rad.

b) $\frac{5\pi}{4}$ rad $= \frac{5 \cdot 180°}{4} = 225°$