EXERCICIOS TRIGONOMETRÍA. RESOLUCIÓN DE TRIÁNGULOS.

1.- Responde:

a) Cantos radiáns corresponden aos 360° dunha circunferencia? 2π radiáns

b) Cantos graos mide 1 radián? $\approx 57^\circ 17' 45''$

$\left.\begin{array}{l} 180° \text{ — } \pi \text{ rad.} \\ x \text{ — } 1 \text{ ''} \end{array}\right\} \quad x = \frac{180}{\pi} \simeq 57^\circ 17' 45''$

c) Cantos graos mide un ángulo que ten $\frac{\pi}{2}$ radiáns? 90°

d) Cantos radiáns equivalen a 270°? $\frac{3\pi}{2}$

2.- Expresa en graos sesaxesimais os seguintes ángulos dados en radiáns:

a) $\frac{\pi}{6} = 30^\circ$ b) $\frac{2\pi}{3} = 120^\circ$ c) $\frac{4\pi}{3} = 240^\circ$

d) $\frac{5\pi}{4} = 225^\circ$ e) $\frac{7\pi}{6} = 210^\circ$ f) $\frac{9\pi}{2} = 810^\circ$

3.- Pasa a radiáns os seguintes ángulos dados en graos. Exprésaos en función de π e en forma decimal.

a) $40° = \frac{2\pi}{9}$

$\left.\begin{array}{l} 180° \text{ — } \pi \text{ rad} \\ 40° \text{ — } x \end{array}\right\} \Rightarrow x = \frac{40\pi}{180} = \frac{2\pi}{9}$

b) $108° = \frac{3\pi}{5}$

$\left.\begin{array}{l} 180° \text{ — } \pi \text{ rad} \\ 108° \text{ — } x \end{array}\right\} \Rightarrow x = \frac{108\pi}{180} = \frac{3\pi}{5}$

c) $135° = \frac{3\pi}{4}$

d) $240° = \frac{4\pi}{3}$

e) $270° = \frac{3\pi}{2}$

f) $126° = \frac{7\pi}{10}$

$\left.\begin{array}{l} 180 \text{ — } \pi \text{ rad} \\ 126 \text{ — } x \end{array}\right\}$

$x = \frac{126\pi}{180} = \frac{63\pi}{90} = \frac{7\pi}{10}$

4.- Calcula as demais razóns trigonométricas do ángulo α ($0° < \alpha < 90°$) utilizando as relacións fundamentais:

a) $sen\,\alpha = \frac{\sqrt{3}}{2}$

$sen^2\alpha + cos^2\alpha = 1 \Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + cos^2\alpha = 1 \Rightarrow$

$\Rightarrow \frac{3}{4} + cos^2\alpha = 1 \Rightarrow cos^2\alpha = \frac{1}{4} \Rightarrow cos\,\alpha = \pm\sqrt{\frac{1}{4}} \xRightarrow{\alpha \text{ no } 1^{\underline{o}} \text{ cuad.}} \boxed{cos\,\alpha = +\frac{1}{2}}$

$\boxed{tan\,\alpha} = \frac{sen\,\alpha}{cos\,\alpha} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2\sqrt{3}}{2} = \boxed{\sqrt{3}}$

$\boxed{sec\,\alpha} = \frac{1}{cos\,\alpha} = \frac{1}{\frac{1}{2}} = \boxed{2}$

$\boxed{cosec\,\alpha} = \frac{1}{sen\,\alpha} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$\boxed{cotan\,\alpha} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

b) $cos\,\alpha = \frac{\sqrt{2}}{2}$

$sen^2\alpha + cos^2\alpha = 1 \Rightarrow sen^2\alpha + \left(\frac{\sqrt{2}}{2}\right)^2 = 1 \Rightarrow$

$\Rightarrow sen^2\alpha + \frac{2}{4} = 1 \Rightarrow sen^2\alpha = \frac{2}{4} \Rightarrow sen\,\alpha = \pm\sqrt{\frac{2}{4}} = \pm\frac{\sqrt{2}}{2} \xRightarrow{\alpha \text{ está no } 1^{\underline{o}} \text{ cuad.}} \boxed{sen\,\alpha = +\frac{\sqrt{2}}{2}}$

$\boxed{tan\,\alpha} = \frac{sen\,\alpha}{cos\,\alpha} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$

c) $tan\,\alpha = \frac{\sqrt{3}}{2}$

$1 + tan^2\alpha = \frac{1}{cos^2\alpha} \Rightarrow 1 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{cos^2\alpha} \Rightarrow 1 + \frac{3}{4} = \frac{1}{cos^2\alpha} \Rightarrow$

$\Rightarrow \frac{7}{4} = \frac{1}{cos^2\alpha} \Rightarrow cos^2\alpha = \frac{4}{7} \Rightarrow \boxed{cos\,\alpha} = \pm\sqrt{\frac{4}{7}} \overset{\alpha \text{ no } 1^{\underline{o}} \text{ cuad.}}{=} +\frac{2}{\sqrt{7}} = \boxed{\frac{2\sqrt{7}}{7}}$

Como $tan\,\alpha = \frac{sen\,\alpha}{cos\,\alpha} \Rightarrow \frac{\sqrt{3}}{2} = \frac{sen\,\alpha}{\frac{2\sqrt{7}}{7}} \Rightarrow sen\,\alpha = \frac{\sqrt{3} \cdot \frac{2\sqrt{7}}{7}}{\frac{2}{1}} = \frac{\sqrt{3} \cdot \cancel{2} \cdot \sqrt{7}}{7 \cdot \cancel{2}}$

$\boxed{sen\,\alpha = \frac{\sqrt{21}}{7}}$

d) $sen\,\alpha = \frac{3}{8}$ $sen^2\alpha + cos^2\alpha = 1 \Rightarrow \left(\frac{3}{8}\right)^2 + cos^2\alpha = 1 \Rightarrow \frac{9}{64} + cos^2\alpha = 1 \Rightarrow$

α no 1º cuadrante

$\Rightarrow cos^2\alpha = 1 - \frac{9}{64} \Rightarrow cos^2\alpha = \frac{55}{64} \Rightarrow \boxed{cos\,\alpha} = \pm\sqrt{\frac{55}{64}} \stackrel{\checkmark}{=} \boxed{+\frac{\sqrt{55}}{8}}$

$\boxed{tan\,\alpha} = \frac{sen\,\alpha}{cos\,\alpha} = \frac{\frac{3}{8}}{\frac{\sqrt{55}}{8}} = \frac{3\cdot\cancel{8}}{\cancel{8}\cdot\sqrt{55}} = \boxed{\frac{3\sqrt{55}}{55}}$

$0'72 = \frac{72}{100} = \frac{36}{50} = \frac{18}{25}$

e) $cos\,\alpha = 0{,}72$ $sen^2\alpha + cos^2\alpha = 1 \Rightarrow sen^2\alpha + (0'72)^2 = 1 \stackrel{\checkmark}{\Rightarrow} sen^2\alpha + \frac{324}{625} = 1$

✓ α no 1º cuadrante

$\Rightarrow sen^2\alpha = \frac{301}{625} \Rightarrow \boxed{sen\,\alpha} = \pm\sqrt{\frac{301}{625}} = \boxed{+\frac{\sqrt{301}}{25}}$

$\boxed{tan\,\alpha} = \frac{sen\,\alpha}{cos\,\alpha} = \frac{\frac{\sqrt{301}}{25}}{\frac{18}{25}} = \frac{2\cancel{5}\cdot\sqrt{301}}{\cancel{2}5\cdot 18} = \boxed{\frac{\sqrt{301}}{18}}$

f) $tan\,\alpha = 3$ Con sistema: $\left.\begin{array}{l} \frac{sen\,\alpha}{cos\,\alpha} = 3 \\ sen^2\alpha + cos^2\alpha = 1 \end{array}\right\} \Rightarrow$ Na E1 $sen\,\alpha = 3\cdot cos\,\alpha$ Substituíndo na E2 *

$* \; (3cos\,\alpha)^2 + cos^2\alpha = 1 \Rightarrow 9\,cos^2\alpha + cos^2\alpha = 1 \Rightarrow 10\,cos^2\alpha = 1 \Rightarrow$

α no 1º cuad.

$\Rightarrow cos^2\alpha = \frac{1}{10} \Rightarrow \boxed{cos\,\alpha} = \pm\sqrt{\frac{1}{10}} \stackrel{\checkmark}{=} +\frac{1}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$

$\boxed{sen\,\alpha} = 3\cdot cos\,\alpha = \boxed{\frac{3\cdot\sqrt{10}}{10}}$

5.- Indica las restantes razones trigonométricas de α (sin usar la calculadora para calcular α). Luego compruébalas utilizando la calculadora.

a) $sen\,\alpha = \frac{-4}{5}$ $\alpha < 270°$ Como $sen\,\alpha = -\frac{4}{5} < 0$ e $\alpha < 270° \Rightarrow \alpha$ está no 3º cuad.

$\Rightarrow cos\,\alpha < 0$ e $tan\,\alpha > 0$

$sen^2\alpha + cos^2\alpha = 1$

$\Downarrow$

$\left(-\frac{4}{5}\right)^2 + cos^2\alpha = 1 \Rightarrow cos^2\alpha = 1 - \frac{16}{25} \Rightarrow cos^2\alpha = \frac{9}{25} \Rightarrow \boxed{cos\,\alpha} = \pm\sqrt{\frac{9}{25}} = \boxed{-\frac{3}{5}}$

$\boxed{tan\,\alpha} = \frac{sen\,\alpha}{cos\,\alpha} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4\cdot 5}{5\cdot 3} = \boxed{\frac{4}{3}}$

b) $cos\,\alpha = \frac{2}{3}$ $tan\,\alpha < 0$ Como $cos\,\alpha = \frac{2}{3} > 0$ e $tan\,\alpha < 0$

α está no 4º cuadrante ⇒ $sen\,\alpha < 0$

✓ no 4º cuadrante

$sen^2\alpha + cos^2\alpha = 1 \Rightarrow sen^2\alpha + \left(\frac{2}{3}\right)^2 = 1 \Rightarrow sen^2\alpha = \frac{5}{9} \Rightarrow \boxed{sen\,\alpha} = -\sqrt{\frac{5}{9}} = \boxed{-\frac{\sqrt{5}}{3}}$

$\boxed{tan\,\alpha} = \frac{sen\,\alpha}{cos\,\alpha} = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\cancel{3}\sqrt{5}}{\cancel{3}\cdot 2} = \boxed{-\frac{\sqrt{5}}{2}}$

c) $\tan\alpha=-3 \quad \alpha<180^\circ$

Como $\tan\alpha<0$ e $\alpha<180^\circ \Rightarrow \alpha$ está no 2º cuadrante

$\Rightarrow \operatorname{sen}\alpha>0$ e $\cos\alpha<0$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \Rightarrow 1+(-3)^2=\frac{1}{\cos^2\alpha} \Rightarrow 10=\frac{1}{\cos^2\alpha} \Rightarrow$

α está no 2º cuad.

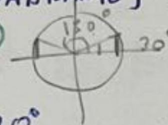
$\Rightarrow \cos^2\alpha=\frac{1}{10} \Rightarrow \cos\alpha=\pm\sqrt{\frac{1}{10}} \Rightarrow \boxed{\cos\alpha} = -\frac{1}{\sqrt{10}} = \boxed{\frac{-\sqrt{10}}{10}}$

Como $\tan\alpha=-3=\frac{\operatorname{sen}\alpha}{\cos\alpha} \Rightarrow \boxed{\operatorname{sen}\alpha} = -3\cdot\cos\alpha = \boxed{+\frac{3\sqrt{10}}{10}}$

6.- Relaciona as razóns trigonométricas dos seguintes ángulos coas dun ángulo do primeiro cuadrante:

a) 150° (NO 2º CUADRANTE) $180-\alpha$

$150^\circ = 180 - 30^\circ$



$\operatorname{sen}150^\circ = +\operatorname{sen}30^\circ$

$\cos 150^\circ = -\cos 30^\circ$

$\tan 150^\circ = -\tan 30^\circ$

b) 135° (NO 2º CUADRANTE) $180-\alpha$

$135^\circ = 180^\circ - 45^\circ$

$\operatorname{sen}135^\circ = +\operatorname{sen}45^\circ$

$\cos 135^\circ = -\cos 45^\circ$

$\tan 135^\circ = -\tan 45^\circ$

c) 210° (NO 3º CUADRANTE) $180+\alpha$

$210^\circ = 180^\circ + 30^\circ$

$\operatorname{sen}210^\circ = -\operatorname{sen}30^\circ$

$\cos 210^\circ = -\cos 30^\circ$

$\tan 210^\circ = +\tan 30^\circ$

d) 225° (NO 3º CUADRANTE) $180+\alpha$

$225^\circ = 180^\circ + 45^\circ$

$\operatorname{sen}225^\circ = -\operatorname{sen}45^\circ$

$\cos 225^\circ = -\cos 45^\circ$

$\tan 225^\circ = +\tan 45^\circ$

e) 315° (NO 4º CUADRANTE) $360-\alpha$

$315 = 360^\circ - 45^\circ$

$\operatorname{sen}315^\circ = -\operatorname{sen}45^\circ$

$\cos 315^\circ = +\cos 45^\circ$

$\tan 315^\circ = -\tan 45^\circ$

f) 120° (NO 2º CUADRANTE) $180-\alpha$

$120^\circ = 180^\circ - 60^\circ$

$\operatorname{sen}120^\circ = +\operatorname{sen}60^\circ$

$\cos 120^\circ = -\cos 60^\circ$

$\tan 120^\circ = -\tan 60^\circ$

g) 340° (NO 4º CUADRANTE) $360-\alpha$

$340^\circ = 360^\circ - 20^\circ$

$\operatorname{sen}340^\circ = -\operatorname{sen}20^\circ$

$\cos 340^\circ = +\cos 20^\circ$

$\tan 340^\circ = -\tan 20^\circ$

h) 200° (NO 3º CUADRANTE) $180^\circ+\alpha$

$200^\circ = 180^\circ + 20^\circ$

$\operatorname{sen}200^\circ = -\operatorname{sen}20^\circ$

$\cos 200^\circ = -\cos 20^\circ$

$\tan 200^\circ = +\tan 20^\circ$

i) 290° (NO 4º CUADRANTE) $360-\alpha$

$290^\circ = 360^\circ - 70^\circ$

$\operatorname{sen}290^\circ = -\operatorname{sen}70^\circ$

$\cos 290^\circ = +\cos 70^\circ$

$\tan 290^\circ = -\tan 70^\circ$

7.- Se $\operatorname{sen}\alpha=0{,}35$, determina:

α NO 1º

Como $\operatorname{sen}^2\alpha+\cos^2\alpha=1 \Rightarrow (0'35)^2+\cos^2\alpha=1 \Rightarrow$

$\Rightarrow \cos^2\alpha = 0'8775 \Rightarrow \cos\alpha \cong 0'9367$

a) $\operatorname{sen}(180^\circ-\alpha)$

$= \operatorname{sen}\alpha = 0'35$

b) $\operatorname{sen}(\alpha+90^\circ)$

$= \cos\alpha = 0'9367$

c) $\operatorname{sen}(180^\circ+\alpha)$

$= -\operatorname{sen}\alpha = -0'35$

d) $\operatorname{sen}(360^\circ-\alpha)$

$= -\operatorname{sen}\alpha =$

$= -0'35$

e) $\operatorname{sen}(90^\circ-\alpha)$

$= \cos\alpha =$

$\cong 0'9367$

f) $\operatorname{sen}(360^\circ+\alpha)$

$= \operatorname{sen}\alpha = 0'35$

8.- Calcula as razóns trigonométricas de 55°, 125°, 145°, 215°, 235°, 305° e 325° a partir das razón trigonométricas de 35°: *sen* 35° = 0,57; *cos* 35° = 0,82; tg 35° = 0,70.

$\boxed{55°} = 90° - 35$

$sen\ 55° = cos\ 35° = 0'82$

$cos\ 55° = sen\ 35° = 0'57$

$tan\ 55° = \frac{1}{tan\ 35°} = \frac{1}{0'7} = \frac{10}{7}$

$125° = 180° - 55°$

$sen\ 125° = sen\ 55° = cos\ 35° = 0'82$

$cos\ 125° = -cos\ 55° = -sen\ 35 = -0'57$

$tan\ 125° = -tan\ 55° = -\frac{1}{tan 35°} = -\frac{10}{7}$

$145° = 180° - 35°$

$sen\ 145° = sen\ 35° = 0'57$

$cos\ 145° = -cos\ 35° = -0'82$

$tan\ 145° = -tan\ 35° = -0'70$

$215° = 180° + 35°$

$sen\ 215° = -sen\ 35° = -0'57$

$cos\ 215° = -cos\ 35° = -0'82$

$tan\ 215° = tan\ 35° = -0'7$

$235° = 180° + 55°$

$sen\ 235° = -sen\ 55° = -cos\ 35° = -0'82$

$cos\ 235° = -cos\ 55° = -sen\ 35° = -0'57$

$tan\ 235° = tan\ 55° = \frac{1}{tan 35°} = \frac{10}{7}$

$305° = 360° - 55°$

$sen\ 305° = -sen\ 55° = -cos\ 35° = -0'8$

$cos\ 305° = cos\ 55° = sen\ 35° = 0'57$

$tan\ 305° = -tan\ 55° = -\frac{1}{tan 35°} = \frac{10}{7}$

$325° = 360 - 35°$

$sen\ 325° = -sen\ 35° = -0'57$

$cos\ 325° = cos\ 35° = 0'82$

$tan\ 325° = -tan\ 35° = -0'7$

9.- Se $\tan\alpha = \frac{2}{3}$ e $0° < \alpha < 90°$, determina: $1 + tan^2\alpha = \frac{1}{cos^2\alpha} \Rightarrow 1 + \frac{4}{9} = \frac{1}{cos^2\alpha} \Rightarrow \frac{13}{9} = \frac{1}{cos^2\alpha}$

$\Downarrow$

$cos^2\alpha = \frac{9}{13}$

α está no 1º cuadrant

$\boxed{cos\ \alpha = +\frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}}$

a) $sen(\alpha)$

$tan\ \alpha = \frac{sen\ \alpha}{cos\ \alpha}$

$\frac{2}{3} = \frac{sen\ \alpha}{\frac{3\sqrt{13}}{13}}$

$\boxed{sen\ \alpha = \frac{2 \cdot \frac{3\sqrt{13}}{13}}{\frac{3}{1}} = \frac{2 \cdot \cancel{3}\sqrt{13}}{\cancel{3} \cdot 13} = \frac{2\sqrt{13}}{13}}$

b) $\cos(\alpha)$

$\boxed{cos\ \alpha = \frac{3\sqrt{13}}{13}}$

c) $\tan(90° - \alpha)$

$\parallel$

$\frac{1}{tan\ \alpha} = \frac{3}{2}$

$\boxed{tan(90 - \alpha) = \frac{3}{2}}$

d) $sen(180° - \alpha)$

$sen(180° - \alpha) = sen\ \alpha$

$\boxed{sen(180° - \alpha) = \frac{2\sqrt{13}}{13}}$

e) $\cos(180° + \alpha)$

$cos(180° + \alpha) = -cos\ \alpha$

$\boxed{cos(180° + \alpha) = -\frac{3\sqrt{13}}{13}}$

f) $\tan(360° - \alpha)$

$tan(360° - \alpha) = -tan\ \alpha$

$\boxed{tan(360° - \alpha) = -\frac{2}{3}}$

10.- Calcula o resultado das seguintes operacións sen utilizar a calculadora:

a) $5\cos\frac{\pi}{2} - \cos 0 + 2\cos\pi - \cos\frac{3\pi}{2} + \cos 2\pi = 5 \cdot cos\ 90° - cos\ 0° + 2\ cos\ 180° - cos\ 270° + cos\ 360$

$= 5 \cdot 0 - 1 + 2 \cdot (-1) - 0 + 1 = 0 - 1 - 2 - 0 + 1 = -2$

b) $5\tan\pi+3\cos\frac{\pi}{2}+2\tan 0+sen\frac{3\pi}{2}-2\,sen\,2\pi$ = $5\cdot\tan 180^\circ + 3\cdot\cos 90 + 2\tan 0 + sen\,270 - 2sen\,360 =$

$= 5\cdot 0 + 3\cdot 0 + 2\cdot 0 + (-1) - 2\cdot 0 = \underline{\underline{-1}}$

c) $\frac{2}{3}sen\frac{\pi}{2}-4\,sen\frac{3\pi}{2}+3\,sen\,\pi-\frac{5}{3}sen\frac{\pi}{2}$ = $\frac{2}{3}sen\,90^\circ - 4\,sen\,270^\circ + 3\,sen\,180^\circ - \frac{5}{3}sen\,90^\circ =$

$= \frac{2}{3}\cdot 1 - 4\cdot(-1) + 3\cdot 0 - \frac{5}{3}\cdot 1 = \frac{2}{3} + 4 + 0 - \frac{5}{3} = \underline{\underline{3}}$

11.- Proba que:

a) $4\,sen\frac{\pi}{6}+\sqrt{2}\cos\frac{\pi}{4}+\cos\pi=2$

$4\cdot sen\frac{\pi}{6} + \sqrt{2}\cos\frac{\pi}{4} + \cos\pi = 4\cdot sen\,30^\circ + \sqrt{2}\cdot\cos 45^\circ + \cos 180^\circ =$

$= 4\cdot\frac{1}{2} + \sqrt{2}\cdot\frac{\sqrt{2}}{2} + (-1) = 2 + 1 - 1 = \underline{\underline{2}}$

b) $2\sqrt{3}\,sen\frac{2\pi}{3}+4\,sen\frac{\pi}{6}-2\,sen\frac{\pi}{2}=3$

REDUCIMOS 1er CUADRANTE

$2\sqrt{3}\,sen\frac{2\pi}{3} + 4\,sen\frac{\pi}{6} - 2\,sen\frac{\pi}{2} = 2\cdot\sqrt{3}\cdot sen\,120^\circ + 4\,sen\,30^\circ - 2\,sen\,90^\circ =$

$= 2\sqrt{3}\cdot sen\,60^\circ + 4\,sen\,30^\circ - 2\,sen\,90^\circ = 2\cdot\sqrt{3}\cdot\frac{\sqrt{3}}{2} + 4\cdot\frac{1}{2} - 2\cdot 1 =$

$\boxed{120^\circ = 180^\circ - 60^\circ}$

$= 3 + 2 - 2 = \underline{\underline{3}}$

12.- Calcula o resultado das seguintes operacións sen utilizar a calculadora:

a) $sen\frac{\pi}{4}+sen\frac{\pi}{2}+sen\,\pi$

$sen\frac{\pi}{4} + sen\frac{\pi}{2} + sen\,\pi = sen\,45^\circ + sen\,90^\circ + sen\,180^\circ =$

$= \frac{\sqrt{2}}{2} + 1 + 0 = 1 + \frac{\sqrt{2}}{2}$

b) $\cos\pi - \cos 0 + \cos\frac{\pi}{2} - \cos\frac{3\pi}{2}$

$\cos\pi - \cos 0 + \cos\frac{\pi}{2} - \cos\frac{3\pi}{2} = \cos 180^\circ - \cos 0^\circ + \cos 90^\circ - \cos 270^\circ =$

$= -1 - 1 + 0 - 0 = \underline{\underline{-2}}$

c) $sen\frac{2\pi}{3} - \cos\frac{7\pi}{6} + \tan\frac{4\pi}{3} + \tan\frac{11\pi}{6}$

$sen\frac{2\pi}{3} - \cos\frac{7\pi}{6} + \tan\frac{4\pi}{3} + \tan\frac{11\pi}{6} = sen\,120^\circ - \cos 210^\circ + \tan 240^\circ + \tan 330^\circ =$

REDUCCIÓN 1º CUADRANTE

$= sen\,60^\circ - (-\cos 30^\circ) + \tan 60^\circ - \tan 30^\circ = \frac{\sqrt3}{2} + \frac{\sqrt3}{2} + \sqrt3 - \frac{\sqrt3}{3} =$

$120^\circ = 180^\circ - 60^\circ \quad 240^\circ = 180^\circ + 60^\circ$

$210^\circ = 180^\circ + 30^\circ \quad 330^\circ = 360^\circ - 30^\circ$

$= \frac{3\sqrt3 + 3\sqrt3 + 6\sqrt3 - 2\sqrt3}{6} = \frac{10\sqrt3}{6}$

$= \frac{5\sqrt3}{3}$

13.- Calcula o resultado das seguintes operacións sen utilizar a calculadora:

a) $sen\frac{5\pi}{4} + \cos\frac{3\pi}{4} - sen\frac{7\pi}{4} = sen\,225^\circ + \cos 135^\circ - sen\,315^\circ =$ REDUCCIÓN 1er CUAD.

$= -sen\,45^\circ + (-\cos 45^\circ) - (-sen\,45^\circ) = -\frac{\sqrt2}{2} - \frac{\sqrt2}{2} + \frac{\sqrt2}{2} = \underline{\underline{\frac{-\sqrt2}{2}}}$

$225^\circ = 180^\circ + 45^\circ \quad 315^\circ = 360^\circ - 45^\circ$

$135^\circ = 180^\circ - 45^\circ$

b) $\cos\frac{5\pi}{3} + \tan\frac{4\pi}{3} - \tan\frac{7\pi}{6} = \cos 300^\circ + \tan 240^\circ - \tan 210^\circ =$

$= \cos 60^\circ + \tan 60^\circ - \tan 30^\circ = +\frac12 + \sqrt3 - \frac{\sqrt3}{3} = \frac{3 + 6\sqrt3 - 2\sqrt3}{6} = \frac{4\sqrt3 + 3}{6}$

$= \underline{\frac{2\sqrt3}{3} + \frac12}$

$300^\circ = 360^\circ - 60^\circ \quad 210^\circ = 180^\circ + 30^\circ$

$240^\circ = 180^\circ + 60^\circ$

c) $\sqrt3\cos\frac{\pi}{6} + sen\frac{\pi}{6} - \sqrt2\cos\frac{\pi}{4} - 2\sqrt3\,sen\frac{\pi}{3} = \sqrt3\cos 30^\circ + sen\,30^\circ - \sqrt2\cos 45^\circ - 2\sqrt3 \cdot sen\,60^\circ =$

$= \sqrt3 \cdot \frac{\sqrt3}{2} + \frac12 - \sqrt2 \cdot \frac{\sqrt2}{2} - \cancel{2}\sqrt3 \cdot \frac{\sqrt3}{\cancel{2}} =$

$= \frac32 + \frac12 - \frac22 - 3 = 2 - 1 - 3 = \underline{\underline{-2}}$

d) $-2\,sen\frac{5\pi}{4} + 2\sqrt6\cos\frac{5\pi}{6} - 2\cos\frac{5\pi}{3} + sen\frac{3\pi}{2} - \cos\pi = -2\,sen\,225^\circ + 2\sqrt6\cos 150^\circ - 2\cos 300^\circ + sen\,270^\circ - \cos 180^\circ$

REDUCCIÓN 1º CUADRANTE

$= -2 \cdot (-sen\,45^\circ) + 2\sqrt6 \cdot (-\cos 30^\circ) - 2 \cdot \cos 60^\circ + (-1) - (-1) =$

$225^\circ = 180^\circ + 45^\circ \quad 300^\circ = 360^\circ - 60^\circ$

$150^\circ = 180^\circ - 30^\circ$

$= -\cancel{2} \cdot \left(-\frac{\sqrt2}{\cancel{2}}\right) + \cancel{2}\sqrt6\left(-\frac{\sqrt3}{\cancel{2}}\right) - 2 \cdot \frac12 - 1 + 1 =$

$= \sqrt2 - 3\sqrt2 - 1 - 1 + 1 =$

$= \underline{\underline{-2\sqrt2 - 1}}$