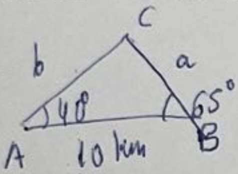
37.- Para localizar unha emisora clandestina, dous receptores, *A* e *B*, que distan entre eles 10 km, orientan as súas antenas cara ao punto onde está a emisora. Estas direccións forman con *AB* ángulos de 40° e 65°. A que distancia de A e B se encontra a emisora?



$\hat{C} = 180° - (A+B) = 180° - (40+65) = 180° - 105° = 75°$

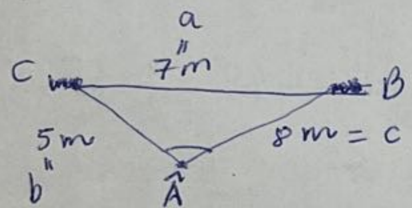
TH. SENO $\quad \dfrac{a}{\operatorname{sen} A} = \dfrac{b}{\operatorname{sen} B} = \dfrac{c}{\operatorname{sen} C}$

$$\frac{10}{\operatorname{sen} 75°} = \frac{a}{\operatorname{sen} 40°} \Rightarrow a = \frac{10 \cdot \operatorname{sen} 40°}{\operatorname{sen} 75°} = 6'65 \text{ km}$$

$$\frac{10}{\operatorname{sen} 75°} = \frac{b}{\operatorname{sen} 65°} \Rightarrow b = \frac{10 \cdot \operatorname{sen} 65°}{\operatorname{sen} 75°} = 9'38 \text{ km}$$

A emisora atópase a 6'65 km de B e a 9'38 km de A.

38.- Nun adestramento de fútbol colócase o balón nun punto situado a 5 m e 8 m de cada un dos postes da portería, cuxo ancho é de 7 m. Baixo que ángulo calculas que se ve a portería desde ese punto?



$$\cos \hat{A} = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 8^2 - 7^2}{2 \cdot 5 \cdot 8}$$

$$\hat{A} = \arccos\left(\frac{40}{80}\right) = \arccos\left(\frac{1}{2}\right) = \underline{\underline{60°}}$$

A portería vese baixo un ángulo de 60°.

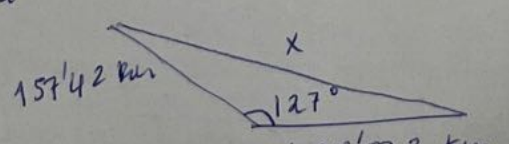
39.- Dous barcos parten dun porto con rumbos distintos que forman un ángulo de 127°. O primeiro sae ás 10 h da mañá cunha velocidade de 17 nós, e o segundo sae ás 11 h 30 min, cunha velocidade de 26 nós. Se o alcance dos seus equipos de radio é de 150 km, poderán poñerse en contacto ás 3 da tarde?

17 nós $= 17 \cdot 1'852$ km/h $= 31'484$ km/h

26 nós $= 26 \cdot 1'852$ km/h $= 48'152$ km/h

10 mañá ás 3 tarde: 5 h $\Rightarrow$ 1º recorreu: $e = v \cdot t = 31'484 \text{ km/h} \cdot 5 \text{ h} = 157'42$ km

11 h 30' mañá ás 3 tarde: 3'5 h $\Rightarrow$ 2º recorreu: $e = v \cdot t = 48'152 \text{ km/h} \cdot 3'5 \text{ h} = 168'532$ km



$x^2 = 157'42^2 + 168'532^2 - 2 \cdot 157'42 \cdot 168'532 \cdot \cos 127°$

$x = 291'72$ km > 150 km

TH. COSENO

NON PODERÁN POÑERSE EN CONTACTO!

FÓRMULAS TRIGONOMÉTRICAS.

40.- Calcula o coseno e a tanxente da suma.

Solución.-

Como $sen(\alpha+\beta)=sen\,\alpha\cos\beta+\cos\alpha\,sen\,\beta$

$$\cos(\alpha+\beta)\overset{\text{ángulos complementarios}}{=}sen(90-(\alpha+\beta))=sen((90-\alpha)+(-\beta))=$$

$$=sen(90-\alpha)\cos(-\beta)+\cos(90-\alpha)sen(-\beta)\overset{\text{ángulos complementarios}}{=}\cos\alpha\cos\beta-sen\,\alpha\,sen\,\beta$$

$$\tan(\alpha+\beta)=\frac{sen(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{sen\,\alpha\cos\beta+\cos\alpha\,sen\,\beta}{\cos\alpha\cos\beta-sen\,\alpha\,sen\,\beta}\overset{\text{dividiendo por}\cos\alpha\cos\beta}{=}\frac{\frac{sen\,\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\,sen\,\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{sen\,\alpha\,sen\,\beta}{\cos\alpha\cos\beta}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

41.- Determina as razóns trigonométricas do ángulo de 75° se sabes que 75° = 30° + 45°.

$75^\circ = 30^\circ + 45^\circ$

$sen(75^\circ) = sen(30^\circ+45^\circ) = sen\,30^\circ\cdot\cos 45^\circ+\cos 30^\circ\cdot sen\,45^\circ = \frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}=$

$=\frac{\sqrt{2}+\sqrt{6}}{4}$

$\cos(75^\circ) = \cos(30^\circ+45^\circ) = \cos 30^\circ\cdot\cos 45^\circ - sen\,30^\circ\cdot sen\,45^\circ = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=$

$=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\tan(75^\circ) = \tan(30^\circ+45^\circ) = \frac{sen\,75^\circ}{\cos 75^\circ} = \frac{\frac{\sqrt{2}+\sqrt{6}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}-\sqrt{2}}$

$=\frac{(\sqrt{2}+\sqrt{6})(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} = \frac{(\sqrt{2}+\sqrt{6})^2}{6-2} = \frac{2+6+2\sqrt{12}}{4} = \frac{8+4\sqrt{3}}{4} =$

$=2+\sqrt{3}$

42.- Calcula o seno, coseno e a tanxente da diferenza.

Solución.-

$$sen(\alpha-\beta)=sen(\alpha+(-\beta))\overset{\text{seno de la suma}}{=}sen\,\alpha\cos(-\beta)+\cos\alpha\,sen(-\beta)=sen\,\alpha\cos(\beta)+\cos\alpha\cdot(-sen(\beta))=$$
$$=sen\,\alpha\cos(\beta)-\cos\alpha\,sen(\beta)$$

$$\cos(\alpha-\beta)=\cos(\alpha+(-\beta))\overset{\text{cos de la suma}}{=}\cos\alpha\cos(-\beta)-sen\,\alpha\,sen(-\beta)=\cos\alpha\cos(\beta)-sen\,\alpha\cdot(-sen(\beta))=$$
$$=\cos\alpha\cos(\beta)+sen\,\alpha\,sen(\beta)$$

$$\tan(\alpha-\beta)=\frac{sen(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{sen\,\alpha\cos\beta-\cos\alpha\,sen\,\beta}{\cos\alpha\cos\beta+sen\,\alpha\,sen\,\beta}\overset{\text{dividiendo por}\cos\alpha\cos\beta}{=}\frac{\frac{sen\,\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\cos\alpha\,sen\,\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{sen\,\alpha\,sen\,\beta}{\cos\alpha\cos\beta}}=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

43.- Calcula as razóns trigonométricas do ángulo dobre.

Solución.-

$$sen(2\alpha)=sen(\alpha+\alpha)\overset{\text{seno de la suma}}{=}sen\,\alpha\cos(\alpha)+\cos\alpha\, sen(\alpha)=2\,sen\,\alpha\cos\alpha$$

$$\cos(2\alpha)=\cos(\alpha+\alpha)=\cos(\alpha+(-\beta))\overset{\text{cos de la suma}}{=}\cos\alpha\cos(\alpha)-sen\,\alpha\, sen(\alpha)=\cos^2\alpha-sen^2\alpha$$

$$\tan(2\alpha)=\tan(\alpha+\alpha)\overset{\text{tanxente da suma}}{=}\frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

44.- Demostra a seguinte igualdade: $\dfrac{\cos(a+b)+\cos(a-b)}{sen(a+b)+sen(a-b)}=\dfrac{1}{\tan a}$

$$\frac{\cos(a+b)+\cos(a-b)}{sen(a+b)+sen(a-b)}\overset{\substack{\text{FÓRMULAS}\\\text{TRIGONOMÉTRICAS}}}{=}\frac{(\cos a\cdot\cos b-sen\,a\cdot sen\,b)+(\cos a\cdot\cos b+sen\,a\,sen\,b)}{(sen\,a\cos b+\cos a\,sen\,b)+(sen\,a\cos b-\cos a\,sen\,b)}$$

$$=\frac{\cancel{2}\cos a\cancel{\cos b}}{\cancel{2}\,sen\,a\cancel{\cos b}}=\frac{\cos a}{sen\,a}=\frac{1}{\tan a}$$

c.q.d.

45.- Demostra que: $\dfrac{2\,sen\,\alpha-sen\,2\,\alpha}{2\,sen\,\alpha+sen\,2\,\alpha}=\dfrac{1-\cos\alpha}{1+\cos\alpha}$

$$\frac{2\,sen\,\alpha-sen(2\alpha)}{2\,sen\,\alpha+sen(2\alpha)}\overset{\text{FÓRM. TRIGON.}}{=}\frac{2\,sen\alpha-2\,sen\alpha\cos\alpha}{2\,sen\alpha+2\,sen\,\alpha\cos\alpha}\overset{\text{SACAR FACTOR COMÚN}}{=}\frac{\cancel{2sen\alpha}(1-\cos\alpha)}{\cancel{2sen\alpha}(1+\cos\alpha)}$$

$$=\frac{1-\cos\alpha}{1+\cos\alpha}$$

c.q.d.

46.- Demostra que:

a) $\dfrac{sen(\alpha+\beta)}{sen(\alpha-\beta)}=\dfrac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

$$\frac{sen(\alpha+\beta)}{sen(\alpha-\beta)}\overset{\text{FÓRM. TRIG.}}{=}\frac{sen\alpha\cdot\cos\beta+\cos\alpha\,sen\,\beta}{sen\,\alpha\cos\beta-\cos\alpha\cdot sen\,\beta}\overset{\substack{\text{DIVIDO NUMERADOR}\\\text{Y DENOMINADOR POR }\cos\alpha\cdot\cos\beta}}{=}\frac{\dfrac{sen\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}}+\dfrac{\cancel{\cos\alpha}\,sen\,\beta}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{sen\,\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}}-\dfrac{\cancel{\cos\alpha}\,sen\,\beta}{\cancel{\cos\alpha}\cos\beta}}$$

$$=\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$$

c.q.d.

b) $\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=\dfrac{1+\tan\alpha\cdot\tan\beta}{1-\tan\alpha\cdot\tan\beta}$

DIVIDO NUMERADOR Y DENOMINADOR ENTRE $\cos\alpha\cdot\cos\beta$

FÓRM. TRIG.

$$\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} \overset{\downarrow}{=} \frac{\cos\alpha\cos\beta+\operatorname{sen}\alpha\cdot\operatorname{sen}\beta}{\cos\alpha\cdot\cos\beta-\operatorname{sen}\alpha\cdot\operatorname{sen}\beta} \overset{\downarrow}{=} \frac{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\operatorname{sen}\alpha\operatorname{sen}\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\operatorname{sen}\alpha\operatorname{sen}\beta}{\cos\alpha\cos\beta}}=$$

$$=\frac{1+\tan\alpha\cdot\tan\beta}{1-\tan\alpha\tan\beta}$$

c.q.d.

47- Simplifica a expresión $\dfrac{sen\,2\,\alpha}{1-\cos^2\alpha}$ e calcula o seu valor para $\alpha=\dfrac{\pi}{4}$

FÓRM. TRIG. SIMPLIFICANDO

$$\frac{\operatorname{sen}(2\alpha)}{1-\cos^2\alpha} \overset{\downarrow}{=} \frac{2\operatorname{sen}\alpha\cos\alpha}{\operatorname{sen}^2\alpha} \overset{\downarrow}{=} \frac{2\cos\alpha}{\operatorname{sen}\alpha} = 2\cdot\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{2}{\tan\alpha}$$

$\cos^2\alpha+\operatorname{sen}^2\alpha=1 \Rightarrow 1-\cos^2\alpha=\operatorname{sen}^2\alpha$

Se $\alpha=\frac{\pi}{4} \Rightarrow \frac{2}{\tan\alpha}=\frac{2}{\tan\frac{\pi}{4}}=\frac{2}{1}=2$

48.- Calcula as razóns trigonométricas do ángulo metade.

Solución.-

$E_1 \quad \cos^2\alpha+sen^2\alpha=1$

$E_2 \quad \cos^2\alpha-sen^2\alpha=\cos(2\alpha)$

Restando E_1-E_2: $\quad 2\,sen^2\alpha=1-\cos(2\alpha)$

Despexando: $sen\,\alpha=\pm\sqrt{\dfrac{1-\cos(2\alpha)}{2}}$ Ou o que é o mesmo,

$sen\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos(\alpha)}{2}}$ Escollemos o signo + ou o - dependendo do cadrante onde se atopa o ángulo $\dfrac{\alpha}{2}$

$E_1 \quad \cos^2\alpha+sen^2\alpha=1$

$E_2 \quad \cos^2\alpha-sen^2\alpha=\cos(2\alpha)$

Sumando E_1+E_2: $\quad 2\cos^2\alpha=1+\cos(2\alpha)$

Despexando: $\cos\alpha=\pm\sqrt{\dfrac{1+\cos(2\alpha)}{2}}$ Ou o que é o mesmo,

$\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1+\cos(\alpha)}{2}}$ Escollemos o signo + ou o - dependendo do cadrante onde se atopa o ángulo $\dfrac{\alpha}{2}$

$$\tan\left(\frac{\alpha}{2}\right)=\frac{sen\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)}=\frac{\pm\sqrt{\frac{1-\cos(\alpha)}{2}}}{\pm\sqrt{\frac{1+\cos(\alpha)}{2}}}=\pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}}$$

49.- Indica as razóns trigonométricas do ángulo de 15° de dous xeitos, considerando:

a) 15° = 45° – 30°

$sen\,15° = sen(45°-30°) = sen\,45°\cdot\cos 30° - \cos 45°\cdot sen\,30° = \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2} - \frac{\sqrt2}{2}\cdot\frac{1}{2} = \frac{\sqrt6-\sqrt2}{4}$

$\cos 15° = \cos(45°-30°) = \cos 45°\cos 30° + sen\,45°\,sen\,30° = \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2} + \frac{\sqrt2}{2}\cdot\frac{1}{2} = \frac{\sqrt6+\sqrt2}{4}$

Racionalizamos

$\tan 15° = \frac{sen\,15°}{\cos 15°} = \frac{\frac{\sqrt6-\sqrt2}{4}}{\frac{\sqrt6+\sqrt2}{4}} = \frac{\sqrt6-\sqrt2}{\sqrt6+\sqrt2} = \frac{(\sqrt6-\sqrt2)(\sqrt6-\sqrt2)}{(\sqrt6+\sqrt2)(\sqrt6-\sqrt2)} = \frac{6+2-2\sqrt{12}}{6-2}$

$\tan 15 = \frac{8-2\sqrt{12}}{4} = \frac{8-4\sqrt3}{4} = 2-\sqrt3$

b) 15° = 30°/2

15° en 1º cuad.

$sen\,15° = sen\left(\frac{30}{2}\right) = +\sqrt{\frac{1-\cos 30}{2}} = +\sqrt{\frac{1-\frac{\sqrt3}{2}}{2}} = +\sqrt{\frac{\frac{2-\sqrt3}{2}}{\frac{2}{1}}} = \sqrt{\frac{2-\sqrt3}{4}}$

$\cos 15° = \cos\left(\frac{30}{2}\right) = +\sqrt{\frac{1+\cos 30}{2}} = +\sqrt{\frac{1+\frac{\sqrt3}{2}}{2}} = +\sqrt{\frac{\frac{2+\sqrt3}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt3}{4}}$

$\tan 15° = \frac{sen\,15°}{\cos 15°} = \frac{\frac{\sqrt{2-\sqrt3}}{2}}{\frac{\sqrt{2+\sqrt3}}{2}} = \frac{\sqrt{2-\sqrt3}}{\sqrt{2+\sqrt3}} = \sqrt{\frac{2-\sqrt3}{2+\sqrt3}} = \sqrt{\frac{4+3-4\sqrt3}{1}}$

Racion.

$= \sqrt{7-4\sqrt3}$

50.- Demostra que: $2\tan\alpha\cdot sen^2\left(\dfrac{\alpha}{2}\right)+sen\,\alpha=\tan\alpha$

$2\tan\alpha\,sen^2\left(\frac{\alpha}{2}\right) + sen\,\alpha = 2\cdot\frac{sen\,\alpha}{\cos\alpha}\left(\sqrt{\frac{1-\cos\alpha}{2}}\right)^2 + sen\,\alpha$

$= \cancel{2}\cdot\frac{sen\,\alpha}{\cos\alpha}\;\frac{1-\cos\alpha}{\cancel{2}} + sen\,\alpha = sen\,\alpha\left(\frac{1-\cos\alpha}{\cos\alpha}+1\right) =$

$= sen\,\alpha\left(\frac{1-\cos\alpha+\cos\alpha}{\cos\alpha}\right) = sen\,\alpha\cdot\frac{1}{\cos\alpha} = \frac{sen\,\alpha}{\cos\alpha} = \tan\alpha$

c.q.d.

51.- Demostra que: $\frac{2\,sen\,\alpha - sen(2\alpha)}{2\,sen\,\alpha + sen(2\alpha)} = \tan^2\left(\frac{\alpha}{2}\right)$

$$\frac{2\,sen\,\alpha - sen(2\alpha)}{2\,sen\,\alpha + sen(2\alpha)} = \frac{2\,sen\,\alpha - 2\,sen\,\alpha\cos\alpha}{2\,sen\,\alpha + 2\,sen\,\alpha\cos\alpha} = \frac{\cancel{2\,sen\,\alpha}(1-\cos\alpha)}{\cancel{2\,sen\,\alpha}(1+\cos\alpha)} =$$

$$= \frac{1-\cos\alpha}{1+\cos\alpha} = \left(\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\right)^2 = \left(\tan\frac{\alpha}{2}\right)^2 = \tan^2\left(\frac{\alpha}{2}\right)$$

c.q.d.

52.- Sabemos que $\cos x = \frac{-3}{4}$ e $sen\,x < 0$. Sen determinar o valor de x, calcula:

a) $sen\,x$ x está no 3º cuadrante $sen^2 x + \cos^2 x = 1 \Rightarrow$ x esta no 3º cuadrante

$\Rightarrow sen^2 x + \left(-\frac{3}{4}\right)^2 = 1 \Rightarrow sen^2 x = 1 - \frac{9}{16} \Rightarrow sen^2 x = \frac{7}{16} \Rightarrow sen\,x = -\frac{\sqrt{7}}{4}$

b) $\cos(\pi + x) = \cos\pi\cdot\cos x - sen\,\pi\cdot sen\,x = (-1)\cdot\cos x - 0\cdot sen\,x = -1\cdot\cos x - 0 =$

$= -\cos x = \frac{3}{4}$

c) $\cos(2x) = \cos^2 x - sen^2 x = \left(-\frac{3}{4}\right)^2 - \left(-\frac{\sqrt{7}}{4}\right)^2 = \frac{9}{16} - \frac{7}{16} = \frac{2}{16} = \frac{1}{8}$

d) $\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \pm\sqrt{\frac{1-\left(-\frac{3}{4}\right)}{1+\left(-\frac{3}{4}\right)}} = \pm\sqrt{\frac{1+\frac{3}{4}}{1-\frac{3}{4}}} = \pm\sqrt{\frac{\frac{7}{4}}{\frac{1}{4}}} =$

$= -\sqrt{7}$

$\frac{x}{2}$ no 2º cuad.

X está no 3º cuad.

$180° < x < 270°$

$90° < \frac{x}{2} < 135°$

$\frac{x}{2}$ no 2º cuad.

$\tan\left(\frac{x}{2}\right)$ é $-$

e) $sen\left(\frac{\pi}{2} - x\right) = sen\,\frac{\pi}{2}\cdot\cos x - \cos\frac{\pi}{2}\cdot sen\,x =$

$= 1\cdot\cos x - 0\cdot sen\,x = \cos x = -\frac{3}{4}$

f) $\cos\left(\pi - \frac{x}{2}\right) = \cos\pi\cdot\cos\frac{x}{2} + sen\,\pi\cdot sen\,\frac{x}{2} = (-1)\cdot\sqrt{\frac{1+\cos x}{2}} + 0\cdot\sqrt{\frac{1-\cos x}{2}} =$

$= -1\cdot\sqrt{\frac{1+\left(-\frac{3}{4}\right)}{2}} + 0 = -\sqrt{\frac{\frac{1}{4}}{\frac{2}{1}}} = -\sqrt{\frac{1}{8}} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$

53.- Demostra as igualdades :

a) $senA+sen\,B=2\,sen\,\frac{A+B}{2}\cos\frac{A-B}{2}$

b) $senA-sen\,B=2\cos\frac{A+B}{2}sen\,\frac{A-B}{2}$

c) $\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

d) $\cos A-\cos B=-2\,sen\frac{A+B}{2}sen\frac{A-B}{2}$

Solución:

Se $A=\alpha+\beta$ e $B=\alpha-\beta$

Entón, $\alpha=\frac{A+B}{2}$ e $\beta=\frac{A-B}{2}$

Como , E_1: $sen(\alpha+\beta)=sen\,\alpha\cos\beta+\cos\alpha\,sen\,\beta$

E_2: $sen(\alpha-\beta)=sen\,\alpha\cos\beta-\cos\alpha\,sen\,\beta$

Sumando E_1+E_2 : $sen(\alpha+\beta)+sen(\alpha-\beta)=2\,sen\,\alpha\cos\beta$

Facendo o cambio : $senA+sen\,B=2\,sen\,\frac{A+B}{2}\cos\frac{A-B}{2}$

Restando E_1-E_2: $sen(\alpha+\beta)-sen(\alpha-\beta)=2\cos\alpha\,sen\,\beta$

Facendo o cambio : $senA-sen\,B=2\cos\frac{A+B}{2}sen\,\frac{A-B}{2}$

Como , E_3: $\cos(\alpha+\beta)=\cos\alpha\cos\beta-sen\,\alpha\,sen\,\beta$

E_4 : $\cos(\alpha-\beta)=\cos\alpha\cos(\beta)+sen\,\alpha\,sen\,\beta$

Sumando E_3+E_4: $\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$

Facendo o cambio : $\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Restando E_3-E_4: $\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\,sen\,\alpha\,sen\,\beta$

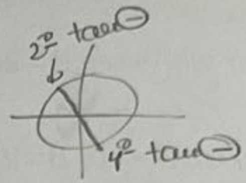
Facendo o cambio : $\cos A-\cos B=-2\,sen\frac{A+B}{2}sen\frac{A-B}{2}$

<u>Exemplo:</u> Transforma en produto $\cos 3x+cosx$

<u>Solución:</u> $\cos 3x+cosx=2\cos 3x+x\,over\,2\cos\frac{3x-x}{2}=2\cos(2x)\cos x$

ECUACIÓNS TRIGONOMÉTRICAS.-

54.- Escribe, en radiáns, a expresión xeral de todos os ángulos que verifican:



a) $\tan x = -\sqrt{3}$ $\Rightarrow$ $x_1 = 120^\circ + 360^\circ k = \frac{2\pi}{3} + 2k\pi$

$x_2 = 300^\circ + 360^\circ k = \frac{5\pi}{3} + 2k\pi \quad k \in \mathbb{Z}$

$\tan 60^\circ = +\sqrt{3}$

$180^\circ - 60^\circ = 120^\circ \leftarrow$ 2º CUAD. tan ⊖

$360^\circ - 60 = 300^\circ \leftarrow$ 4º CUAD. tan ⊖

b) $sen\, x = \cos x$ $\Rightarrow \frac{sen\, x}{\cos x} = 1 \Rightarrow \tan x = 1$

(1º cuad.) tan ⊕

$x_1 = 45^\circ + 360^\circ k = \frac{\pi}{4} + 2k\pi \quad k \in \mathbb{Z}$

$x_2 = 225^\circ + 360^\circ k = \frac{5\pi}{4} + 2k\pi$

$180 + 45^\circ$ (3º cuad) tan ⊕

c) $sen^2 x = 1 \Rightarrow sen\, x = \pm 1$

$\Rightarrow$ $x_1 = 90^\circ + 360^\circ k = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$

$x_2 = 270^\circ + 360 k = \frac{3\pi}{2} + 2k\pi$

d) $senx = \tan x$ $\Rightarrow sen\, x = \frac{sen\, x}{\cos x} \Rightarrow sen x \cdot \cos x - sen\, x = 0 \Rightarrow$

SACAMOS FACTOR COMÚN

$sen\, x (\cos x - 1) = 0$

$sen\, x = 0$ $\rightarrow x_1 = 0^\circ + 360^\circ k = 2k\pi$

$\rightarrow x_2 = 180^\circ + 360^\circ k = \pi + 2k\pi$

$\cos x - 1 = 0 \Rightarrow \cos x = 1 \longrightarrow x_3 = 0^\circ + 360^\circ k = 2k\pi \quad k \in \mathbb{Z}$

$\boxed{x = k\pi \quad k \in \mathbb{Z}}$

55.- Resolve estas ecuacións:

a) $2\cos^2 x + cosx - 1 = 0$

Facendo $t = \cos x$ $\quad 2t^2 + t - 1 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+8}}{2 \cdot 2} = \frac{-1 \pm 3}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$

Desfago o cambio, $t = \cos x = \frac{1}{2}$ $\rightarrow x_1 = 60^\circ + 360^\circ k = \frac{\pi}{3} + 2k\pi$

$\rightarrow x_2 = 300^\circ + 360^\circ k = \frac{5\pi}{3} + 2k\pi \quad k \in \mathbb{Z}$

$t = \cos x = -1 \rightarrow x_3 = 180^\circ + 360^\circ k = \pi + 2k\pi$

b) $2sen^2 x - 1 = 0$ $\Rightarrow 2 sen^2 x = 1 \Rightarrow sen^2 x = \frac{1}{2} \Rightarrow sen\, x = \pm\sqrt{\frac{1}{2}} \Rightarrow$

$\Rightarrow sen\, x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

$x_1 = 45^\circ + 360^\circ k = \frac{\pi}{4} + 2k\pi$

$x_2 = 135^\circ + 360^\circ k = \frac{3\pi}{4} + 2k\pi$

$x_3 = 225^\circ + 360^\circ k = \frac{5\pi}{4} + 2k\pi$

$x_4 = 315^\circ + 360^\circ k = \frac{7\pi}{4} + 2k\pi \quad k \in \mathbb{Z}$

c) $\tan^2 x + tanx = 0$

SACAMOS FACTOR COMÚN $\Rightarrow \tan x (\tan x + 1) = 0$

$\tan x = 0$ $\rightarrow x_1 = 0^\circ + 360^\circ k = 2k\pi$

$\rightarrow x_2 = 180^\circ + 360^\circ k = \pi + 2k\pi$

$\tan x = -1$ $\rightarrow x_3 = 135^\circ + 360^\circ k = \frac{3\pi}{4} + 2k\pi$

2º CUA.

$\rightarrow x_4 = 315^\circ + 360^\circ k = \frac{7\pi}{4} + 2k\pi \quad k \in \mathbb{Z}$

4º CUA.

$sen^2 x = 1 - cos^2 x$

d) $2\,sen^2 x + 3\cos x = 3 \Longrightarrow 2\cdot(1-\cos^2 x) + 3\cos x = 3 \Rightarrow 2 - 2\cos^2 x + 3\cos x = 3 \Rightarrow$

$-2\cos^2 x + 3\cos x - 1 = 0 \Longrightarrow 2\cos^2 x - 3\cos x + 1 = 0 \underset{t=\cos x}{\Longrightarrow} 2t^2 - 3t + 1 = 0 \Rightarrow$

$\Rightarrow t = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3\pm 1}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$ Desfago cambio, $t=1=\cos x \Rightarrow x_1 = 0^\circ + 360k = 2k\pi$

$t = \frac{1}{2} = \cos x \Rightarrow x_2 = 60^\circ + 360^\circ k = \frac{\pi}{3} + 2k\pi$

$x_3 = 300^\circ + 360^\circ k = \frac{5\pi}{3} + 2k\pi$

$k \in \mathbb{Z}$

Solucións: a) 60°,180° e 300°; b) -45°, 45°, 135°, 225° e 315°; c) 0°, 45°, 180° e 225°; d) -60°, 0°, 60°
e 300°

$\cos(2x) = \cos^2 x - sen^2 x$

56.- Resolve:

$sen^2 x = 1 - \cos^2 x$

a) $4\cos 2x + 3\cos x = 1 \Rightarrow 4(\cos^2 x - sen^2 x) + 3\cos x = 1 \Rightarrow 4\cos^2 x - 4 sen^2 x + 3\cos x = 1 \Rightarrow$

$\Rightarrow 4\cos^2 x - 4(1-\cos^2 x) + 3\cos x = 1 \Rightarrow 8\cos^2 x + 3\cos x - 5 = 0 \underset{t=\cos x}{\Longrightarrow}$

$\Rightarrow 8t^2 + 3t - 5 = 0$

$t_1 = \frac{5}{8} = \cos x \rightarrow x_1 = 51^\circ\, 19'\, 4'' + 360^\circ k$

$x_2 = 308^\circ\, 40'\, 56'' + 360^\circ k$

$t_2 = \cos x = -1 \rightarrow x_3 = 180^\circ + 360^\circ k$ $k \in \mathbb{Z}$

b) $\tan 2x + 2\cos x = 0 \Rightarrow \frac{2\tan x}{1-\tan^2 x} + 2\cos x = 0$ $1 - \tan^2 x \neq 0$ (solucións non válidas)

Multiplicando $1-\tan^2 x$

$\Rightarrow 2\tan x + 2\cos x(1-\tan^2 x) = 0 \Rightarrow 2\tan x + 2\cos x - 2\cos x \tan^2 x = 0 \Rightarrow$

$\Rightarrow 2\frac{sen x}{\cos x} + 2\cos x - 2\cos x \frac{sen^2 x}{\cos^2 x} = 0 \underset{\cos x \neq 0}{\overset{\text{Multipl. cos x}}{\Longrightarrow}} 2 sen x + 2\cos^2 x - 2 sen^2 x = 0 \Rightarrow$

$\Rightarrow 2 sen x + 2(1 - sen^2 x) - 2 sen^2 x = 0 \Rightarrow -4 sen^2 x + 2 sen x + 2 = 0 \underset{t = sen x}{\Longrightarrow}$

$\Rightarrow +4t^2 - 2t - 2 = 0$

$t = 1 = sen x \rightarrow x_1 = 90^\circ + 360^\circ k$ Se $\cos x = 0 \rightarrow x_4 = 270^\circ + 360^\circ k$

SOLUCIÓN VÁLIDA

$t = -\frac{1}{2} = sen x \rightarrow x_2 = 210^\circ + 360^\circ k$ $k \in \mathbb{Z}$

$x_3 = 330^\circ + 360^\circ k$

c) $\sqrt{2}\cos\left(\frac{x}{2}\right) - \cos x = 1$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\Rightarrow \sqrt{2}\cdot\left(\pm\sqrt{\frac{1+\cos x}{2}}\right) - \cos x = 1 \Rightarrow \pm\sqrt{1+\cos x} - \cos x = 1 \Rightarrow \left(\pm\sqrt{1+\cos x}\right)^2 = (1+\cos x)^2$

$\Rightarrow 1 + \cos x = 1 + \cos^2 x + 2\cos x \Rightarrow \cos^2 x + \cos x = 0 \underset{\text{COMÚN}}{\overset{\text{SACAR FACT.}}{\Longrightarrow}} \cos x(\cos x + 1) = 0$

$\cos x = 0 \Rightarrow x_1 = 90^\circ + 360^\circ k$

$x_2 = 270^\circ + 360^\circ k \rightarrow$ Non válida $k \in \mathbb{Z}$

$\cos x + 1 = 0 \Rightarrow \cos x = -1 \Rightarrow x_3 = 180^\circ + 360^\circ k$

Hai que comprobar as solucións pq. ecuación con raíces!

d) $2\,sen\,x\cos^2 x - 6\,sen^3 x = 0$

$2 sen x(\cos^2 x - 3 sen^2 x) = 0$

$sen x = 0 \Rightarrow x_1 = 0^\circ + 360^\circ k$ $k \in \mathbb{Z}$

$x_2 = 180^\circ + 360^\circ k$

$\cos^2 x - 3 sen^2 x = 0 \Rightarrow 1 - sen^2 x - 3 sen^2 x = 0 \Rightarrow$

$\cos^2 x = 1 - sen^2 x$

$\Rightarrow 1 - 4 sen^2 x = 0 \Rightarrow sen^2 x = \frac{1}{4}$

$\Downarrow$

$sen x = \pm\frac{1}{2}$

$x_3 = 30^\circ + 360^\circ k$

$x_4 = 150^\circ + 360^\circ k$ $k \in \mathbb{Z}$

$x_5 = 210^\circ + 360^\circ k$

$x_6 = 330^\circ + 360^\circ k$

Solucións: a) -51° 19' 4", 51° 19' 4" e 180° ; b) -30°, 90°, 210°, 270° e 330°; c) 90° e 180° ; d) 30°, 150°, 210° e 330°

57.- Resolve as seguintes ecuacións:

a) $2\cos^2 x - sen^2 x + 1 = 0 \xrightarrow{\cos^2 x = 1 - sen^2 x} 2\cdot(1-sen^2 x) - sen^2 x + 1 = 0 \Rightarrow$

$\Rightarrow 2 - 2sen^2 x - sen^2 x + 1 = 0 \Rightarrow -3sen^2 x + 3 = 0 \Rightarrow 3sen^2 x = 3 \Rightarrow$

$\Rightarrow sen^2 x = 1 \Rightarrow sen x = \pm 1$

$x_1 = 90° + 360° k$
$x_2 = 270° + 360° k$ $\quad k \in \mathbb{Z}$

Tamén se poden expresar todas as solucións como:

$\boxed{x = 90° + 180° k = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}}$

b) $sen^2 x - sen x = 0$

$sen x\,(sen x - 1) = 0$

$sen x = 0 \rightarrow x_1 = 0° + 360° k$, $x_2 = 180° + 360° k$

$sen x - 1 = 0 \Rightarrow sen x = 1 \Rightarrow x_3 = 90° + 360° k$

$k \in \mathbb{Z}$

Tamén se poden expresar todas as solucións como:

$\boxed{x = 90° + 180° k \quad k \in \mathbb{Z}}$

c) $2\cos^2 x - \sqrt{3}\cos x = 0 \Rightarrow \cos x\,(2\cos x - \sqrt{3}) = 0$

$\cos x = 0 \Rightarrow x_1 = 90° + 360° k$, $x_2 = 270° + 360° k$ $\quad k \in \mathbb{Z}$

$2\cos x - \sqrt{3} = 0 \Rightarrow \cos x = \frac{\sqrt{3}}{2}$ → $x_3 = 30° + 360° k$, $x_4 = 330° + 360° k$

58.- Resolve:

a) $sen^2 x - \cos^2 x = 1 \xrightarrow{\cos^2 x = 1 - sen^2 x} sen^2 x - (1 - sen^2 x) = 1 \Rightarrow 2sen^2 x - 2 = 0 \Rightarrow$

$\Rightarrow sen^2 x = 1 \Rightarrow sen x = \pm 1$ → $x_1 = 90° + 360° k$, $x_2 = 270° + 360° k$ $\quad k \in \mathbb{Z}$

Pódense expresar: $\boxed{x = 90° + 180° k \quad k \in \mathbb{Z}}$

b) $\cos^2 x - sen^2 x = 0$

$\Downarrow \leftarrow sen^2 x = 1 - \cos^2 x$

$\cos^2 x - (1 - \cos^2 x) = 0 \Rightarrow 2\cos^2 x - 1 = 0 \Rightarrow \cos^2 x = \frac{1}{2} \Rightarrow$

$\Rightarrow \cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$ → $x_1 = 45° + 360° k$, $x_2 = 135° + 360° k$, $x_3 = 225° + 360° k$, $x_4 = 315° + 360° k$ $\quad k \in \mathbb{Z}$

$\cos^2 x = 1 - \text{sen}^2 x$

c) $2\cos^2 x + senx = 1 \Rightarrow 2(1-\text{sen}^2 x) + \text{sen}\,x = 1 \Rightarrow -2\,\text{sen}^2 x + \text{sen}\,x + 1 = 0$

$\Rightarrow 2\,\text{sen}^2 x - \text{sen}\,x - 1 = 0 \xRightarrow{t=\text{sen}\,x} 2t^2 - t - 1 = 0 \to t = 1,\ t = -\frac{1}{2}$

$t = \text{sen}\,x = 1 \Rightarrow x_1 = 90° + 360° k$

$t = \text{sen}\,x = -\frac{1}{2} \Rightarrow x_2 = 210° + 360° k \quad k \in \mathbb{Z}$

$x_3 = 330° + 360° k$

O sen é ⊖ no 3º e 4º cuadrante

d) $3\tan^2 x - \sqrt{3}\,tanx = 0 \xRightarrow{\text{SACAR FACT. COMÚN}} 3\tan x\left(\tan x - \frac{\sqrt{3}}{3}\right) = 0$

$\tan x = 0 \to x_1 = 0° + 360° k,\ x_2 = 180° + 360° k \quad k \in \mathbb{Z}$

$\tan x - \frac{\sqrt{3}}{3} = 0 \Rightarrow \tan x = \frac{\sqrt{3}}{3} \to x_3 = 30° + 360° k,\ x_4 = 210° + 360° k$

Non entra no exame!

59.-

a) Transforma en produto $sen3x - senx$ e resolve despois a ecuación $sen3x - senx = 0$.

Coa FÓRMULA: $\text{sen}\,A - \text{sen}\,B = 2\cos\left(\frac{A+B}{2}\right)\text{sen}\left(\frac{A-B}{2}\right)$

$\text{sen}(3x) - \text{sen}\,x = 2\cos\left(\frac{3x+x}{2}\right)\cdot \text{sen}\left(\frac{3x-x}{2}\right)$

$\text{sen}(3x) - \text{sen}\,x = 2\cos(2x)\cdot\text{sen}(x)$

Se $2\cos(2x)\cdot\text{sen}\,x = 0$:

$\cos(2x) = 0 \to 2x = 90° + 360° k \to x_3 = 45° + 180° k$; $2x = 270° + 360° k \to x_4 = 135° + 180° k$

$\text{sen}\,x = 0 \to x_1 = 0° + 360° k,\ x_2 = 180° + 360° k \quad k \in \mathbb{Z}$

b) Transforma en produto $\cos 3x + cosx$ e resolve despois a ecuación $\cos 3x + cosx = 0$.

Coa FÓRMULA: $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\cos 3x + \cos x = 2\cdot\cos(2x)\cdot\cos x$

Se $2\cos(2x)\cdot\cos x = 0$:

$\cos x = 0 \to x_1 = 90° + 360° k,\ x_2 = 270° + 360° k \quad k \in \mathbb{Z}$

$\cos(2x) = 0 \to x_3 = 45° + 180° k,\ x_4 = 135° + 180° k$

60.- Resolve:

a) $sen\left(\frac{\pi}{6} - x\right) + \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{2}$

$\left(\text{sen}\frac{\pi}{6}\cdot\cos x - \cos\frac{\pi}{6}\cdot\text{sen}\,x\right) + \left(\cos\frac{\pi}{3}\cos x + \text{sen}\frac{\pi}{3}\cdot\text{sen}\,x\right) = \frac{1}{2}$

$\frac{1}{2}\cos x - \cancel{\frac{\sqrt{3}}{2}\text{sen}\,x} + \frac{1}{2}\cos x + \cancel{\frac{\sqrt{3}}{2}\text{sen}\,x} = \frac{1}{2}$

$\cos x = \frac{1}{2} \to x_1 = 60° + 360° k,\ x_2 = 300° + 360° k \quad k \in \mathbb{Z}$

cos ⊕ no 1º e 4º cuadrante!

b) $sen\,2x - 2\cos^2 x = 0 \xrightarrow[\downarrow]{sen(2x) = 2\,senx\cos x} 2\,senx\cos x - 2\cos^2 x = 0 \xrightarrow[\downarrow]{\text{SACAR FACTOR COMÚN}}$

$2\cos x\,(senx - \cos x) = 0$

$\rightarrow \cos x = 0 \;\Rightarrow\; x_1 = 90° + 360°k$, $\; x_2 = 270° + 360°k$

$\rightarrow sen\,x = \cos x \;\Rightarrow\; x_3 = 45° + 360°k$, $\; x_4 = 225° + 360°k$

$k \in \mathbb{Z}$

c) $\cos 2x - 3\,sen\,x + 1 = 0 \xrightarrow[\downarrow]{\cos(2x) = \cos^2 x - sen^2 x} (\cos^2 x - sen^2 x) - 3\,sen\,x + 1 = 0 \xrightarrow[\downarrow]{\cos^2 x = 1 - sen^2 x}$

$\Rightarrow 1 - sen^2 x - sen^2 x - 3\,sen\,x + 1 = 0 \Rightarrow -2\,sen^2 x - 3\,sen\,x + 2 = 0 \Rightarrow$

$\Rightarrow 2\,sen^2 x + 3\,sen\,x - 2 = 0 \xrightarrow[\downarrow]{t = sen\,x} 2t^2 + 3t - 2 = 0$

$t_1 = \frac{1}{2} = sen\,x$

$t_2 = \boxed{-2 = sen\,x}$ NON porque $-1 \leq sen\,x \leq 1$

$t = \frac{1}{2} = sen\,x \;\Rightarrow\; x_1 = 30° + 360°k$, $\; x_2 = 150° + 360°k$, $\; k \in \mathbb{Z}$

d) $sen\left(\frac{\pi}{4} + x\right) - \sqrt{2}\,senx = 0 \Rightarrow sen\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\,sen\,x - \sqrt{2}\,sen\,x = 0$

$\Rightarrow \frac{\sqrt{2}}{2}\cos x + \underbrace{\frac{\sqrt{2}}{2}\,sen\,x - \sqrt{2}\,sen\,x}_{} = 0 \Rightarrow \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\,sen\,x = 0$

$\Rightarrow \frac{\sqrt{2}}{2}(\cos x - sen\,x) = 0 \Rightarrow \cos x = sen\,x \Rightarrow \tan x = 1 \Rightarrow$

tan (+) no 1º e 3º cuadrante!

$x_1 = 45° + 360°k$, $\; x_2 = 225° + 360°k$, $\; k \in \mathbb{Z}$

61.-

a) Demostra que $\frac{sen\alpha + sen\,2\alpha}{1 + \cos\alpha + \cos 2\alpha} = \tan\alpha$

$$\frac{sen\,\alpha + sen(2\alpha)}{1 + \cos\alpha + \cos(2\alpha)} = \frac{sen\,\alpha + 2\,sen\alpha\cos\alpha}{1 + \cos\alpha + \cos^2\alpha - sen^2\alpha} \overset{sen^2\alpha = 1 - \cos^2\alpha}{=} \frac{sen\alpha + 2\,sen\alpha\cos\alpha}{\cancel{1} + \cos\alpha + \cos^2\alpha \cancel{- 1} + \cos^2\alpha}$$

$$= \frac{sen\,\alpha\,(1 + 2\cos\alpha)}{2\cos^2\alpha + \cos\alpha} = \frac{sen\,\alpha\,\cancel{(1 + 2\cos\alpha)}}{\cos\alpha\,\cancel{(2\cos\alpha + 1)}} = \frac{sen\,\alpha}{\cos\alpha} = \tan\alpha$$

c.q.d.

b) Calcula sen utilizar a calculadora: $2\,sen\frac{5\pi}{4}+2\sqrt{6}\cos\frac{11\pi}{6}-4\,sen\frac{2\pi}{3}-2\cos\frac{4\pi}{3}+sen\frac{3\pi}{2}$

$2\,sen 225^\circ + 2\sqrt{6}\cdot\cos 330^\circ - 4\cdot sen 120^\circ - 2\cos 240^\circ + sen 270^\circ =$

$= 2\cdot(-sen 45^\circ) + 2\sqrt{6}\cdot\cos 30^\circ - 4\cdot sen 60^\circ - 2\cdot(-\cos 60) + sen 270^\circ =$

$= 2\cdot\left(-\frac{\sqrt{2}}{2}\right) + 2\sqrt{6}\cdot\frac{\sqrt{3}}{2} - 4\cdot\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} + (-1) =$

$= -\sqrt{2} + \sqrt{18} - 2\sqrt{3} + 1 - 1 = -\sqrt{2} + 3\sqrt{2} - 2\sqrt{3} + 0 = 2\sqrt{2} - 2\sqrt{3}$

62.- Sabendo que $sen\alpha=\frac{3}{5}$ e α é un ángulo do primeiro cuadrante, e que $\beta=180^\circ+\alpha$, utiliza as fórmulas trigonométricas para calcular **expresando o resultado con fraccións ou radicais**:

a) Coseno e tanxente de α

$\cos^2\alpha + sen^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - sen^2\alpha \Rightarrow \cos^2\alpha = 1-\left(\frac{3}{5}\right)^2 \Rightarrow \cos^2\alpha = 1-\frac{9}{25}$

$\Rightarrow \cos^2\alpha = \frac{16}{25} \Rightarrow \cos\alpha = +\sqrt{\frac{16}{25}} = +\frac{4}{5}$

α no 1º cuad.

$\tan\alpha = \frac{sen\alpha}{\cos\alpha} = \frac{3/5}{4/5} = \frac{3}{4}$

b) Seno, coseno e tanxente de β

$sen(\underbrace{180^\circ+\alpha}_{\beta}) = sen 180^\circ\cdot\cos\alpha + \cos 180^\circ\cdot sen\alpha = 0\cdot\cos\alpha - 1\cdot sen\alpha = -\frac{3}{5} = sen\beta$

$\cos(180^\circ+\alpha) = \cos 180^\circ\cdot\cos\alpha - sen 180^\circ\cdot sen\alpha = -1\cos\alpha - 0\cdot sen\alpha = -\frac{4}{5} = \cos\beta$

$\tan\beta = \tan(180^\circ+\alpha) = \frac{sen\beta}{\cos\beta} = \frac{-3/5}{-4/5} = +\frac{3}{4}$

c) $\cos 2\alpha$

$\cos(2\alpha) = \cos^2\alpha - sen^2\alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

d) $\tan\left(\frac{\beta}{2}\right)$

no 1º cua.

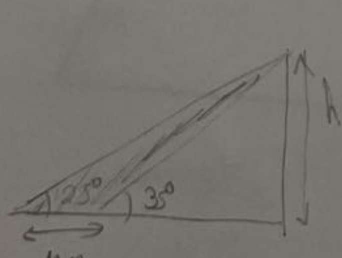
$0 \le \alpha \le 90^\circ \Rightarrow 180^\circ \le \underbrace{180+\alpha}_{\beta} \le 270^\circ \Rightarrow 90^\circ \le \frac{\beta}{2} \le 135^\circ$

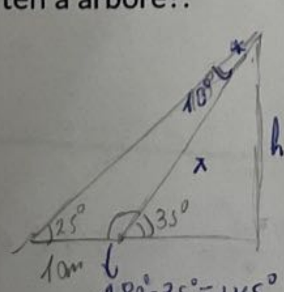
$\Rightarrow \frac{\beta}{2}$ está no 2º cuadrante
entón $\tan\left(\frac{\beta}{2}\right)$ é $\ominus$

$\tan\frac{\beta}{2} = -\sqrt{\frac{1-\cos\beta}{1+\cos\beta}} = -\sqrt{\frac{1-\left(-\frac{4}{5}\right)}{1+\left(-\frac{4}{5}\right)}} = -\sqrt{\frac{\frac{9}{5}}{\frac{1}{5}}}$

$\tan\frac{\beta}{2} = -\sqrt{9} = -3$

63.- Unha árbore e un observador están en beiras opostas dun río. O observador mide o ángulo que forma súa visual co punto máis alto da árbore e obtén 35°. Retrocede 10 metros e ao medir de novo o ángulo obtén 25°. Que altura ten a árbore?.





$\frac{10}{sen 10^\circ} = \frac{x}{sen 25^\circ} \Rightarrow x = \frac{10\cdot sen 25^\circ}{sen 10^\circ} = 24'34\,m$

$sen 35^\circ = \frac{h}{x} \Rightarrow h = x\cdot sen 35^\circ =$

$= 24'34\cdot sen 35^\circ =$

$= 13'96\,m$

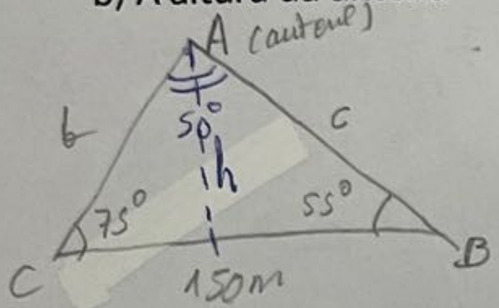
$*\ 180^\circ - (145+25) = 180^\circ - 170^\circ = 10^\circ$

A altura da árbore é 13'96 m.

64.-Unha antena reprodutora de sinais de radio é observada desde dous puntos do chan separados entre eles 150 metros. Os ángulos que forman as visuais á parte superior da antena coa horizontal son 75° e 55°. Calcula:

a) As distancias de cada punto de observación á parte superior da antena.

b) A altura da antena



$$\hat{A} = 180° - (75° + 55°) = 180° - 130° = 50°$$

$$\frac{150}{\text{sen } 50°} = \frac{c}{\text{sen } 75°} \Rightarrow c = \frac{150 \text{ sen } 75°}{\text{sen } 50°} = 189'14 \text{ m}$$

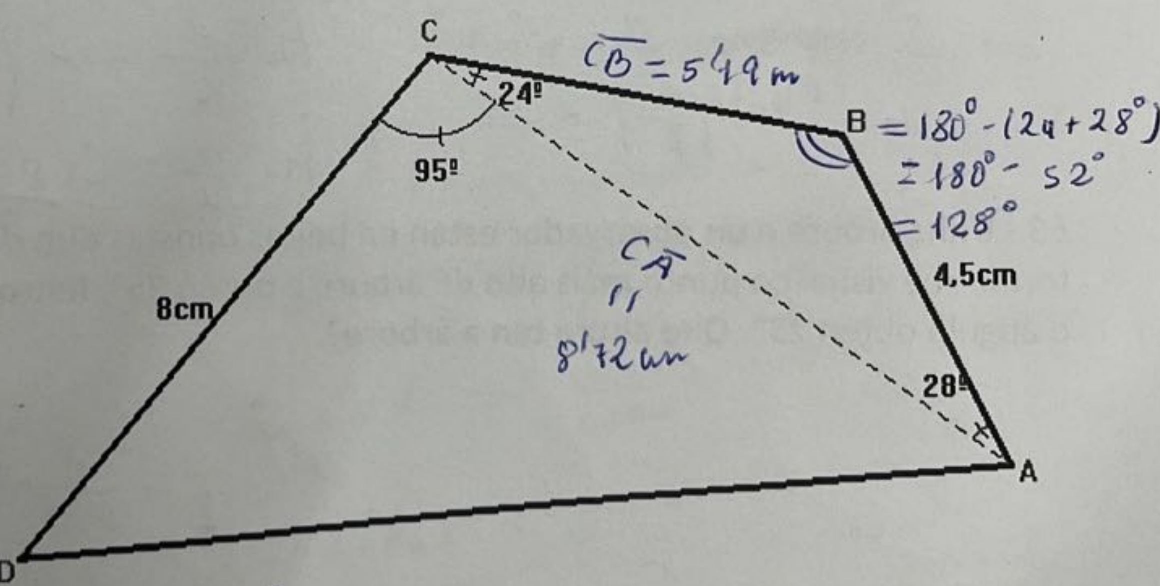
$$\frac{150}{\text{sen } 50°} = \frac{b}{\text{sen } 55°} \Rightarrow b = \frac{150 \cdot \text{sen } 55°}{\text{sen } 50°} = 160'40 \text{ m}$$

a) A distancia ao punto C é 160'40 m e ao punto B é 189'14 m.

b) $\text{sen } 55° = \frac{h}{c} = \frac{h}{189'14} \Rightarrow h = 189'14 \cdot \text{sen } 55° = 154'93 \text{ m}$

A altura da antena é 154'93 m.

65.- Calcula o perímetro do cuadrilátero adxunto :



Por th. seno, en $\hat{CBA}$

$$\frac{\overline{CB}}{\text{sen } 28°} = \frac{4'5}{\text{sen } 24°} \Rightarrow \overline{CB} = \frac{4'5 \cdot \text{sen } 28°}{\text{sen } 24°} = 5'19 \text{ cm}$$

$$\frac{\overline{CA}}{\text{sen}\,128^\circ} = \frac{4'5}{\text{sen}\,24^\circ} \Rightarrow \overline{CA} = \frac{4'5 \cdot \text{sen}\,128^\circ}{\text{sen}\,24^\circ} = 8'72 \text{ cm}$$

Por Th. coseno en $\overset{\Delta}{DCA}$:

$$\overline{DA}^2 = 8^2 + 8'72^2 - \underline{2 \cdot 8 \cdot 8'72 \cdot \cos 95^\circ} = 64 + 76'0384 + 12'15997$$

$$\overline{DA}^2 = 152'19837 \Rightarrow \overline{DA} \cong 12'34$$

Perímetro $= \overline{DC} + \overline{CB} + \overline{BA} + \overline{DA} = 8 + 5'19 + 4'5 + 12'34$

Perímetro cuadrilátero $= 30'03$ cm

66.- Resolve as seguintes ecuacións trigonométricas

a) $\cos 2x = 1 + 4\cos x$

$\Downarrow \leftarrow \cos(2x) = \cos^2 x - \text{sen}^2 x$

$\cos^2 x - \text{sen}^2 x = 1 + 4\cos x \overset{\text{sen}^2 x = 1 - \cos^2 x}{\Longrightarrow} \cos^2 x - (1 - \cos^2 x) = 1 + 4\cos x \Rightarrow$

$\Rightarrow -1 + 2\cos^2 x = 1 + 4\cos x \Longrightarrow 2\cos^2 x - 4\cos x - 2 = 0 \Longrightarrow$

$\overset{t = \cos x}{\Longrightarrow} 2t^2 - 4t - 2 = 0 \Rightarrow t^2 - 2t - 1 = 0 \nearrow t_1 = 1 - \sqrt{2} = \cos x$

$\searrow t_2 = \boxed{1 + \sqrt{2} = \cos x}$ Non! $-1 \leq \cos x \leq 1$

$\cos x = 1 - \sqrt{2} \Rightarrow$ $\boxed{\begin{array}{l} x_1 = \arccos(1 - \sqrt{2}) \simeq 114^\circ\, 28'\, 11'' + 360^\circ k \\ x_2 = 245^\circ\, 31'\, 49'' + 360^\circ k \quad k \in \mathbb{Z} \end{array}}$

$-0'4142\ldots$

$\hookrightarrow 180^\circ + \underbrace{(180^\circ - x_1)}_{\alpha \text{ no } 1^\circ \text{ cuadrante } = 65^\circ 31' 49''} = 245^\circ\, 31'\, 49''$ $\swarrow \cos\theta$ no ~~2º e~~ 3º

b) $2\tan x \cos^2\left(\dfrac{x}{2}\right) - \text{sen}\,x = 1$

$\Downarrow \leftarrow \cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}} \Rightarrow \cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2}$

$\cancel{2}\tan x \cdot \left(\frac{1+\cos x}{\cancel{2}}\right) - \text{sen}\,x = 1 \Rightarrow \frac{\text{sen}\,x}{\cos x}(1+\cos x) - \text{sen}\,x = 1$

$\overset{\text{Multiplicamos por } \cos x \neq 0}{\Longrightarrow} \text{sen}\,x(1+\cos x) - \text{sen}\,x\cos x = \cos x \Longrightarrow$

$\Rightarrow \text{sen}\,x + \cancel{\text{sen}\,x\cos x} - \cancel{\text{sen}\,x\cos x} - \cos x = 0 \Longrightarrow$

$\Rightarrow \text{sen}\,x = \cos x$ $\boxed{\begin{array}{l} x_1 = 45^\circ + 360^\circ k \quad k \in \mathbb{Z} \\ x_2 = 225^\circ + 360^\circ k \end{array}}$

Se $\cos x = 0 \rightarrow x_3 = 90^\circ + 360^\circ k$ (Non válida)

$\rightarrow x_4 = 270^\circ + 360^\circ k$ (Non válida)