RAMAS INFINITAS. ASÍNTOTAS

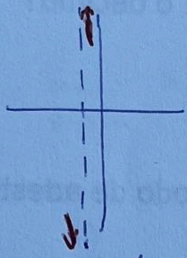
37.- Determina as asíntotas verticais e sitúa a curva respecto a elas:

a) $y=\dfrac{x^2+3x+11}{x+1}$

$x+1=0 \Rightarrow x=-1$

$\lim\limits_{x\to -1} \dfrac{x^2+3x+11}{x+1} = \dfrac{1-3+11}{0} = \dfrac{9}{0} = \pm\infty \Rightarrow$ $\boxed{\text{EN } x=-1 \text{ HAI A.V.}}$

$\lim\limits_{x\to -1^-} \dfrac{x^2+3x+11}{x+1} \overset{x=-1'01}{=} -\infty \quad \dfrac{\oplus}{\ominus}$; $\lim\limits_{x\to -1^+} \dfrac{x^2+3x+11}{x+1} \overset{x=-0'99}{=} +\infty \quad \dfrac{\oplus}{\oplus}$



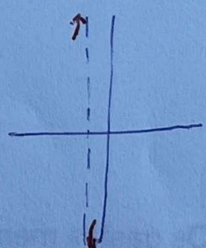
$x=-1$

b) $y=\dfrac{x^2+3x}{x+1}$

$x+1=0 \Rightarrow x=-1$

$\lim\limits_{x\to -1} \dfrac{x^2+3x}{x+1} = \dfrac{1-3}{0} = \dfrac{-2}{0} = \pm\infty \Rightarrow$ $\boxed{\text{EN } x=-1 \text{ HAI A.V.}}$

$\lim\limits_{x\to -1^-} \dfrac{x^2+3x}{x+1} \overset{x=-1'01}{=} +\infty \quad \dfrac{\ominus}{\ominus}$; $\lim\limits_{x\to -1^+} \dfrac{x^2+3x}{x+1} \overset{x=-0'99}{=} -\infty \quad \dfrac{\ominus}{\oplus}$



$x=-1$

38.- Determina as asíntotas e sitúa a curva respecto a elas:

a) $y=\dfrac{x^2+2}{x^2-2x}$

$x^2-2x=0 \Rightarrow x(x-2)=0 \;\; x=0,\; x=2$

$\boxed{\text{EN } y=1 \text{ A.H.}}$

A.V. $\lim\limits_{x\to 0} \dfrac{x^2+2}{x^2-2x} = \dfrac{2}{0} = \pm\infty \Rightarrow$ $\boxed{\text{EN } x=0 \text{ A.V.}}$ $\quad x(x-2)$

A.H. $\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^2-2x} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1$

$\lim\limits_{x\to 0^-} \dfrac{x^2+2}{x^2-2x} \overset{x=-0'01}{=} +\infty \quad \dfrac{\oplus}{\oplus}$; $\lim\limits_{x\to 0^+} \dfrac{x^2+2}{x^2-2x} \overset{x=+0'01}{=} -\infty \quad \dfrac{\oplus}{\ominus}$

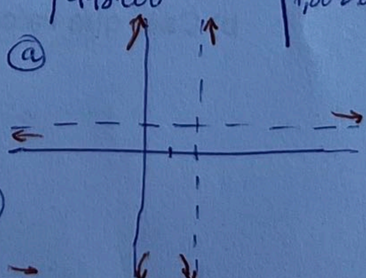
$\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^2-2x} = 1^+$; $\lim\limits_{x\to -\infty} \dfrac{x^2+2}{x^2-2x} = 1^-$

x	1000	-1000
f(x)	$\dfrac{1.000.002}{998.000} \simeq 1'002$	$\dfrac{1.000.002}{1.002.000} \simeq 0'9$

A.V. $\lim\limits_{x\to 2} \dfrac{x^2+2}{x^2-2x} = \dfrac{6}{0} = \pm\infty \Rightarrow$ $\boxed{\text{EN } x=2 \text{ A.V.}}$ $\quad x(x-2)$

$\lim\limits_{x\to 2^-} \dfrac{x^2+2}{x(x-2)} \overset{x=1'99}{=} -\infty \quad \dfrac{\oplus}{\oplus\ominus}$; $\lim\limits_{x\to 2^+} \dfrac{x^2+2}{x(x-2)} \overset{x=2'01}{=} +\infty \quad \dfrac{\oplus}{\oplus\oplus}$

(a)



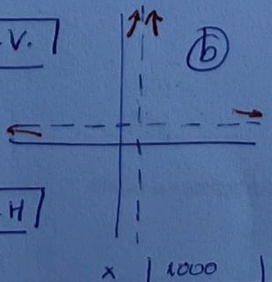
$x=0 \quad x=2$

b) $y=\dfrac{x^2+2}{x^2-2x+1}$

$x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x=1$

A.V. $\lim\limits_{x\to 1} \dfrac{x^2+2}{(x-1)^2} = \dfrac{3}{0} = \pm\infty \Rightarrow$ $\boxed{\text{EN } x=1 \text{ A.V.}}$

(b)



$\lim\limits_{x\to 1^-} \dfrac{x^2+2}{(x-1)^2} = +\infty \quad \dfrac{\oplus}{\oplus}$; $\lim\limits_{x\to 1^+} \dfrac{x^2+2}{(x-1)^2} = +\infty \quad \dfrac{\oplus}{\oplus}$

A.H. $\lim\limits_{x\to\infty} \dfrac{x^2+2}{(x-1)^2} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1 \Rightarrow$ $\boxed{y=1 \text{ A.H}}$

$\lim\limits_{x\to\infty} \dfrac{x^2+2}{(x-1)^2} = 1^+$; $\lim\limits_{x\to -\infty} \dfrac{x^2+2}{(x-1)^2} = 1^-$

x	1000	-1000
f(x)	$\dfrac{1000002}{999^2}$ [illegible]	$\dfrac{1000002}{(-1001)^2} = 0'998\ldots$

39.- Determina as ramas destas funcións. Sitúa a curva respecto da asíntota:

ASÍNTOTAS VERTICAIS

a) $y=\dfrac{x}{1+x^2}$

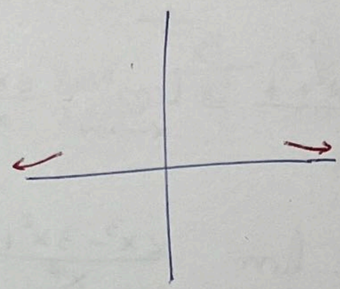
$1+x^2=0 \Rightarrow x^2=-1$ NON SOLUCIÓN REAL $\Rightarrow \boxed{\text{NON HAI A.V.}}$

A.H.

$\lim\limits_{x\to\infty} \dfrac{x}{1+x^2} = \lim\limits_{x\to\infty} \dfrac{x}{x^2} = \lim\limits_{x\to\infty} \dfrac{1}{x} = \dfrac{1}{\infty} = 0$

$\Rightarrow \boxed{\text{EN } y=0 \text{ A.H.}}$

$\lim\limits_{x\to\infty} \dfrac{x}{1+x^2} = 0^+$



$\lim\limits_{x\to-\infty} \dfrac{x}{1+x^2} = 0^-$

x	1000	-1000
$f(x)$	$\frac{1000}{1.000.001} \simeq 0'009...$	$\frac{-1000}{1000.001} \simeq -0'009...$

b) $y=\dfrac{x^3}{1+x^2}$

$1+x^2=0 \Rightarrow x^2=-1$ NON SOLUCIÓN REAL $\Rightarrow \boxed{\text{NON HAI A.V.}}$

A.H.

$\lim\limits_{x\to\infty} \dfrac{x^3}{1+x^2} = \lim\limits_{x\to\infty} \dfrac{x^3}{x^2} = \lim\limits_{x\to\infty} x = \infty \Rightarrow$ NON HAI A.H.

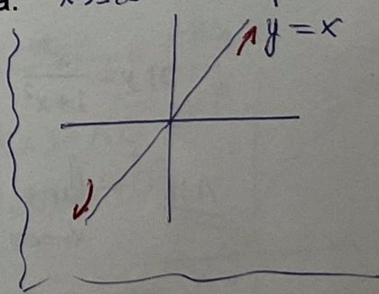
A.O.

$\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x^3}{x+x^3} = \lim\limits_{x\to\infty} \dfrac{x^3}{x^3} = 1 = m \neq 0 \Rightarrow$ HAI A.O. e $m=1$

$n = \lim\limits_{x\to\infty} (f(x)-mx) = \lim\limits_{x\to\infty} \left(\dfrac{x^3}{1+x^2} - 1\cdot x\right) = \lim\limits_{x\to\infty} \dfrac{\cancel{x^3}-x-\cancel{x^3}}{1+x^2} = \lim\limits_{x\to\infty} \dfrac{-x}{x^2} = \lim\limits_{x\to\infty} \dfrac{-1}{x} = 0$

$\boxed{y=x \text{ é A.O.}}$

$f(x)-AO = \dfrac{x^3}{1+x^2} - x = \dfrac{-x}{1+x^2}$ $\xrightarrow{x\to\infty} 0^- \Rightarrow f(x) < AO$; $\xrightarrow{x\to-\infty} 0^+ \Rightarrow f(x) > AO$



40.- Determina as ramas destas funcións. Sitúa a curva respecto da asíntota:

a) $y=\dfrac{x^2+2}{x^2-2x}$

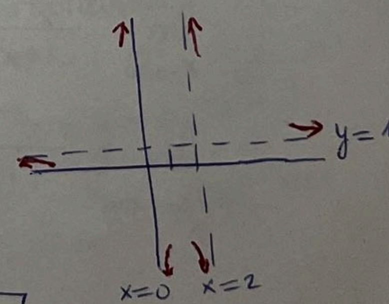
A.V. $x^2-2x=0 \Rightarrow x(x-2)=0 \rightarrow x=0$; $x=2$

$\lim\limits_{x\to 0} \dfrac{x^2+2}{x^2-2x} = \dfrac{2}{0} = \pm\infty$; $\lim\limits_{x\to 0^-} \dfrac{x^2+2}{x^2-2x} = +\infty$; $\lim\limits_{x\to 0^+} \dfrac{x^2+2}{x^2-2x} = -\infty$

$\boxed{\text{EN } x=0 \text{ HAI A.V.}}$

$\lim\limits_{x\to 2} \dfrac{x^2+2}{x^2-2x} = \dfrac{6}{0} = \pm\infty$; $\lim\limits_{x\to 2^-} \dfrac{x^2+2}{x^2-2x} = -\infty$; $\lim\limits_{x\to 2^+} \dfrac{x^2+2}{x^2-2x} = +\infty$

$\boxed{\text{EN } x=2 \text{ HAI A.V.}}$



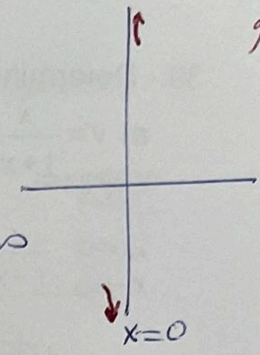
A.H. $\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^2-2x} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1 \Rightarrow \boxed{\text{EN } y=1 \text{ A.H.}}$

$\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^2-2x} = 1^+$

$\lim\limits_{x\to-\infty} \dfrac{x^2+2}{x^2-2x} = 1^-$

x	1000	-1000
$f(x)$	1'002...	0'998...

b) $y=\dfrac{2x^3-3x^2+7}{x}$



A.V. $x=0$ $\lim\limits_{x\to 0} \dfrac{2x^3-3x^2+7}{x} = \dfrac{7}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=0 \text{ A.V.}}$

$\lim\limits_{x\to 0^-} \dfrac{2x^3-3x^2+7}{x} = -\infty$; $\lim\limits_{x\to 0^+} \dfrac{2x^3-3x^2+7}{x} = +\infty$

A.H. $\lim\limits_{x\to\infty} \dfrac{2x^3-3x^2+7}{x} = \lim\limits_{x\to\infty} \dfrac{2x^3}{x} = \lim\limits_{x\to\infty} 2x^2 = 2\infty^2 = 2\infty = \infty$

$\Rightarrow$ NON HAI A.H.

A.O. $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{2x^3-3x^2+7}{x^2} = \lim\limits_{x\to\infty} \dfrac{2x^3}{x^2} = \lim\limits_{x\to\infty} 2x = \infty$

$\Rightarrow$ NON HAI A.O.

RAMAS INFINITAS

$\lim\limits_{x\to\infty} \dfrac{2x^3-3x^2+7}{x} = \lim\limits_{x\to\infty} 2x^2 = \infty$

$\lim\limits_{x\to-\infty} \dfrac{2x^3-3x^2+7}{x} = \lim\limits_{x\to-\infty} 2x^2 = \infty$

41.- Determina as ramas infinitas, e se teñen asíntotas, sitúa a curva respecto a elas:

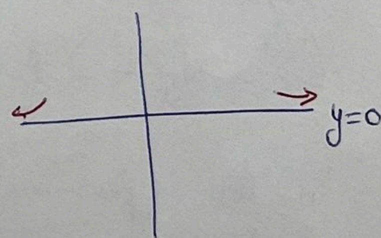
a) $y=\dfrac{1}{x^2+1}$ $x^2+1=0 \Rightarrow x^2=-1$ NON SOLUCIÓN REAL $\Rightarrow \boxed{\text{NON A.V.}}$

A.H. $\lim\limits_{x\to\infty} \dfrac{1}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{1}{x^2} = \dfrac{1}{\infty} = 0 \Rightarrow \boxed{\text{EN } y=0 \text{ A.H.}}$

$\lim\limits_{x\to\infty} \dfrac{1}{x^2+1} = 0^+$

$\lim\limits_{x\to-\infty} \dfrac{1}{x^2+1} = 0^+$

x	1000	-1000
f(x)	0'00...9	0'00...9



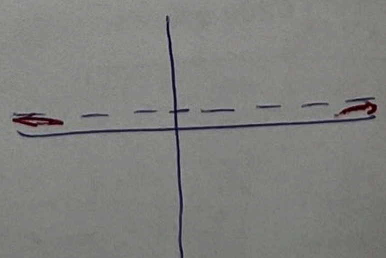
b) $y=\dfrac{x^2}{1+x^2}$ $1+x^2=0 \Rightarrow x^2=-1$ NON SOLUCIÓN REAL $\Rightarrow \boxed{\text{NON A.V.}}$

A.H. $\lim\limits_{x\to\infty} \dfrac{x^2}{1+x^2} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1 \Rightarrow \boxed{\text{EN } y=1 \text{ A.H.}}$

$\lim\limits_{x\to\infty} \dfrac{x^2}{1+x^2} = 1^-$

$\lim\limits_{x\to-\infty} \dfrac{x^2}{1+x^2} = 1^-$

x	1000	-1000
f(x)	$\dfrac{1000000}{1000001} = 0'99...$	$\dfrac{1000000}{1000001} = 0'99...$



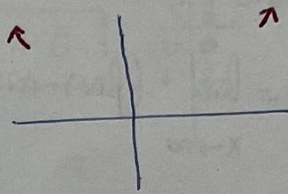
42.-Determina as ramas infinitas destas funcións, e se teñen asíntotas, sitúa a curva respecto a elas :

a) $y=\dfrac{x^4}{x^2+1}$ $x^2+1=0 \Rightarrow x^2=-1$ NON SOLUCIÓN REAL $\Rightarrow$ $\boxed{\text{NON A.V.}}$

A.H. $\lim\limits_{x\to\infty} \dfrac{x^4}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{x^4}{x^2} = \lim\limits_{x\to\infty} x^2 \overset{*}{=} \infty \Rightarrow$ $\boxed{\text{NON A.H.}}$

A.O. $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x^4}{x^3+x} = \lim\limits_{x\to\infty} \dfrac{x^4}{x^3} = \lim\limits_{x\to\infty} x = \infty \Rightarrow$

$\Rightarrow$ $\boxed{\text{NON A.O}}$



RAMAS INFINITAS $\lim\limits_{x\to\infty} \dfrac{x^4}{x^2+1} \overset{*}{=} \infty$

$\lim\limits_{x\to-\infty} \dfrac{x^4}{x^2+1} = \lim\limits_{x\to-\infty} x^2 = (-\infty)^2 = \infty$

b) $y=\dfrac{x^2+3x}{x+1}$ $x+1=0 \Rightarrow x=-1$

$\lim\limits_{x\to-1^-} \dfrac{x^2+3x}{x+1} = +\infty$ $\dfrac{\ominus}{\ominus}$

$\lim\limits_{x\to-1^+} \dfrac{x^2+3x}{x+1} = -\infty$ $\dfrac{\ominus}{\oplus}$

A.V. $\lim\limits_{x\to-1} \dfrac{x^2+3x}{x+1} = \dfrac{1-3}{0} = \dfrac{-2}{0} = \pm\infty$ $\boxed{\text{EN } x=-1 \text{ A.V.}}$

A.H. $\lim\limits_{x\to\infty} \dfrac{x^2+3x}{x+1} = \lim\limits_{x\to\infty} \dfrac{x^2}{x} = \lim\limits_{x\to\infty} x = \infty \Rightarrow$ $\boxed{\text{NON HAI A.H.}}$

A.O. $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x^2+3x}{x^2+x} = 1 = m \neq 0$ VAI HABER A.O. e $m=1$

$n = \lim\limits_{x\to\infty} (f(x)-mx) = \lim\limits_{x\to\infty} \left(\dfrac{x^2+3x}{x+1} - 1\cdot x\right) = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}+3x-\cancel{x^2}-x}{x+1} = \lim\limits_{x\to\infty} \dfrac{2x}{x+1} = 2$

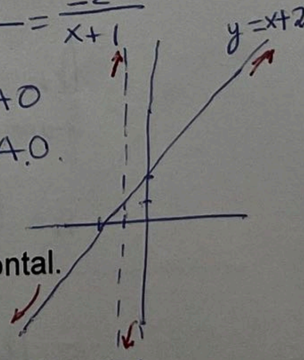
$\boxed{y = x+2 \text{ A.O.}}$

x	y
0	2
1	3
-2	0

$f(x) - AO = \dfrac{x^2+3x}{x+1} - (x+2) = \dfrac{\cancel{x^2}+\cancel{3x}-\cancel{x^2}-\cancel{2x}-\cancel{x}-2}{x+1} = \dfrac{-2}{x+1}$

$f(x) - AO = \dfrac{-2}{x+1}$ $\xrightarrow{x\to\infty} 0^- \Rightarrow f(x) < AO$

$\xrightarrow[x\to-\infty]{} 0^+ \Rightarrow f(x) > A.O.$



43.- Proba que a función $f(x)=\dfrac{x^2-4}{x^2-2x}$ só ten unha asíntota vertical e outra horizontal.

$x^2-2x=0 \Rightarrow x(x-2)=0$ $\begin{cases} x=0 \\ x=2 \end{cases}$

$\lim\limits_{x\to 0} \dfrac{x^2-4}{x^2-2x} = \dfrac{-4}{0} = \pm\infty$

$\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-2x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 2} \dfrac{(x+2)\cancel{(x-2)}}{x\cancel{(x-2)}} = \dfrac{4}{2} = 2$ $\rightarrow$ EN $x=2$ NON HAI A.V.

$\boxed{\text{EN } x=0 \text{ A.V.}}$

$\lim\limits_{x\to\infty} \dfrac{x^2-4}{x^2-2x} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1 \Rightarrow$ $\boxed{\text{EN } y=1 \text{ A.H}}$

44.- Cada unha das seguintes funcións ten unha asíntota oblicua. Determínaa e estuda a posición da curva respecto a ela:

a) $f(x)=\frac{3x^2}{x+1}$

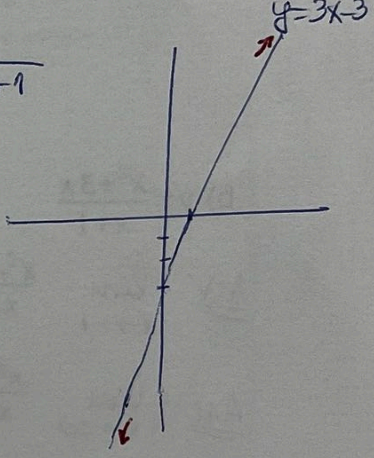
$\lim_{x\to\infty} \frac{3x^2}{x+1} = \lim_{x\to\infty} \frac{3x^2}{x} = \lim_{x\to\infty} 3x = \infty \Rightarrow$ NON A.H.

$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{3x^2}{x^2+x} = \lim_{x\to\infty} \frac{3x^2}{x^2} = 3 = m \neq 0$ VAI HABER A.O. e $m=3$

$n = \lim_{x\to\infty} (f(x)-mx) = \lim_{x\to\infty} \left(\frac{3x^2}{x+1} - 3x\right) = \lim_{x\to\infty} \frac{\cancel{3x^2} - \cancel{3x^2} - 3x}{x+1} =$

$= \lim_{x\to\infty} \frac{-3x}{x+1} = -3$ $\boxed{y = 3x-3 \text{ é A.O.}}$

x	y
0	-3
1	0



$f(x) - A.O. = \frac{3x^2}{x+1} - (3x-3) = \frac{\cancel{3x^2} - \cancel{3x^2} - \cancel{3x} + \cancel{3x} + 3}{x+1} = \frac{3}{x+1}$

$f(x) - AO = \frac{3}{x+1}$ $\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x) > AO$

$\xrightarrow{x\to-\infty} 0^- \Rightarrow f(x) < AO$

b) $f(x)=\frac{3+x-x^2}{x}$

$\lim_{x\to\infty} \frac{3+x-x^2}{x} = \lim_{x\to\infty} \frac{-x^2}{x} = \lim_{x\to\infty} -x = -\infty \Rightarrow$ NON HAI A.H.

$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{3+x-x^2}{x^2} = \lim_{x\to\infty} \frac{-x^2}{x^2} = -1 = m \neq 0$ VAI HABER A.O. e $m=-1$

$n = \lim_{x\to\infty} (f(x)-mx) = \lim_{x\to\infty} \left(\frac{3+x-x^2}{x} - (-1)x\right) =$

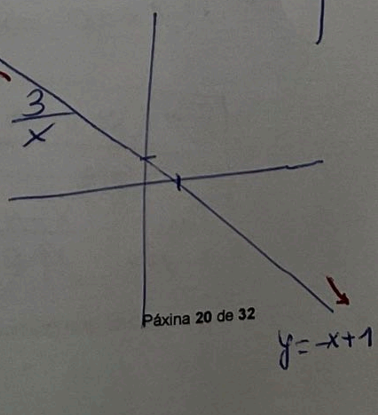
$= \lim_{x\to\infty} \frac{3+x-\cancel{x^2}+\cancel{x^2}}{x} = \lim_{x\to\infty} \frac{x}{x} = 1 \Rightarrow \boxed{y = -x+1 \text{ é A.O.}}$

x	y
0	1
1	0

$f(x) - A.O. = \frac{3+x-x^2}{x} - (-x+1) = \frac{3+\cancel{x}-\cancel{x^2}+\cancel{x^2}-\cancel{x}}{x} = \frac{3}{x}$

$f(x) - AO = \frac{3}{x}$ $\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x) > AO$

$\xrightarrow{x\to-\infty} 0^- \Rightarrow f(x) < AO$



c) $f(x)=\dfrac{4x^2-3}{2x}$

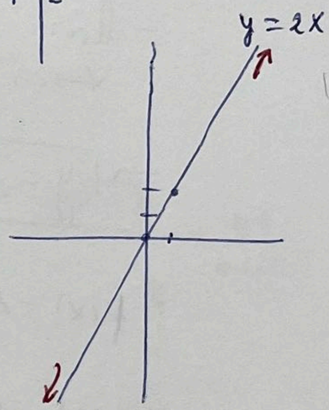
$$\lim_{x\to\infty}\frac{4x^2-3}{2x}=\lim_{x\to\infty}\frac{4x^2}{2x}=\lim_{x\to\infty}2x=\infty \Rightarrow \text{NON HAI A.H.}$$

$$\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{4x^2-3}{2x^2}=\lim_{x\to\infty}\frac{4x^2}{2x^2}=2=m\neq 0 \Rightarrow \text{VAI HABER A.O. e } m=2$$

$$n=\lim_{x\to\infty}(f(x)-mx)=\lim_{x\to\infty}\left(\frac{4x^2-3}{2x}-2x\right)=\lim_{x\to\infty}\frac{\cancel{4x^2}-3-\cancel{4x^2}}{2x}=$$

$$=\lim_{x\to\infty}\frac{-3}{2x}=\frac{-3}{\infty}=0 \Rightarrow \boxed{y=2x \text{ é A.O.}}$$

x	y
0	0
1	2



$$f(x)-A.O.=\frac{4x^2-3}{2x}-2x=\frac{\cancel{4x^2}-3-\cancel{4x^2}}{2x}=\frac{-3}{2x}$$

$$f(x)-A.O.=\frac{-3}{2x}\ \begin{cases}\xrightarrow{x\to\infty} 0^- \Rightarrow f(x)<AO\\ \xrightarrow{x\to-\infty} 0^+ \Rightarrow f(x)>A.O.\end{cases}$$

d) $f(x)=\dfrac{x^2+x-2}{x-3}$

$$\lim_{x\to\infty}\frac{x^2+x-2}{x-3}=\lim_{x\to\infty}\frac{x^2}{x}=\lim_{x\to\infty}x=\infty \Rightarrow \text{NON HAI A.H.}$$

$$\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2+x-2}{x^2-3x}=\lim_{x\to\infty}\frac{x^2}{x^2}=1=m\neq 0$$

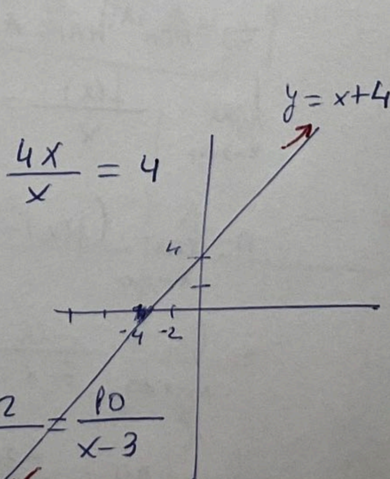
$\Downarrow$ VAI HABER AO e $m=1$

$$n=\lim_{x\to\infty}(f(x)-mx)=\lim_{x\to\infty}\left(\frac{x^2+x-2}{x-3}-x\right)=$$

$$=\lim_{x\to\infty}\frac{\cancel{x^2}+x-2-\cancel{x^2}+3x}{x-3}=\lim_{x\to\infty}\frac{4x-2}{x-3}=\lim_{x\to\infty}\frac{4x}{x}=4$$

x	y
0	4
-4	0

$$\Rightarrow \boxed{y=x+4 \text{ é A.O.}}$$



$$f(x)-A.O.=\frac{x^2+x-2}{x-3}-(x+4)=\frac{\cancel{x^2}+\cancel{x}-2-\cancel{x^2}-\cancel{4x}+\cancel{3x}+12}{x-3}=\frac{10}{x-3}$$

$$f(x)-A.O.=\frac{10}{x-3}\ \begin{cases}\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x)>AO\\ \xrightarrow{x\to-\infty} 0^- \Rightarrow f(x)<AO\end{cases}$$

e) $f(x)=\dfrac{2x^3-3}{x^2-2}$

$$\lim_{x\to\infty}\frac{2x^3-3}{x^2-2}=\lim_{x\to\infty}\frac{2x^3}{x^2}=\lim_{x\to\infty}2x=\infty \Rightarrow \text{NON HAÍ A.H.}$$

$$\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{2x^3-3}{x^3-2x}=\lim_{x\to\infty}\frac{2x^3}{x^3}=2=m\neq 0$$ VAI HABER A.O. e $m=2$

$$n=\lim_{x\to\infty}(f(x)-mx)=\lim_{x\to\infty}\left(\frac{2x^3-3}{x^2-2}-2x\right)=$$

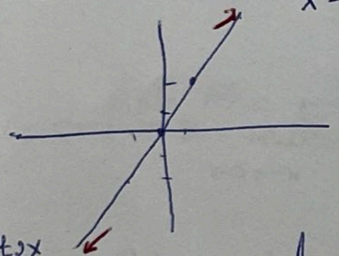
$$=\lim_{x\to\infty}\frac{\cancel{2x^3}-3\cancel{-2x^3}+4x}{x^2-2}=\lim_{x\to\infty}\frac{4x-3}{x^2-2}=\lim_{x\to\infty}\frac{4x}{x^2}=\lim_{x\to\infty}\frac{4}{x}=\frac{4}{\infty}=0$$

$$\Rightarrow \boxed{y=2x \text{ é A.O.}}$$

x	y
0	0
1	2

$$f(x)-A.O.=\frac{2x^3-3}{x^2-2}-2x=\frac{\cancel{2x^3}-3\cancel{-2x^3}+4x}{x^2-2}=\frac{4x-3}{x^2-2}$$

$$f(x)-AO=\frac{4x-3}{x^2-2}\ \begin{cases}\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x)>AO\\ \xrightarrow{x\to-\infty} 0^- \Rightarrow f(x)<AO\end{cases}$$



y=2x

f) $f(x)=\dfrac{-2x^2+3}{2x-2}$

$$\lim_{x\to\infty}\frac{-2x^2+3}{2x-2}=\lim_{x\to\infty}\frac{-2x^2}{2x}=\lim_{x\to\infty}-x=-\infty$$

$\Rightarrow$ NON HAÍ A.H.

$$\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{-2x^2+3}{2x^2-2x}=\lim_{x\to\infty}\frac{-2x^2}{2x^2}=-1=m\neq 0$$ VAI HABER A.O. e $m=-1$

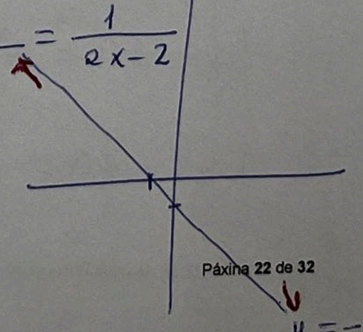
$$n=\lim_{x\to\infty}(f(x)-mx)=\lim_{x\to\infty}\left(\frac{-2x^2+3}{2x-2}-(-1)x\right)=\lim_{x\to\infty}\frac{\cancel{-2x^2}+3\cancel{+2x^2}-2x}{2x-2}=$$

$$=\lim_{x\to\infty}\frac{-2x}{2x}=-1 \Rightarrow \boxed{y=-x-1 \text{ é A.O.}}$$

x	y
0	-1
-1	0

$$f(x)-AO=\frac{-2x^2+3}{2x-2}-(-x-1)=\frac{\cancel{-2x^2}+3\cancel{+2x^2}\cancel{-2x}\cancel{+2x}-2}{2x-2}=\frac{1}{2x-2}$$

$$f(x)-AO=\frac{1}{2x-2}\ \begin{cases}\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x)>AO\\ \xrightarrow{x\to-\infty} 0^- \Rightarrow f(x)<AO\end{cases}$$



y=-x-1

45.- Determina as asíntotas das seguintes funcións e sitúa a curva respecto a cada unha de elas:

a) $y=\frac{(3-x)^2}{2x+1}$ $\quad 2x+1=0 \Rightarrow x=-\frac{1}{2}$ $\quad \lim_{x\to -\frac{1}{2}} \frac{(3-x)^2}{2x+1} = \frac{(\frac{7}{2})^2}{0} = \pm\infty$

$\boxed{\text{EN } x=-\frac{1}{2} \text{ A.V.}}$ $\quad \lim_{x\to (-\frac{1}{2})^-} \frac{(3-x)^2}{2x+1} = -\infty$; $\lim_{x\to (-\frac{1}{2})^+} \frac{(3-x)^2}{2x+1} = +\infty$

A.H. $\lim_{x\to\infty} \frac{(3-x)^2}{2x+1} = \lim_{x\to\infty} \frac{x^2}{2x} = \lim_{x\to\infty} \frac{x}{2} = \infty \Rightarrow \boxed{\text{NON HAI A.H.}}$

A.O. $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{9+x^2-6x}{2x^2+x} = \lim_{x\to\infty} \frac{x^2}{2x^2} = \frac{1}{2} = m \neq 0$

VAI HABER A.O. e $m=\frac{1}{2}$

$n = \lim_{x\to\infty} (f(x)-mx) = \lim_{x\to\infty} \left(\frac{9+x^2-6x}{2x+1} - \frac{1}{2}x \right) =$

$= \lim_{x\to\infty} \frac{18+\cancel{2x^2}-12x-\cancel{2x^2}-x}{2(2x+1)} = \lim_{x\to\infty} \frac{-13x+18}{4x+2} = -\frac{13}{4} = n$

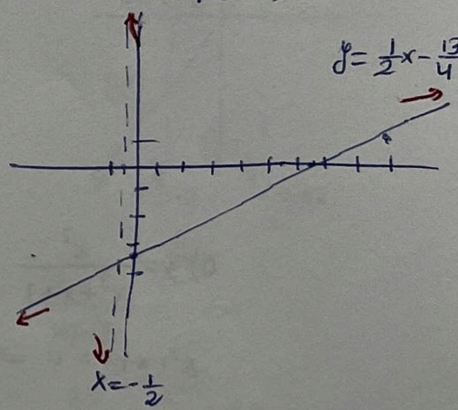
$\Rightarrow \boxed{y=\frac{1}{2}x-\frac{13}{4} \text{ é A.O.}}$

x	y
0	$-\frac{13}{4}=-3'25$
4	$2-\frac{13}{4}=-1'25$
8	$4-\frac{13}{4}=0'75$

$f(x) - A.O. = \frac{9+x^2-6x}{2x+1} - \left(\frac{1}{2}x - \frac{13}{4}\right) = \frac{36+\cancel{4x^2}-\cancel{24x}-\cancel{4x^2}-\cancel{2x}+\cancel{26x}+13}{4\cdot(2x+1)} = \frac{49}{4\cdot(2x+1)}$

$f(x) - AO = \frac{49}{4\cdot(2x+1)}$ $\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x) > AO$

$\qquad\qquad\qquad\qquad \xrightarrow{x\to-\infty} 0^- \Rightarrow f(x) < AO$



b) $y=\frac{5x-2}{2x-7}$ $\quad 2x-7=0 \Rightarrow x=\frac{7}{2}$

$\lim_{x\to\frac{7}{2}} \frac{5x-2}{2x-7} = \frac{\frac{31}{2}}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=\frac{7}{2} \text{ HAI A.V.}}$

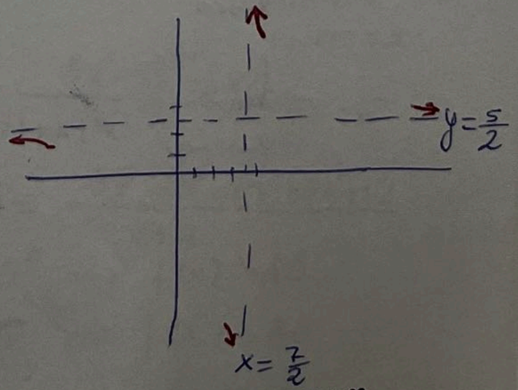
$\lim_{x\to(\frac{7}{2})^-} \frac{5x-2}{2x-7} = -\infty$; $\lim_{x\to(\frac{7}{2})^+} \frac{5x-2}{2x-7} = +\infty$

A.H. $\lim_{x\to\infty} \frac{5x-2}{2x-7} = \lim_{x\to\infty} \frac{5x}{2x} = \frac{5}{2} \Rightarrow \boxed{\text{EN } y=\frac{5}{2} \text{ HAI A.H.}}$

$\lim_{x\to\infty} \frac{5x-2}{2x-7} = \left(\frac{5}{2}\right)^+$

x	1000	−1000
f(x)	2'507...	2'4922...

$\lim_{x\to-\infty} \frac{5x-2}{2x-7} = \left(\frac{5}{2}\right)^-$



c) $y=\frac{x+2}{x^2-1}$ $\quad x^2-1=0 \Rightarrow x=\pm 1$

$\lim_{x\to 1} \frac{x+2}{x^2-1} = \frac{3}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=1 \text{ A.V.}}$

$\lim_{x\to 1^-} \frac{x+2}{x^2-1} = \frac{\oplus}{\ominus} = -\infty$

$\lim_{x\to 1^+} \frac{x+2}{x^2-1} = \frac{\oplus}{\oplus} = +\infty$

$\lim_{x\to -1} \frac{x+2}{x^2-1} = \frac{1}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=-1 \text{ A.V.}}$

$\lim_{x\to -1^-} \frac{x+2}{x^2-1} = \frac{\oplus}{\oplus} = +\infty$

$\lim_{x\to -1^+} \frac{x+2}{x^2-1} = \frac{\oplus}{\ominus} = -\infty$

A.H.

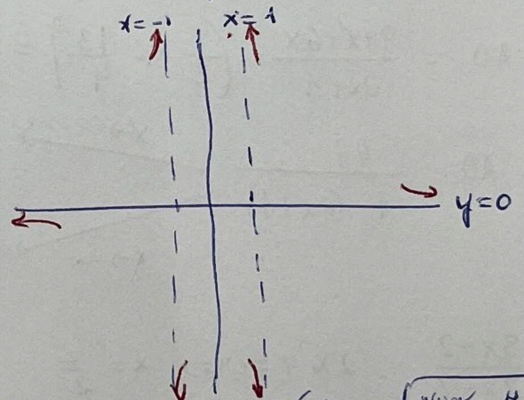
$\lim_{x\to \infty} \frac{x+2}{x^2-1} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

$\Rightarrow \boxed{\text{EN } y=0 \text{ A.H.}}$

$\lim_{x\to \infty} \frac{x+2}{x^2-1} = 0^+$

$\lim_{x\to \infty} \frac{x+2}{x^2-1} = 0^-$

x	1000	-1000
f(x)	0'001002...	-0'0009



d) $y=\frac{x^2}{x^2+x+1}$

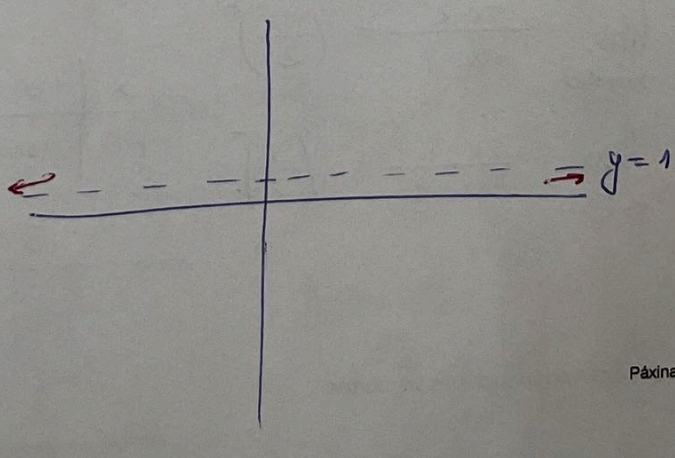
$x^2+x+1=0 \Rightarrow x=\frac{-1\pm\sqrt{1-4\cdot1\cdot1}}{2} \notin \mathbb{R} \Rightarrow$ NON SOLUCIÓN REAL $\Rightarrow \boxed{\text{NON HAI A.V.}}$

A.H. $\lim_{x\to \infty} \frac{x^2}{x^2+x+1} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1 \Rightarrow \boxed{\text{EN } y=1 \text{ A.H.}}$

$\lim_{x\to \infty} \frac{x^2}{x^2+x+1} = 1^-$

x	1000	-1000
f(x)	0'9990...	1'0009...

$\lim_{x\to -\infty} \frac{x^2}{x^2+x+1} = 1^+$



e) $f(x)=\dfrac{x^3}{x^2-4}$ $\quad x^2-4=0 \Rightarrow x=\pm 2$

$$\lim_{x\to 2} \frac{x^3}{x^2-4} = \frac{8}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=2 \text{ A.V.}}$$

$$\lim_{x\to -2} \frac{x^3}{x^2-4} = \frac{-8}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=-2 \text{ A.V.}}$$

$$\lim_{x\to 2^-} \frac{x^3}{x^2-4} \overset{\oplus}{\underset{\ominus}{=}} -\infty \qquad \lim_{x\to 2^+} \frac{x^3}{x^2-4} \overset{\oplus}{\underset{\oplus}{=}} +\infty$$

$$\lim_{x\to -2^-} \frac{x^3}{x^2-4} \overset{\ominus}{\underset{\oplus}{=}} -\infty \qquad \lim_{x\to -2^+} \frac{x^3}{x^2-4} \overset{\ominus}{\underset{\ominus}{=}} +\infty .$$

A.H. $\lim_{x\to\infty} \dfrac{x^3}{x^2-4} = \lim_{x\to\infty} \dfrac{x^3}{x^2} = \lim_{x\to\infty} x = \infty$

$\Rightarrow \boxed{\text{NON HAI A.H.}}$

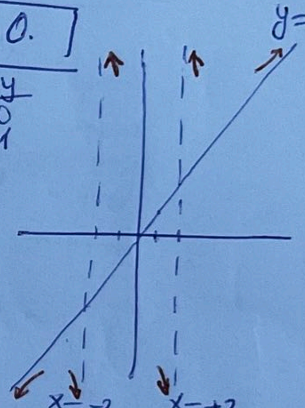
A.O. $\lim_{x\to\infty} \dfrac{f(x)}{x} = \lim_{x\to\infty} \dfrac{x^3}{x^3-4x} = \lim_{x\to\infty} \dfrac{x^3}{x^3} = 1 = m \neq 0$ VAI HABER A.O. e $m=1$

$n = \lim_{x\to\infty} (f(x)-mx) = \lim_{x\to\infty} \left(\dfrac{x^3}{x^2-4} - 1\cdot x\right) \overset{*}{=} \lim_{x\to\infty} \dfrac{\cancel{x^3}-\cancel{x^3}+4x}{x^2-4} =$

$= \lim_{x\to\infty} \dfrac{4x}{x^2} = \lim_{x\to\infty} \dfrac{4}{x} = \dfrac{4}{\infty} = 0 \Rightarrow \boxed{y = x \text{ A.O.}}$

x	y
0	0
1	1

$f(x) - A.O. \overset{*}{=} \dfrac{4x}{x^2-4}$ $\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x) > A.O$; $\xrightarrow{x\to -\infty} 0^- \Rightarrow f(x) < A.O$



f) $f(x)=\dfrac{3x^2}{x+2}$ $\quad x+2=0 \Rightarrow x=-2$

$$\lim_{x\to -2} \frac{3x^2}{x+2} = \frac{12}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=-2 \text{ A.V.}}$$

$$\lim_{x\to -2^-} \frac{3x^2}{x+2} \overset{\oplus}{\underset{\ominus}{=}} -\infty \qquad \lim_{x\to -2^+} \frac{3x^2}{x+2} \overset{\oplus}{\underset{\oplus}{=}} +\infty$$

A.H. $\lim_{x\to\infty} \dfrac{3x^2}{x+2} = \lim_{x\to\infty} \dfrac{3x^2}{x} = \lim_{x\to\infty} 3x = \infty \Rightarrow \boxed{\text{NON HAI A.H.}}$

A.O. $\lim_{x\to\infty} \dfrac{f(x)}{x} = \lim_{x\to\infty} \dfrac{3x^2}{x^2+2x} = \lim_{x\to\infty} \dfrac{3x^2}{x^2} = 3 = m \neq 0$ VAI HABER A.O. e $m=3$

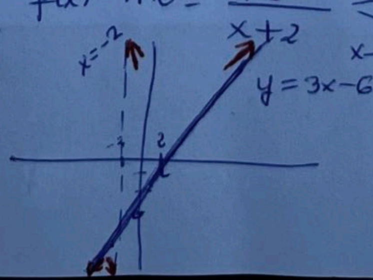
$n = \lim_{x\to\infty} (f(x)-mx) = \lim_{x\to\infty} \left(\dfrac{3x^2}{x+2} - 3x\right) = \lim_{x\to\infty} \dfrac{\cancel{3x^2}-\cancel{3x^2}-6x}{x+2} = -6$

$\boxed{y = 3x-6 \text{ A.O.}}$

x	y
2	0
0	-6

$f(x) - A.O. = \dfrac{3x^2}{x+2} - (3x-6) = \dfrac{\cancel{3x^2}-\cancel{3x^2}\cancel{-6x}\cancel{+6x}+12}{x+2}$

$f(x) - A.O = \dfrac{12}{x+2}$ $\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x) > A.O$; $\xrightarrow{x\to -\infty} 0^- \Rightarrow f(x) < A.O$



46.- Determina as ramas infinitas destas funcións. Cando teñan asíntotas, sitúa a curva:

a) $y=\frac{x^4-1}{x^2}$ $\quad x^2=0 \Rightarrow x=0$

$\lim_{x\to 0} \frac{x^4-1}{x^2} = \frac{-1}{0} = \pm\infty$ $\boxed{\text{EN } x=0 \text{ HAI A.V.}}$

$\lim_{x\to 0^-} \frac{x^4-1}{x^2} \overset{\frac{-}{+}}{=} -\infty$

$\lim_{x\to 0^+} \frac{x^4-1}{x^2} \overset{\frac{-}{+}}{=} -\infty$

A.H.

$\lim_{x\to\infty} \frac{x^4-1}{x^2} = \lim_{x\to\infty} \frac{x^4}{x^2} = \lim_{x\to\infty} x^2 \overset{*}{=} +\infty \Rightarrow$ NON HAI A.H.

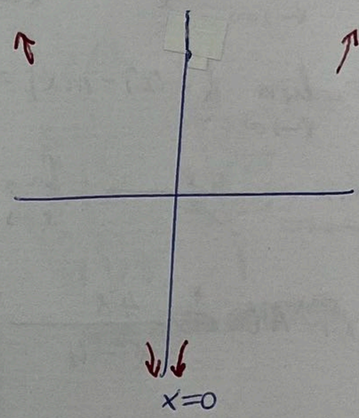
A.O.

$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^4-1}{x^3} = \lim_{x\to\infty} \frac{x^4}{x^3} = \lim_{x\to\infty} x = \infty \Rightarrow$ NON HAI A.O.

RAMAS INFINITAS

$\lim_{x\to\infty} \frac{x^4-1}{x^2} \overset{*}{=} +\infty$

$\lim_{x\to-\infty} \frac{x^4-1}{x^2} = \lim_{x\to-\infty} x^2 = (-\infty)^2 = +\infty$



b) $y=\frac{(x+3)^2}{(x+1)^2}$ $\quad (x+1)^2=0 \Rightarrow x=-1$

$\lim_{x\to -1} \frac{(x+3)^2}{(x+1)^2} = \frac{4}{0} = \pm\infty \Rightarrow$ $\boxed{\text{EN } x=-1 \text{ A.V.}}$

$\lim_{x\to -1^-} \frac{(x+3)^2}{(x+1)^2} \overset{\frac{+}{+}}{=} +\infty$

$\lim_{x\to -1^+} \frac{(x+3)^2}{(x+1)^2} \overset{\frac{+}{+}}{=} +\infty$

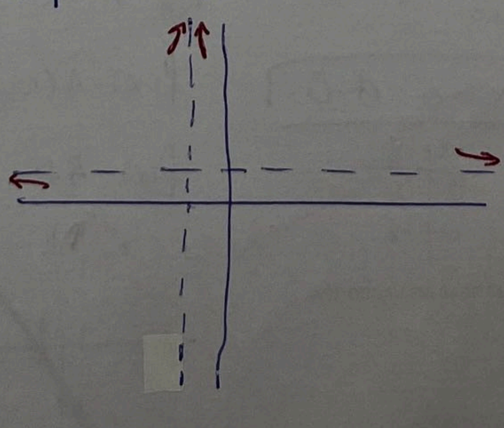
A.H.

$\lim_{x\to\infty} \frac{(x+3)^2}{(x+1)^2} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1 \Rightarrow$ $\boxed{\text{EN } y=1 \text{ A.H.}}$

$\lim_{x\to\infty} \frac{(x+3)^2}{(x+1)^2} = 1^+$

x	1000	-1000
f(x)	1'0039...	0'996

$\lim_{x\to-\infty} \frac{(x+3)^2}{(x+1)^2} = 1^-$



c) $y=\dfrac{1}{9-x^2}$ $\quad 9-x^2=0 \Rightarrow x^2=9 \Rightarrow x=\pm 3$

$$\lim_{x\to -3} \frac{1}{9-x^2} = \frac{1}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=-3 \text{ A.V.}}$$

$$\lim_{x\to -3^-} \frac{1}{9-x^2} = \frac{\oplus}{\ominus} = -\infty$$

$$\lim_{x\to -3^+} \frac{1}{9-x^2} = \frac{\oplus}{\oplus} = +\infty$$

$$\lim_{x\to 3} \frac{1}{9-x^2} = \frac{1}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=3 \text{ A.V.}}$$

$$\lim_{x\to 3^-} \frac{1}{9-x^2} = \frac{\oplus}{\oplus} = +\infty$$

$$\lim_{x\to 3^+} \frac{1}{9-x^2} = \frac{\oplus}{\ominus} = -\infty$$

A.H.

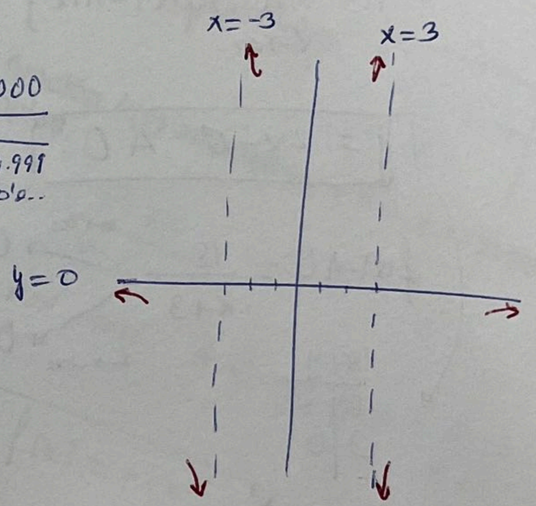
$$\lim_{x\to\infty} \frac{1}{9-x^2} = \lim_{x\to\infty} \frac{1}{-x^2} = \frac{1}{-\infty^2} = \frac{1}{-\infty} = 0$$

$$\Rightarrow \boxed{\text{EN } y=0 \text{ A.H.}}$$

x	1000	−1000
f(x)	$\frac{1}{-999991} = -0'0\ldots$	$\frac{1}{-999.991} = -0'0\ldots$

$$\lim_{x\to\infty} \frac{1}{9-x^2} = 0^-$$

$$\lim_{x\to-\infty} \frac{1}{9-x^2} = 0^-$$



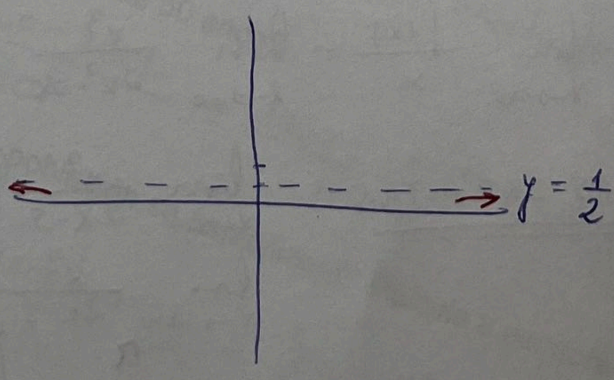
d) $y=\dfrac{x^2-1}{2x^2+1}$ $\quad 2x^2+1=0 \Rightarrow 2x^2=-1$ NON SOLUCIÓN REAL $\Rightarrow \boxed{\text{NON HAI A.V.}}$

A.H. $$\lim_{x\to\infty} \frac{x^2-1}{2x^2+1} = \lim_{x\to\infty} \frac{x^2}{2x^2} = \frac{1}{2} \Rightarrow \boxed{\text{EN } y=\tfrac{1}{2} \text{ A.H.}}$$

$$\lim_{x\to\infty} \frac{x^2-1}{2x^2+1} = \left(\frac{1}{2}\right)^-$$

x	1000	−1000
f(x)	0'499...	0'499...

$$\lim_{x\to-\infty} \frac{x^2-1}{2x^2+1} = \left(\frac{1}{2}\right)^-$$



e) $y=\frac{2x^2}{x+3}$ $x+3=0 \Rightarrow x=-3$

$\lim_{x\to -3} \frac{2x^2}{x+3} = \frac{18}{0} = \pm\infty \Rightarrow \boxed{\text{EN } x=-3 \text{ A.V.}}$

$\lim_{x\to -3^-} \frac{2x^2}{x+3} \overset{\frac{+}{-}}{=} -\infty$

$\lim_{x\to -3^+} \frac{2x^2}{x+3} \overset{\frac{+}{+}}{=} +\infty$

A.H. $\lim_{x\to\infty} \frac{2x^2}{x+3} = \lim_{x\to\infty} \frac{2x^2}{x} = \lim_{x\to\infty} 2x = \infty \Rightarrow \boxed{\text{NON HAI A.H}}$

A.O. $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{2x^2}{x^2+3x} = \lim_{x\to\infty} \frac{2x^2}{x^2} = 2 = m \neq 0$ VAI HABER A.O. e $m=2$

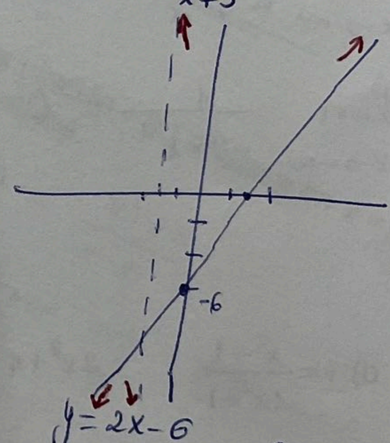
$n = \lim_{x\to\infty} (f(x)-mx) = \lim_{x\to\infty} \left(\frac{2x^2}{x+3} - 2x\right) = \lim_{x\to\infty} \frac{\cancel{2x^2}-\cancel{2x^2}-6x}{x+3} = -6$

$\boxed{y = 2x-6 \text{ A.O.}}$ $f(x) - \text{A.O.} = \frac{2x^2}{x+3} - (2x-6) = \frac{\cancel{2x^2}-\cancel{2x^2}-\cancel{6x}+\cancel{6x}+18}{x+3} = \frac{18}{x+3}$

$f(x) - \text{A.O.} = \frac{18}{x+3}$ $\xrightarrow{x\to\infty} 0^+ \Rightarrow f(x) > AO$

$\xrightarrow{x\to -\infty} 0^- \Rightarrow f(x) < AO$

x	y
0	-6
3	0



f) $y=\frac{x^3}{2x-5}$ $2x-5=0 \Rightarrow x=\frac{5}{2}$

$\lim_{x\to \frac{5}{2}} \frac{x^3}{2x-5} = \frac{\left(\frac{5}{2}\right)^3}{0} = \pm\infty$

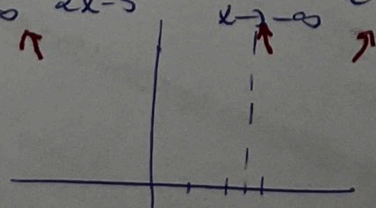
$\boxed{\text{EN } x=\frac{5}{2} \text{ A.V.}}$ $\lim_{x\to \left(\frac{5}{2}\right)^-} \frac{x^3}{2x-5} \overset{\frac{+}{-}}{=} -\infty$; $\lim_{x\to \left(\frac{5}{2}\right)^+} \frac{x^3}{2x-5} \overset{\frac{+}{+}}{=} +\infty$

A.H. $\lim_{x\to\infty} \frac{x^3}{2x-5} \overset{(*)}{=} \lim_{x\to\infty} \frac{x^3}{2x} = \lim_{x\to\infty} \frac{x^2}{2} = \infty \Rightarrow \boxed{\text{NON HAI A.H}}$

A.O. $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^3}{2x^2-5x} = \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \infty \Rightarrow \boxed{\text{NON HAI A.O.}}$

RAMAS INFINITAS $\lim_{x\to\infty} \frac{x^3}{2x-5} \overset{(*)}{=} \infty$

$\lim_{x\to -\infty} \frac{x^3}{2x-5} = \lim_{x\to -\infty} \frac{x^2}{2} = \frac{(-\infty)^2}{2} = \infty$



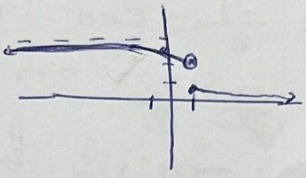
47.- Pode ter unha función máis de dúas asíntotas verticais? E máis de dúas asíntotas horizontais?. Pon exemplos.

Si. Por ex. $f(x)=\dfrac{1}{9-x^2}$ $x=3$ e $x=-3$ A.V.

Si. Unha función a trozos: $f(x)=\begin{cases}\dfrac{1}{x-2}+3 & x<1\\ \left(\dfrac{1}{2}\right)^x & x\geq 1\end{cases}$

Ten cando $\underline{x\to-\infty}$ $\underline{y=3 \text{ A.H.}}$

Ten cando $\underline{x\to\infty}$ $\underline{y=0 \text{ A.H.}}$



x	y
1	2
0	2'5
-2	2'75
-3	2'8

x	y
1	0'5
2	0'25
3	0'125

48.- O denominador dunha función *f* (*x*) anúlase en *x* = *a*. Existe necesariamente unha asíntota vertical en *x* = *a*? Pon exemplos.

NON. Ex. $f(x)=\dfrac{x^2-2x}{x-2}$

$\underline{f(x)}$ CANDO $x=2$ NON TEN A.V. porque $\lim\limits_{x\to2}\dfrac{x^2-2x}{x-2} \overset{\frac{0}{0}}{=} \lim\limits_{x\to2}\dfrac{x\cancel{(x-2)}}{\cancel{x-2}} = \lim\limits_{x\to2} x = 2$

$\boxed{x=a \text{ é A.V.} \Leftrightarrow \lim\limits_{x\to a} f(x)=\pm\infty}$

49.- Nunha cidade faise un censo inicial e sábese que o número de habitantes evoluciona segundo a función :

$$P(t)=\frac{t^2+500t+2500}{(t+50)^2}$$ *onde t é o número de anos transcurridos dende que se fai o censo*

e P(t) o número de habitantes en millóns.

a) **Cantos habitantes hai cando se realiza o censo inicial?**

$P(0)=\dfrac{2500}{50^2}=\dfrac{2500}{2500}=1$ millón de habitantes

b) **Cantos habitantes habrá dentro de 50 anos?**

$P(50)=\dfrac{50^2+500\cdot50+2500}{(50+50)^2}=\dfrac{2500+25000+2500}{10000}=\dfrac{30000}{10000}=3$ millóns de habitantes

c) **Co paso do tempo, cara que poboación se estabilizará? Calcula a asíntota horizontal para comprobalo.**

$\lim\limits_{t\to\infty}P(t)=\lim\limits_{t\to\infty}\dfrac{t^2+500t+2500}{(t+50)^2}=\lim\limits_{t\to\infty}\dfrac{t^2}{t^2}=1$

$\underline{P=1 \text{ A.H.}}$

Co paso do tempo a poboación tenderá a 1 millón de habitantes.

50.- Calcula o límite da función $y=\dfrac{x^2-9}{x^2-3x}$ nos puntos nos que non é continua e interprétao graficamente.

$x^2-3x=0 \Rightarrow x(x-3)=0 \begin{cases} x=0 \\ x=3 \end{cases}$

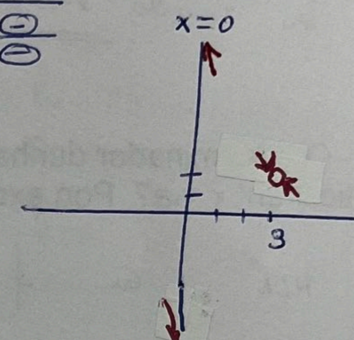
f(x) DESCONTINUA EN $x=0$ e $x=3$ $\quad \nexists f(0)$, $\nexists f(3)$

$\boxed{x=0}$

$\lim\limits_{x\to 0} \dfrac{x^2-9}{x^2-3x} = \dfrac{-9}{0} = \pm\infty$ EN $x=0$ DESCONTINUIDADE DE SALTO INFINITO

$\lim\limits_{x\to 0^-} \dfrac{x^2-9}{\underbrace{x^2-3x}_{x(x-3)}} \underset{x=-0'01}{=} -\infty \qquad \dfrac{\ominus}{\ominus\ominus} = \dfrac{\ominus}{\oplus}$

$\lim\limits_{x\to 0^+} \dfrac{x^2-9}{\underbrace{x^2-3x}_{x(x-3)}} \underset{x=0'01}{=} +\infty \qquad \dfrac{\ominus}{\oplus\ominus} = \dfrac{\ominus}{\ominus}$



$\boxed{x=3}$

INDETERMINACIÓN

$\lim\limits_{x\to 3} \dfrac{x^2-9}{x^2-3x} = \dfrac{0}{0} = \lim\limits_{x\to 3} \dfrac{(x+3)\cancel{(x-3)}}{x\cancel{(x-3)}} = \lim\limits_{x\to 3} \dfrac{x+3}{x} = \dfrac{6}{3} = 2$

x	2'99	3'01
f(x)	2'0033...	1'99...

EN $x=3$ $\left.\begin{array}{l} \nexists f(3) \\ \exists \lim\limits_{x\to 3} f(x) \end{array}\right\} \Rightarrow$ EN $x=3$ DESCONTINUIDADE EVITABLE

51.- Calcula o valor de a para que: $\lim\limits_{x\to\infty} \dfrac{ax^2+3x}{2x^2-5}=3$.

$\lim\limits_{x\to\infty} \dfrac{ax^2+3x}{2x^2-5} = \lim\limits_{x\to\infty} \dfrac{a\cancel{x^2}}{2\cancel{x^2}} = \dfrac{a}{2}$

Como $\lim\limits_{x\to\infty} \dfrac{ax^2+3x}{2x^2-5} = 3$

$\Downarrow$

$\dfrac{a}{2} = 3 \Rightarrow \boxed{a=6}$

52.- Calcula o valor de m e n para que a seguinte función sexa continua en todo $\mathbb{R}$.

$$f(x)=\begin{cases} x^2 & se\ x\leq -1 \\ mx+n & se\ -1<x<2 \\ \dfrac{2}{x} & se\ x\geq 2 \end{cases}$$

f CONTINUA EN $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$ por ser polinómica e racional continuas nos intervalos nos que están definidas.

¿EN $x=-1$?

$\lim_{x\to -1^-} f(x)=\lim_{x\to -1} x^2 = 1$

$\lim_{x\to -1^+} f(x)=\lim_{x\to -1} (mx+n)=-m+n$

$f(-1)=(-1)^2=1$

Para f CONT. EN $x=-1$

$\boxed{-m+n=1}$

¿EN $x=2$?

$\lim_{x\to 2^-} f(x)=\lim_{x\to 2} (mx+n)=2m+n$

$\lim_{x\to 2^+} f(x)=\lim_{x\to 2} \frac{2}{x}=\frac{2}{2}=1$

$f(2)=\frac{2}{2}=1$

Para f CONT. EN $x=2 \Rightarrow 2m+n=1$

Resolvendo o stma: $\begin{cases} -m+n=1 \\ 2m+n=1 \end{cases}$

$$\begin{array}{r} +m-n=-1 \\ 2m+n=1 \\ \hline 3m=0 \Rightarrow m=0 \Rightarrow n=1 \end{array}$$

f CONTINUA EN $\mathbb{R}$ SE $m=0$ e $n=1$

53.- Calcula o valor de m e n para que a seguinte función sexa continua en todo $\mathbb{R}$.

$$f(x)=\begin{cases} 2^x & se\ x\leq 1 \\ mx+n & se\ 1<x<2 \\ \log_2 x & se\ x\geq 2 \end{cases}$$

f CONTINUA EN $(-\infty,1)\cup(1,2)\cup(2,\infty)$ por ser exponencial, polinómica e logarítmica continuas nos intervalos nos que están definidas.

¿EN $x=1$?

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1} 2^x=2$

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}(mx+n)=m+n$

$f(1)=2^1=2$

¿EN $x=2$?

$\lim_{x\to 2^-} f(x)=\lim_{x\to 2}(mx+n)=2m+n$

$\lim_{x\to 2^+} f(x)=\lim_{x\to 2}\log_2 x=\log_2 2=1$

$f(2)=\log_2 2=1$

Para f CONT. EN $x=1 \Rightarrow m+n=2$

Para f CONT EN $x=2 \Rightarrow 2m+n=1$

Resolvendo o sistema $\Longrightarrow$

$$\begin{array}{r} m+n=2 \\ -2m-n=-1 \\ \hline -m=1 \\ m=-1 \\ n=2-m=3 \end{array}$$

f CONTINUA EN $\mathbb{R}$ SE $m=-1$ e $n=3$

54.- Unha determinada especie evoluciona segundo a función:

$f(t) = 5 + 2^{-t}$ *onde t é onumero de anos e* $f(t)$ *os millóns de unidades existentes*

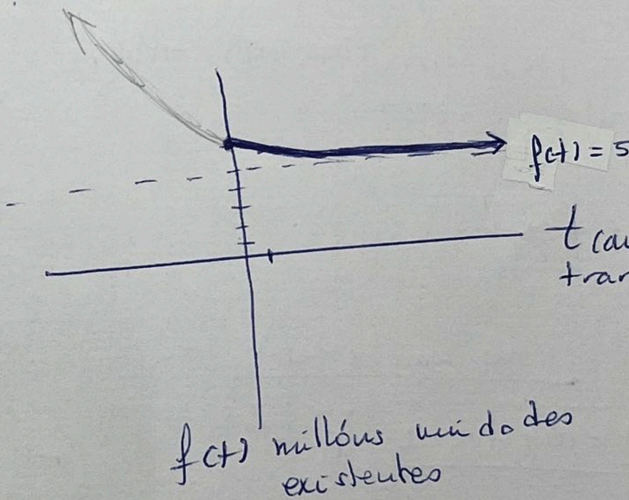
Representa a gráfica e contesta: ¿a especie está en vías de extinción?

$$\lim_{t \to \infty} f(t) = \lim_{t \to \infty} 5 + 2^{-t} = 5 + 2^{-\infty} = 5 + \frac{1}{2^{\infty}}$$

$$= 5 + \frac{1}{\infty} = 5 + 0 = 5 \text{ millóns de unidades}$$

Cando $t \to \infty$, a función tende a estabilizarse en 5 millóns de unidades. Non está en vías de extinción.

t	$f(t)$
$\boxed{0}$	$\boxed{6} = 5 + 2^0 = 5 + 1$
$\boxed{1}$	$5 + 2^{-1} = 5 + \frac{1}{2} = \boxed{5'5}$
$\boxed{2}$	$5 + 2^{-2} = 5 + \frac{1}{4} = 5'25$
$\boxed{3}$	$\boxed{5'125}$



55.- Calcula:

a) $\lim\limits_{x \to -\infty} \dfrac{5 - 4x^3}{2x^3 + 3x - 1} = \lim\limits_{x \to -\infty} \dfrac{-4x^3}{2x^3} = \lim\limits_{x \to -\infty} \dfrac{-4}{2} = -2$

b) $\lim\limits_{x \to \infty} \left(\dfrac{2x^2 - 1}{2x} - \dfrac{x^2 - x + 3}{x} \right) = \lim\limits_{x \to \infty} \dfrac{\cancel{2x^2} - 1 - \cancel{2x^2} + 2x - 6}{2x} = \lim\limits_{x \to \infty} \dfrac{2x - 7}{2x} = \lim\limits_{x \to \infty} \dfrac{2x}{2x} = 1$

c) $\lim\limits_{x \to 3} \dfrac{x^2 + 4x + 3}{x^2 + x - 6} = \dfrac{3^2 + 4 \cdot 3 + 3}{3^2 + 3 - 6} = \dfrac{9 + 12 + 3}{9 + 3 - 6} = \dfrac{24}{6} = 4$