NÚMEROS COMPLEXOS

EXERCICIOS.-

1.- Efectúa las siguientes operaciones en forma binómica:

a) $(6-5i)+(2-i)-2(-5+6i)$ S. $18-18i$

$(6-5i)+(2-i)-2\cdot(-5+6i) = 6-5i+2-i+10-12i =$

$= 18-18i$

b) $(2-3i)-(5+4i)+\frac{1}{2}(6-4i)$ S. $-9i$

$(2-3i)-(5+4i)+\frac{1}{2}\cdot(6-4i) = 2-3i-5-4i+3-2i =$

$= 0-9i = -9i$

c) $(3+2i)(4-2i)$ S. $16+2i$

$i^2=-1$

$(3+2i)\cdot(4-2i) = 12-6i+8i-4i^2 = 12-6i+8i-4\cdot(-1) =$

$= 16+2i$

d) $(2+3i)(5-6i)$ S. $28+3i$

$i^2=-1$

$(2+3i)\cdot(5-6i) = 10-12i+15i-18i^2 = 10-12i+15i-18\cdot(-1) =$

$= 28+3i$

* MULTIPLICAMOS NUMERADOR E DENOMINADOR POLO CONXUGADO DO DENOMINADOR

e) $\frac{2+4i}{4-2i} \overset{*}{=} \frac{2+4i}{4-2i}\cdot\frac{4+2i}{4+2i} = \frac{(2+4i)(4+2i)}{(4-2i)\cdot(4+2i)} =$ S. i

$= \frac{8+4i+16i+8i^2}{16+\cancel{8i}-\cancel{8i}-4i^2} = \frac{8+4i+16i-8}{16+4} = \frac{20i}{20} = i$

$i^2=-1$

f) $\frac{1+5i}{3+4i} \overset{*}{=} \frac{1+5i}{3+4i}\cdot\frac{3-4i}{3-4i} = \frac{(1+5i)\cdot(3-4i)}{(3+4i)(3-4i)} =$ S. $\frac{23}{25}+\frac{11}{25}i$

$= \frac{3-4i+15i-20i^2}{9-\cancel{12i}+\cancel{12i}-16i^2} = \frac{3-4i+15i+20}{9+16} = \frac{23+11i}{25} = \frac{23}{25}+\frac{11}{25}i$

$i^2=-1$

$(a+b)^0 \longrightarrow 1$

$(a+b)^1 \longrightarrow 1 \quad 1$

$(a+b)^2 \longrightarrow 1 \quad 2 \quad 1$

$1 \quad 3 \quad 3 \quad 1 \longleftarrow (a+b)^3$

$1 \quad 4 \quad 6 \quad 4 \quad 1 \longleftarrow (a+b)^4$

TRIÁNGULO PASCAL

* POLO CONXUGADO

g) $\frac{4-2i}{i} = \frac{4-2i}{i} \cdot \frac{(-i)}{(-i)} = \frac{(4-2i)\cdot(-i)}{i\cdot(-i)}$ S. $-2-4i$

$= \frac{-4i+2i^2}{-i^2} = \frac{-4i-2}{1} = -2-4i$

$i^2=-1$

h) $(2+i)^3 = 1\cdot 2^3\cdot i^0 + 3\cdot 2^2\cdot i^1 + 3\cdot 2^1\cdot i^2 + 1\cdot 2^0\cdot i^3 =$ S. $2+11i$

$= 8+12i+6i^2+i^3 = 8+12i-6-i = 2+11i$

$i^2=-1$

$i^3=-i$

i) $(2-3i)^4 = 1\cdot 2^4\cdot(-3i)^0 + 4\cdot 2^3\cdot(-3i)^1 + 6\cdot 2^2\cdot(-3i)^2 + 4\cdot 2^1\cdot(-3i)^3 + 1\cdot 2^0\cdot(-3i)^4 =$ S. $-119+120i$

$(2+(-3i))^4 = 1\cdot 16\cdot 1 + 4\cdot 8\cdot(-3)\cdot i + 6\cdot 4\cdot 9\cdot i^2 + 4\cdot 2\cdot(-27)\cdot i^3 + 1\cdot 1\cdot 81\cdot i^4$

$= 16 - 96i - 216 - 216i + 81 = -119+120i$

$i^2=-1$

$i^3=-i \;;\; i^4=1$

k) $(1+i)^3 = 1\cdot 1^3\cdot i^0 + 3\cdot 1^2\cdot i^1 + 3\cdot 1^1\cdot i^2 + 1\cdot 1^0\cdot i^3 =$ S. $-2+2i$

$i^2 \to -1$, $i^3 \to -i$

$= 1\cdot 1\cdot 1 + 3\cdot 1\cdot i + 3\cdot 1\cdot(-1) + 1\cdot 1(-i) =$

$= 1+3i-3-i = -2+2i$

l) $(2-3i)^3 = 1\cdot 2^3\cdot(-3i)^0 + 3\cdot 2^2\cdot(-3i)^1 + 3\cdot 2^1\cdot(-3i)^2 + 1\cdot 2^0\cdot(-3i)^3 =$ S. $-46-9i$

$(2+(-3i)^3) = 1\cdot 8\cdot 1 + 3\cdot 4\cdot(-3)\cdot i + 3\cdot 2\cdot 9\cdot i^2 + 1\cdot 1\cdot(-27)\cdot i^3$

$i^2 \to -1$, $i^3 \to -i$

$= 8 - 36i + 54\cdot(-1) - 27\cdot(-i) =$

$= 8 - 36i - 54 + 27i = -46-9i$

$i^7 = i^4\cdot i^3 = 1\cdot(-i) = -i$

m) $\frac{i^7-1}{1+i} = \frac{-i-1}{1+i} \cdot \frac{1-i}{1-i} = \frac{-i+i^2-1+i}{1\cancel{-i}\cancel{+i}-i^2}$ S. -1

$= \frac{\cancel{-i}-1-1\cancel{+i}}{1+1} = \frac{-2}{2} = -1$

n) $(-i)^{30}-(-i)^{31}-(-i^3) = (-1)^{30}\cdot i^{30} - (-1)^{31}\cdot i^{31} + i^3$ S. $-1-2i$

$= 1\cdot i^{30} - (-1)\cdot i^{31} + i^3 = 1\cdot i^{28}\cdot i^2 + 1\cdot i^{28}\cdot i^3 + i^3 =$

$= 1\cdot 1\cdot i^2 + 1\cdot 1\cdot i^3 + i^3 = -1-i-i = -1-2i$

$i^{28} = i^4 = 1 \;;\; i^3=-i \;;\; i^2=-1$

2.- Expresa los siguientes números complejos en forma polar:

a) $4+4\sqrt{3}i$ $\quad a=4\,;\ b=4\sqrt{3}$

S. $8_{60°}$

$r=|z|=\text{Módulo}=\sqrt{a^2+b^2}=\sqrt{4^2+(4\sqrt{3})^2}=\sqrt{16+48}=\sqrt{64}=8$

$\tan\alpha=\dfrac{b}{a}=\dfrac{\cancel{4}\sqrt{3}}{\cancel{4}}=\sqrt{3}\ \underset{\uparrow}{\Longrightarrow}\ \alpha=60°=\text{Argumento}$

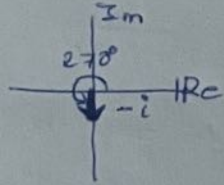
z está no 1º cuadrante

Entón, $z=4+4\sqrt{3}\,i=8_{60°}=r_\alpha$

b) $-i$ $\quad a=0\,;\ b=-1$

S. $1_{270°}$

$-i=z=1_{270°}$



Módulo $(-i)=1$

Argumento $(-i)=270°$

Tamén:

$r=|z|=\text{Módulo}=\sqrt{a^2+b^2}=\sqrt{0^2+(-1)^2}=1$

$\tan\alpha=\dfrac{-1}{0}=\pm\infty\ \underset{\uparrow}{\Longrightarrow}\ \alpha=270°$

α está

c) $3-3\sqrt{3}i$ $\quad a=3\,;\ b=-3\sqrt{3}$

S. $6_{300°}$

$r=|z|=\text{Módulo}=\sqrt{a^2+b^2}=\sqrt{3^2+(-3\sqrt{3})^2}=\sqrt{9+27}=\sqrt{36}=6$

$\tan\alpha=\dfrac{b}{a}=\dfrac{-3\sqrt{3}}{3}=-\sqrt{3}\ \underset{\uparrow}{\Longrightarrow}\ \alpha=-60°=300°=\text{Argumento}$

α está 4º cuadrante

Entón, $z=3-3\sqrt{3}\,i=6_{300°}$

d) $3+4i$ $\quad a=3\,;\ b=4$

S. $5_{53°8'}$

$r=|z|=\text{Módulo}=\sqrt{a^2+b^2}=\sqrt{3^2+4^2}=\sqrt{25}=5$

$\tan\alpha=\dfrac{b}{a}=\dfrac{4}{3}\ \underset{\uparrow}{\Longrightarrow}\ \alpha=\arctan\left(\dfrac{4}{3}\right)\cong 53°\,8'=\text{Argumento}$

α está 1º cuadrante

Entón, $z=3+4i=5_{53°8'}$

e) $3-4i$

S. $5_{306°52'}$

$a=3\,;\ b=-4$

$r=|z|=\text{Módulo}=\sqrt{3^2+(-4)^2}=\sqrt{25}=5$

$\tan\alpha=\dfrac{b}{a}=\dfrac{-4}{3}\ \underset{\uparrow}{\Longrightarrow}\ \alpha=\arctan\left(-\dfrac{4}{3}\right)=-53°\,7'\,48''=306°\,52'\,12''$

α está 4º cuadrante

Entón, $z=3-4i=5_{306°\,52'\,12''}$

f) $1-i$ $a=1$; $b=-1$

S. $\sqrt{2}_{315°}$

$r=|z|=$ Módulo $=\sqrt{1^2+(-1)^2}=\sqrt{2}$

$\tan\alpha=\frac{b}{a}=\frac{-1}{1}=-1 \Rightarrow \alpha=-45°=360°-45°=315°$

α está 4º cuadrante

Entón, $z=1-i=\sqrt{2}_{315°}$

g) $-2-5i$

S. $\sqrt{29}_{248°12'}$

$a=-2$; $b=-5$

$r=|z|=$ Módulo $=\sqrt{(-2)^2+(-5)^2}=\sqrt{4+25}=\sqrt{29}$

$\tan\alpha=\frac{-5}{-2}=\frac{b}{a}=\frac{5}{2} \Rightarrow \alpha=68°11'55''$ NON VALE ¡ESTÁ NO 1º CUADRANTE!

α está 3º cuadrante

Entón, $\alpha=180°+68°11'55''=248°11'55''$

$z=-2-5i=\boxed{\sqrt{29}}_{248°11'55''}$

h) -2 $a=-2$; $b=0$

S. $2_{180°}$

$z=-2=2_{180°}$

3.- Expresa los siguientes números complejos en forma binómica (Fórmula de Moivre):

a) $4_{30°}$

S. $2\sqrt{3}+2i$

$4_{30°}=4\cdot(\cos 30°+i\,\text{sen}\,30°)=4\cdot\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=2\sqrt{3}+2i$

b) $1_{30°}$

S. $\frac{\sqrt{3}}{2}+\frac{i}{2}$

$1_{30°}=1\cdot(\cos 30°+i\,\text{sen}\,30°)=1\cdot\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)$

$=\frac{\sqrt{3}}{2}+\frac{i}{2}$

c) $2_{180°}$

S. -2

$2_{180°}=2\cdot(\cos 180°+i\,\text{sen}\,180°)=2\cdot(-1+i\cdot 0)=-2$

d) $(\sqrt{3})_{\frac{5\pi}{6}}$

S. $\frac{-3}{2}+\frac{\sqrt{3}}{2}i$

$$(\sqrt{3})_{\frac{5\pi}{6}} = \sqrt{3}\cdot\left(\cos \overset{150^\circ}{\frac{5\pi}{6}} + i\,\text{sen}\,\overset{150^\circ}{\frac{5\pi}{6}}\right) = \sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)$$

$$= -\frac{3}{2}+\frac{\sqrt{3}}{2}i$$

$\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

$\text{sen}\,150^\circ = \text{sen}\,30^\circ = \frac{1}{2}$

e) 4_{120°

S. $-2+2\sqrt{3}i$

$$4_{120^\circ} = 4\cdot\left(\cos 120^\circ + i\,\text{sen}\,120^\circ\right) =$$

$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$

$\text{sen}\,120^\circ = \text{sen}\,60^\circ = \frac{\sqrt{3}}{2}$

$$= 4\cdot\left(-\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right) = -2+2\sqrt{3}\,i$$

f) 2_{100°

S.- $-0,3473+1,9696i$

$$2_{100^\circ} = 2\cdot\left(\cos 100^\circ + i\,\text{sen}\,100^\circ\right) =$$

$$= 2\cdot\left(-0'1736+i\,0'9248\right) = -0'3473+1'9696i$$

4.- Dados los números complejos $z=5_{45^\circ}$, $w=2_{15^\circ}$, $t=4i$, obtén en forma polar:

a) $z\cdot w$

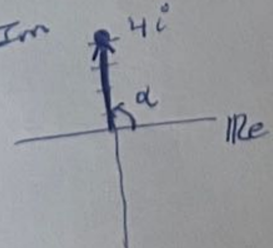
S. 10_{60°

$$z\cdot w = 5_{45^\circ}\cdot 2_{15^\circ} = (5\cdot 2)_{45^\circ+15^\circ} = 10_{60^\circ}$$

b) $\frac{z}{w^2} = \frac{5_{45^\circ}}{(2_{15^\circ})^2} = \frac{5_{45^\circ}}{4_{30^\circ}} = \left(\frac{5}{4}\right)_{45^\circ-30^\circ} = \left(\frac{5}{4}\right)_{15^\circ}$

S. $\left(\frac{5}{4}\right)_{15^\circ}$

$$t = 4i = 4_{90^\circ}$$



c) $\frac{z^3}{w\cdot t^2}$

S. $\left(\frac{125}{32}\right)_{300°}$

$$\frac{z^3}{wt^2} = \frac{(5_{45°})^3}{2_{15°}\cdot(4_{90°})^2} = \frac{5^3_{45°\cdot 3}}{2_{15°}\cdot 4^2_{90°\cdot 2}} = \frac{125_{135°}}{2_{15°}\cdot 16_{180°}}$$

$$= \frac{125_{135°}}{(2\cdot 16)_{15°+180°}} = \frac{125_{135°}}{32_{195°}} = \left(\frac{125}{32}\right)_{135°-195°} = \left(\frac{125}{32}\right)_{-60°} = \left(\frac{125}{32}\right)_{30}$$

d) $\frac{z\cdot w^3}{t}$

S.10

$$\frac{z\cdot w^3}{t} = \frac{5_{45°}\cdot(2_{15°})^3}{4_{90°}} = \frac{5_{45°}\cdot 8_{45°}}{4_{90°}} = \frac{40_{90°}}{4_{90°}} = \left(\frac{40}{4}\right)_{90°-90°}$$

$$= 10_{0°} = 10$$

5.- Calcula as seguintes potencias e expresa o resultado en forma binómica:

a) $(1+i)^2 = 1 + 2i + i^2 = 1+2i - 1 = 2i$

S. $2i$

Pasando a polar: $|z| = \sqrt{a^2+b^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$a=1$; $b=1$ $\quad \tan\alpha = \frac{b}{a} = \frac{1}{1} = 1 \xRightarrow[\alpha \text{ está } 1^{er} \text{ cuadrante}]{} \alpha = 45°$

$1+i = z = (\sqrt{2})_{45°}$

$$(1+i)^2 = \left[(\sqrt{2})_{45°}\right]^2 = (\sqrt{2})^2_{2\cdot 45°} = 2_{90°} = 2(\cos 90° + i \operatorname{sen} 90°)$$

$$= 2\cdot(0+i\cdot 1) = 2i$$

b) $(\sqrt{3}-i)^5$

S. $-16\sqrt{3}-16i$

Pasando a polar: $|z| = \sqrt{a^2+b^2} = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$a=\sqrt{3}$; $b=-1$ $\quad \tan\alpha = \frac{b}{a} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3} \xRightarrow[\alpha \text{ está } 4^{o} \text{ cuadrante}]{} \alpha = -30° = 330°$

$\sqrt{3} - i = 2_{330°}$

$1650° \,|\, \underline{360}$
$210° \quad 4$

$$(\sqrt{3}-i)^5 = (2_{330°})^5 = (2^5)_{330\cdot 5} = 32_{1650°} = 32_{4\cdot 360°+210°} =$$

$$= 32_{210°} = 32\cdot(\cos 210° + i \operatorname{sen} 210°) = 32\cdot\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = *$$

$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\operatorname{sen} 210° = -\operatorname{sen} 30° = -\frac{1}{2}$

$* = -16\sqrt{3} - 16i$

g) $(-2+i)^5$

$z=-2+i$ está $2^{\underline{o}}$ cuadrante

S. $38+41i$

$|z|=\sqrt{(-2)^2+1^2}=\sqrt{5}$

$\tan\alpha=\frac{b}{a}=\frac{1}{-2}=-\frac{1}{2}\Rightarrow\alpha=180^\circ-26^\circ33'54''\cong153^\circ26'6''$

α está no $2^{\underline{o}}$

$\alpha=-26^\circ33'54''$ NON VALE ESTÁ NO $4^{\underline{o}}$

Entón, $(-2+i)^5=\left((\sqrt{5})_{153^\circ26'6''}\right)^5=$

$=(\sqrt{5})^5_{767^\circ10'29''}=25\sqrt{5}\left[\cos(767^\circ10'29'')+i\,\mathrm{sen}\,(767^\circ10'29'')\right]$

$\cong 38+41i$

h) $(-2+2\sqrt{3}i)^4$

S. $-128+128\sqrt{3}i$

$z=-2+2\sqrt{3}i$ está no $2^{\underline{o}}$ cuadrante

$|z|=\sqrt{(-2)^2+(2\sqrt{3})^2}=\sqrt{4+12}=\sqrt{16}=4$

$\tan\alpha=\frac{b}{a}=\frac{2\sqrt{3}}{-2}=-\sqrt{3}\Rightarrow\alpha=180^\circ-60^\circ=120^\circ$

α está no $2^{\underline{o}}$ $\quad\alpha=-60^\circ$ NON VALE ESTÁ NO $2^{\underline{o}}$

$480^\circ=360^\circ+120^\circ$

Entón, $(-2+2\sqrt{3}i)^4=(4_{120^\circ})^4=(4^4)_{4\cdot120^\circ}=256_{480^\circ}=256_{120^\circ}=$

$=256\cdot(\underbrace{\cos 120}_{(-\cos 60)}+i\underbrace{\mathrm{sen}\,120^\circ}_{\mathrm{sen}\,60^\circ})=256\cdot\left(-\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)=-128+128\sqrt{3}i$

6.- Calcula as seguintes raíces :

a) $\sqrt[4]{1+i}$

S. $\sqrt[8]{2}_{11,25^\circ};\sqrt[8]{2}_{101,25^\circ};\sqrt[8]{2}_{191,25^\circ};\sqrt[8]{2}_{281,25^\circ}$

$z=1+i\quad a=1;\ b=1$

$|z|=\sqrt{1^2+1^2}=\sqrt{2}$

$\tan\alpha=\frac{1}{1}=1\Rightarrow\alpha=45^\circ$

$1^{\underline{o}}$ cuad.

$1+i=(\sqrt{2})_{45^\circ}$

$\sqrt[4]{1+i}=\sqrt[4]{(\sqrt{2})_{45^\circ}}=\left(\sqrt[4]{\sqrt{2}}\right)_{\frac{45^\circ+360k}{4}}=\sqrt[8]{2}_{\frac{45^\circ+360k}{4}}$

$k=0,1,2,3$

$k=0\rightarrow(\sqrt[8]{2})_{11'25^\circ}$

$k=1\rightarrow(\sqrt[8]{2})_{101'25^\circ}$

$k=2\rightarrow(\sqrt[8]{2})_{191'25^\circ}$

$k=3\rightarrow(\sqrt[8]{2})_{281'25^\circ}$

b) $\sqrt[3]{1-i}$

S. $\sqrt[6]{2}_{105°}; \sqrt[6]{2}_{225°}; \sqrt[6]{2}_{345°}$

$z = 1 - i$ (4º cuadrante) $a = 1,\ b = -1$

$|z| = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\tan\alpha = \frac{b}{a} = \frac{-1}{1} = -1 \Rightarrow \alpha = \arctan(-1) = -45° = 315°$

α en 4º

$\sqrt[3]{1-i} = \sqrt[3]{(\sqrt{2})_{315°}} = \left(\sqrt[3]{\sqrt{2}}\right)_{\frac{315+360k}{3}} = \left(\sqrt[6]{2}\right)_{\frac{315+360k}{3}}$ $k=0,1,2$

$k=0$: $\left(\sqrt[6]{2}\right)_{105°}$

$k=1$: $\left(\sqrt[6]{2}\right)_{225}$

$k=2$: $\left(\sqrt[6]{2}\right)_{345°}$

c) $\sqrt[3]{-i} = \sqrt[3]{1_{270°}} = \left(\sqrt[3]{1}\right)_{\frac{270°+360k}{3}}$ $k=0,1,2$

S. $i; -\frac{\sqrt{3}}{2} \pm \frac{1}{2}i$

$k=0$: $1_{90°} = 1\cdot(\cos 90 + i\,\text{sen}\,90) = i$

$k=1$: $1_{210} = 1\cdot(\cos 210 + i\,\text{sen}\,210) =$
$= -\cos 30 - i\,\text{sen}\,30° =$
$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$k=2$: $1_{330°} = 1\cdot(\cos 330° + i\,\text{sen}\,330°) =$
$= \cos 30° - i\,\text{sen}\,30° =$
$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

d) $\sqrt[3]{8i}$

S. $-2i; \pm\sqrt{3}+i$

$\sqrt[3]{8i} = \sqrt[3]{8_{90'}} = \left(\sqrt[3]{8}\right)_{\frac{90+360k}{3}} = 2_{\frac{90+360k}{3}}$ $k=0,1,2$

$k=0$: $2_{30} = 2\cdot(\cos 30 + i\,\text{sen}\,30) = 2\cdot\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + i$

$k=1$: $2_{150°} = 2\cdot(\underbrace{\cos 150}_{-\cos 30} + i\,\underbrace{\text{sen}\,150}_{\text{sen}\,30}) = 2\cdot\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\sqrt{3} + i$

$k=2$: $2_{270} = -2i$

e) $\sqrt[3]{8}$

S. $2; -1 \pm \sqrt{3}i$

$\sqrt[3]{8} = \sqrt[3]{8_{0°}} = \left(\sqrt[3]{8}\right)_{\frac{0°+360k}{3}} = 2_{\frac{360k}{3}} = 2_{120k}$ $k=0,1,2$

$k=0$: $2_{0} = 2$

$k=1$: $2_{120°} = 2\cdot(\overbrace{\cos 120}^{-\cos 60} + i\,\overbrace{\text{sen}\,120}^{\text{sen}\,60}) = 2\cdot\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + \sqrt{3}i$

$k=2$: $2_{240°} = 2\cdot(\overbrace{\cos 240}^{-\cos 60} + i\,\overbrace{\text{sen}\,240}^{-\text{sen}\,60}) = 2\cdot\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -1 - \sqrt{3}i$

f) $\sqrt[3]{4-4\sqrt{3}i}$

S. $2_{100°}; 2_{220°}; 2_{340°}$

$z = 4-4\sqrt{3}\,i$ está 4º cuadrante

$|z| = \sqrt{4^2+(-4\sqrt{3})^2} = \sqrt{16+48} = \sqrt{64} = 8$

$\tan\alpha = \frac{b}{a} = \frac{-4\sqrt{3}}{4} = -\sqrt{3} \Rightarrow \alpha = 300°$

α está 4º

$$\sqrt[3]{4-4\sqrt{3}\,i} = \sqrt[3]{8_{300°}} = \left(\sqrt[3]{8}\right)_{\frac{300°+360k}{3}} = 2_{\frac{300+360k}{3}} = \begin{cases} k=0 \rightarrow 2_{100°} \\ k=1 \rightarrow 2_{220°} \\ k=3 \rightarrow 2_{340°} \end{cases}$$

$k=0,1,2$ $\quad$ $k=0,1,2$

g) $\sqrt[5]{-243}$ $= \sqrt[5]{243_{180}} = \left(\sqrt[5]{243}\right)_{\frac{180°+360k}{5}}$

S. $3_{36°}; 3_{108°}; 3_{180°}; 3_{252°}; 3_{324°}$

$k=0,1,2,3,4$

$= 3_{\frac{180°+360k}{5}}$ $\quad k=0,1,2,3,4$

$k=0$ $\quad 3_{36°}$

$k=1$ $\quad 3_{\frac{180+360}{5}} = 3_{\frac{540}{5}} = 3_{108°}$

$k=2$ $\quad 3_{\frac{180+720}{5}} = 3_{180°} = -3$

$k=3$ $\quad 3_{\frac{180+1080}{5}} = 3_{252°}$

$k=4$ $\quad 3_{\frac{180+1440}{5}} = 3_{324°}$

h) $\sqrt[4]{-8+8\sqrt{3}i}$

S. $\sqrt{3}+i; -1+\sqrt{3}i; -\sqrt{3}-i; 1-\sqrt{3}i$

$z = -8+8\sqrt{3}\,i$ $\quad z$ está 2º

$|z| = \sqrt{(-8)^2+(8\sqrt{3})^2} = \sqrt{64+192} = \sqrt{256} = 16$

$\tan\alpha = \frac{8\sqrt{3}}{-8} = -\sqrt{3} \Rightarrow \alpha = 120°$

α está 2º

$$\sqrt[4]{-8+8\sqrt{3}\,i} = \sqrt[4]{16_{120°}} = \left(\sqrt[4]{16}\right)_{\frac{120+360k}{4}} = 2_{\frac{120+360k}{4}} = *$$

$k=0,1,2,3$

$*$

$k=0$ $\quad 2_{30} = 2(\cos 30 + i \operatorname{sen} 30) = 2\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \sqrt{3}+i$

$k=1$ $\quad 2_{120} = 2(\underbrace{\cos 120}_{-\cos 60} + i\underbrace{\operatorname{sen} 120}_{\operatorname{sen} 60}) = 2\cdot\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1+\sqrt{3}\,i$

$k=2$ $\quad 2_{210} = 2(\underbrace{\cos 210}_{-\cos 30} + i\underbrace{\operatorname{sen} 210}_{-\operatorname{sen} 30}) = 2\cdot\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = -\sqrt{3}-i$

$k=3$ $\quad 2_{300} = 2(\underbrace{\cos 300}_{\cos 60} + i\underbrace{\operatorname{sen} 300}_{-\operatorname{sen} 60}) = 2\cdot\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 1-\sqrt{3}\,i$

7.- Resuelve las ecuaciones :

a) $z^3+27=0$ S. $-3; \frac{3}{2}\pm\frac{3\sqrt{3}}{2}i$

$z^3=-27 \Rightarrow z=\sqrt[3]{-27}=\sqrt[3]{27_{180}}=$

$=(\sqrt[3]{27})_{\frac{180+360k}{3}} = 3_{\frac{180+360k}{3}}$

$k=0,1,2$

$k=0$ $3_{60}=3\cdot(\cos 60+i\,\text{sen}\,60)=3\cdot\left(\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)=\frac{3}{2}+\frac{3\sqrt{3}}{2}i=z_1$

$k=1$ $3_{\frac{180+360}{3}}=3_{180^\circ}=3(\cos 180^\circ+i\,\text{sen}\,180^\circ)=3(-1+i\cdot 0)=-3=z_2$

$k=2$ $3_{\frac{180+720}{3}}=3_{300}=3(\cos 300^\circ+i\,\text{sen}\,300^\circ)=3(\cos 60^\circ+i(\text{sen}\,60^\circ))$

$=3\cdot\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$

$=\frac{3}{2}-\frac{3\sqrt{3}}{2}i=z_3$

b) $z^4+1=0$ S. $\frac{\pm\sqrt{2}}{2}\pm\frac{\sqrt{2}}{2}i$

$z^4=-1 \Rightarrow z=\sqrt[4]{-1}=\sqrt[4]{1_{180^\circ}}=(\sqrt[4]{1})_{\frac{180^\circ+360k}{4}}$ $k=0,1,2,3$

$z=1_{\frac{180^\circ+360k}{4}}$ $k=0,1,2,3$

$k=0$ $1_{45^\circ}=1\cdot(\cos 45+i\,\text{sen}\,45)=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}=z_1$

$k=1$ $1_{135^\circ}=1\cdot(\cos 135^\circ+i\,\text{sen}\,135^\circ)=1\cdot(-\cos 45^\circ+i\,\text{sen}\,45^\circ)=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i=z$

$k=2$ $1_{225^\circ}=1\cdot(\cos 225^\circ+i\,\text{sen}\,225^\circ)=1\cdot(-\cos 45^\circ-i\,\text{sen}\,45^\circ)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i=z$

$k=3$ $1_{315^\circ}=1\cdot(\cos 315^\circ+i\,\text{sen}\,315^\circ)=1\cdot(\cos 45^\circ-i\,\text{sen}\,45^\circ)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i=z$

c) $z^6+64=0$ S: $2i; -\sqrt{3}\pm i; \sqrt{3}\pm i$

$z^6=-64 \Rightarrow z=\sqrt[6]{-64}=\sqrt[6]{64_{180^\circ}}=(\sqrt[6]{64})_{\frac{180^\circ+360k}{6}}$ $k=0,1,2,3,4,5$

$z=2_{\frac{180+360k}{6}}$ $k=0,1,2,3,4,5$

$k=0$ $2_{30}=2\cdot(\cos 30+i\,\text{sen}\,30)=2\cdot\left(\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)=\sqrt{3}+i=z_1$

$k=1$ $2_{90}=2(\cos 90+i\,\text{sen}\,90)=2\cdot(0+i\cdot 1)=2i=z_2$

$k=2$ $2_{150}=2\cdot(\cos 150+i\,\text{sen}\,150)=2\cdot(-\cos 30+i\,\text{sen}\,30)=$

$=2\cdot\left(-\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)=-\sqrt{3}+i=z_3$

$k=3$ $2_{210}=2\cdot(\cos 210+i\,\text{sen}\,210)=2\cdot(-\cos 30-i\,\text{sen}\,30)=$

$=2\cdot\left(-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=-\sqrt{3}-i=z_4$

$k=4$ $2_{270^\circ}=-2i=z_5$

$k=5$ $2_{330^\circ}=2\cdot(\cos 330+i\,\text{sen}\,330)=2\cdot(\cos 30-i\,\text{sen}\,30)=2\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=\sqrt{3}-i=z_6$

8.- Obtén polinomios coas seguintes raíces :

a) $2+\sqrt{3}i$ e $2-\sqrt{3}i$

S. x^2-4x+7

$(x-(2+\sqrt{3}i))\cdot(x-(2-\sqrt{3}i)) = (x-2-\sqrt{3}i)(x-2+\sqrt{3}i) =$

$= x^2 - 2x + \cancel{\sqrt{3}ix} - 2x + 4 - \cancel{2\sqrt{3}i} - \cancel{\sqrt{3}ix} + \cancel{2\sqrt{3}i} - 3i^2 =$

$= x^2 - 4x + 7$

$= x^2 - 4x + 7$

$P(x) = x^2 - 4x + 7$

b) $-3i$ e $3i$

S. x^2+9

$(x-3i)(x+3i) = x^2 \cancel{-3ix} + \cancel{3ix} - 3^2i^2$

$= x^2 + 9$

$P(x) = x^2 + 9$