[illegible]

Sist. equivalente: $\begin{cases} x+y+z=2 \\ -11y-11z=-5 \\ 7z=21 \end{cases}$

$\underline{\underline{E_3}}$: $7z=21$
$z=3$

$\underline{\underline{E_2}}$ $-11y-12z=-5 \Rightarrow -11y-36=-6 \longrightarrow 11y=-30 \Rightarrow y=-2$

$\underline{\underline{E_1}}$ $x+y+z=2 \Rightarrow x-2+3=2 \Rightarrow x=1$

Sol. $x=1,\ y=-2,\ z=3$ S.C.D. (una única solución)

1 2 $\begin{cases} 2x-y+z=2 \\ x-y-z=-1 \\ x-2y+3z=4 \end{cases}$

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 1 & -1 & -1 & -1 \\ 1 & -2 & 3 & 4 \end{array}\right) \xrightarrow{F_1 \leftrightarrow F_2} \left(\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 2 & -1 & 1 & 2 \\ 1 & -2 & 3 & 4 \end{array}\right)$$

$$\xrightarrow[\substack{F_2 \leftarrow F_2 - 2F_1 \\ F_3 \leftarrow F_3 - F_1}]{F_1 \leftarrow F_1} \left(\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & -1 & 4 & 5 \end{array}\right)$$

$$\xrightarrow[F_3 \leftarrow F_3 + F_2]{\substack{F_1 \leftarrow F_1 \\ F_2 \leftarrow F_2}} \left(\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 7 & 9 \end{array}\right)$$

$\begin{cases} x-y-z=-1 \\ y+3z=4 \\ 7z=9 \end{cases}$

E_3: $7z=9 \Rightarrow z=\frac{9}{7}$

E_2: $y+3z=4 \Rightarrow y+\frac{3\cdot 9}{7}=4$
$y=4-\frac{27}{7}=\frac{1}{7}$

S.C.D. (sistema compatible determinado) (una única solución)

E_1: $x-y-z=-1$
$x-\frac{1}{7}-\frac{9}{7}=-1 \Rightarrow x=-1+\frac{1}{7}+\frac{9}{7}$
$x=\frac{3}{7}$

Sol: $x=\frac{3}{7};\ y=\frac{1}{7};\ z=\frac{9}{7}$

22.09

a) $\begin{cases} 2x-y+z=-7 \\ x+2y-z=9 \\ -3x+y+2z=-8 \end{cases}$

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & -7 \\ 1 & 2 & -1 & 9 \\ -3 & 1 & 2 & -8 \end{array}\right) \xrightarrow{E_1 \leftrightarrow E_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 2 & -1 & 1 & -7 \\ -3 & 1 & 2 & -8 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3+3E_1]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow E_2-2E_1}} \left(\begin{array}{cccc} 1 & 2 & -1 & 9 \\ 2-2\cdot 1 & -1-2\cdot 2 & 1-2(-1) & -7-2\cdot 9 \\ -3+3\cdot 1 & 1+3\cdot 2 & 2+3(-1) & -8+3\cdot 9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 9 \\ 0 & -5 & 3 & -25 \\ 0 & 7 & -1 & 19 \end{array}\right) \xrightarrow[E_3 \leftrightarrow 5E_3]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow 7E_2}} \left(\begin{array}{cccc} 1 & 2 & -1 & 9 \\ 0 & 35 & -21 & 175 \\ 0 & 35 & -5 & 95 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3-E_2]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow E_2}} \left(\begin{array}{cccc} 1 & 2 & -1 & 9 \\ 0 & 35 & -21 & 175 \\ 0 & 0 & 16 & -80 \end{array}\right)$$

$\begin{cases} x+2y-z=9 \\ 35y-21z=175 \\ 16z=-80 \end{cases}$

$E_3: \; 16z=-80 \Rightarrow z=\frac{-80}{16}=-5$

$E_2: \; 35y-21z=175 \Rightarrow 35y+105=175$

$35y=70$

$y=2$

$E_1: \; x+2y-z=9 \Rightarrow x+4+5=9 \Rightarrow x=0$

Sol. $x=0, \; y=2, \; z=-5$ S.C.D. (una única solució)

b) $\left.\begin{array}{l} x-y+3z=-4 \\ x+y+z=2 \\ x+2y-z=6 \end{array}\right\}$ $\left(\begin{array}{ccc|c} 1 & -1 & 3 & -4 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & 6 \end{array}\right) \xrightarrow{E_1 \leftrightarrow E_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 3 & -4 \\ 1 & 2 & -1 & 6 \end{array}\right)$

$$\xrightarrow[E_3 \leftrightarrow E_3-E_1]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow E_2-E_1}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 2 & -6 \\ 0 & 1 & -2 & 4 \end{array}\right) \xrightarrow[E_3 \leftrightarrow 2E_3]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow E_2}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 2 & -6 \\ 0 & 2 & -4 & 8 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3+E_2]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow E_2}}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 2 & -6 \\ 0 & 0 & -2 & 2 \end{array}\right)$$

Sistema de ecuaciones equivalente $\longrightarrow$ $\left.\begin{array}{l} x+y+z=2 \\ -2y+2z=-6 \\ -2z=2 \end{array}\right\}$

$x+y+z=2 \rightarrow$ (*)

$-2y+2z=-6 \rightarrow -2y+2\cdot(-1)=-6$

$-2y=-4$

$-y=-2$

$\boxed{y=2}$

$-2z=2$

$-z=1$

$\boxed{z=-1}$

(*) $x+2-1=2$

$x=2-1$

$\boxed{x=1}$

Sol. $(x=1; \; y=2; \; z=-1)$

S.C.D

24.09.

ⓓ $\begin{cases} x+y+z=7 \\ 3x-2y-z=3 \\ -2x+y+2z=0 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 3 & -2 & -1 & 3 \\ -2 & 1 & 2 & 0 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3+2E_1]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow E_2-3E_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -5 & -4 & -18 \\ 0 & 3 & 4 & 14 \end{array}\right) \xrightarrow[E_3 \leftrightarrow 5E_2]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow -3E_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -15 & -12 & -54 \\ 0 & 15 & 20 & 20 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3-E_2]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow E_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -15 & -12 & -54 \\ 0 & 0 & 8 & 16 \end{array}\right) \sim \begin{cases} x+y+z=7 \\ 15y+12z=54 \\ 8z=16 \end{cases}$$

E_3: $8z=16 \Rightarrow \boxed{z=2}$

E_2: $15y+12\cdot 2=54 \Rightarrow$

$\Rightarrow 15y+24=54 \Rightarrow$

$\Rightarrow 15y=30 \Rightarrow \boxed{y=2}$

E_1: $x+2+2=7 \Rightarrow \boxed{x=3}$

Sol: $x=3, y=2, z=2$

S.C.D. (sistema compatible determinado)

Nota: $\begin{cases} x+y+z=3 \\ 2x+y-z=2 \\ 3x+2y=4 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & -1 & 2 \\ 3 & 2 & 0 & 4 \end{array}\right) \xrightarrow[E_2 \leftrightarrow E_2-2E_1,\ E_3 \leftrightarrow E_3-3E_1]{E_1 \leftrightarrow E_1} \left(\begin{array}{cccc} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -4 \\ 0 & -1 & -3 & -5 \end{array}\right)$$

$$\xrightarrow[E_3 \leftrightarrow E_3-E_2]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow E_2} \left(\begin{array}{cccc} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -4 \\ \boxed{0\ 0\ 0} & & & \boxed{-1} \end{array}\right) \Rightarrow \begin{cases} x+y+z=3 \\ -y-3z=-4 \\ 0=-1 \end{cases}$$

¡IMPOSIBLE!
No tiene solución
S.I. (Sistema incompatible)

ⓔ $\begin{cases} 3x-4y+2z=1 \\ -2x-3y+z=2 \\ 5x-y+z=5 \end{cases}$

$$\left(\begin{array}{ccc|c} 3 & -4 & 2 & 1 \\ -2 & -3 & 1 & 2 \\ 5 & -1 & 1 & 5 \end{array}\right) \xrightarrow[E_2 \leftrightarrow 3E_2+2E_1,\ E_3 \leftrightarrow 3E_3-5E_1]{E_1 \leftrightarrow E_1}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 3 & -4 & 2 & 1 \\ 0 & -17 & 7 & 8 \\ 0 & 17 & -7 & 10 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3+E_2]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow E_2} \left(\begin{array}{ccc|c} 3 & -4 & 2 & 1 \\ 0 & -17 & 7 & 8 \\ 0 & 0 & 0 & 18 \end{array}\right)$$

$0=18$
¡Imposible!

S.I. (Sistema incompatible)
Sin solución!

Nota: $\begin{cases} x+y+z=3 \\ 2x+y-z=2 \\ 3x+2y=5 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & -1 & 2 \\ 3 & 2 & 0 & 5 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3-3E_1]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow E_2-2E_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -4 \\ 0 & -1 & -3 & -4 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3-E_2]{E_1 \leftrightarrow E_1,\ E_2 \leftrightarrow E_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -4 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right)$$

S.C.I. (∞s solucións)

$\begin{cases} x+y+z=3 \\ y+3z=4 \end{cases}$

Imos facer $z=\lambda$ (λ é un parámetro e pode valer calquera valor de $\mathbb{R}$)
$\lambda \in \mathbb{R}$

E_2: $y=4-3z \Rightarrow y=4-3\lambda$

E_1: $x=3-y-z=3-(4-3\lambda)-\lambda=-1+3\lambda-\lambda$
$x=2\lambda-1$

Solucións $(2\lambda-1,\ 4-3\lambda,\ \lambda)\ \lambda \in \mathbb{R}$

$\lambda=0 \rightarrow (-1, 4, 0)$
$\lambda=1 \rightarrow (1, 1, 1)$ $\dots\infty$

f) $\begin{cases} x-2y=-3 \\ -2x+3y+z=4 \\ 2x+y-5z=4 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ -2 & 3 & 1 & 4 \\ 2 & 1 & -5 & 4 \end{array}\right) \xrightarrow[E_3 \leftrightarrow E_3-2E_1]{\substack{E_1 \leftrightarrow E_1 \\ E_2 \leftrightarrow E_2+2E_1}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & -1 & 1 & -2 \\ 0 & 5 & -5 & 10 \end{array}\right) \to$$

$$\xrightarrow[E_3 \leftrightarrow E_3+5E_2]{\substack{E_1 \leftrightarrow E_1+E_2 \\ E_2 \leftrightarrow E_2}} \left(\begin{array}{ccc|c} 1 & -3 & 1 & -5 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \begin{cases} x-3y+z=-5 \\ -y+z=-2 \end{cases}$$

$z=\lambda$

$E_2 \to y=\lambda+2$

$E_1 \to x-3(\lambda+2)+\lambda=-5 \to$

$\to x-3\lambda-6+\lambda=-5 \to x=2\lambda+1$

S.C.I. $\lambda \in \mathbb{R}$

$(2\lambda+1, \lambda+2, \lambda)_{\lambda\in\mathbb{R}}$

g) $\begin{cases} x+y+z=6 \\ x-y-z=-4 \\ 3x+y+z=8 \end{cases}$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & -4 \\ 3 & 1 & 1 & 8 \end{pmatrix} \xrightarrow[E_3 \leftrightarrow E_3-3E_1]{E_2 \leftrightarrow E_2-1E_1} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & -2 & -2 & -10 \end{pmatrix} \xrightarrow{E_2-E_3}$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{cases} x+y+z=6 \\ -2y-2z=-10 \end{cases}$$

$z=\lambda$

$E_2: -2y-2z=-10 \Rightarrow -2y=-10+2z \to y=5-\lambda$

$E_3: x+5-\lambda+\lambda=6 \Rightarrow$

$\to x+5=6 \Rightarrow x=1$

Sol: $x=1$; $y=5-\lambda$; $z=\lambda$ $\lambda\in\mathbb{R}$