CÁLCULO DE FUNCIÓNS DERIVADAS.MATI.
REGRA DA CADEA.

Nome:____ Grupo:________

Acha a función derivada das seguintes funcións:

1.- $y = e^{3x-1}$

2.- $y = \ln\left(x^2 - 5x + 1\right)$

3.- $y = sen\left(4x^3 - 5x + 7\right)$

4.- $y = \sqrt[3]{(5x+3)^2} = (5x+3)^{2/3}$

5.- $y = \cos(3x - \pi)$

6.- $y = \left(x^4 - 3x^3 + 2x^2 + \sqrt{2}\right)^7$

7.- $y = 2^{senx}$

8.- $y = \sqrt[3]{1+2x} = (1+2x)^{1/3}$

9.- $y = 3\left(senx - \cos x\right)^4$

10.- $y = \arctan\left(\frac{1}{x}\right) = \arctan\left(x^{-1}\right)$

11.- $y = \left(\ln x + e^x\right)^4$

12.- $y = \sqrt{x^2 - 4x + 1} = \left(x^2 - 4x + 1\right)^{1/2}$

13.- $y = e^{-x^2+\pi}$

14.- $y = \sqrt{3e^x - \ln x} = \left(3e^x - \ln x\right)^{1/2}$

15.- $y = \cos\left(x^3 - 3x^2 + 5x - 1\right)$

16.- $y = \log\left(3x^4 - \frac{2}{x}\right)$

17.- $y=\ln(\ln x)$

18.- $y=\cos(senx)$

19.- $y=\tan(2x-3)$

20.- $y=arcsen(-x+10)$

21.- $y=sen^2\left(3x+\frac{\pi}{2}\right)$

22.- $y=sen\,x^3=sen(x^3)$

23.- $y=sen^3x=(senx)^3$

24.- $y=\left(\frac{3x}{2}-5\right)^4$

25.- $y=\ln\left(\frac{x+1}{x}\right)$

26.- $y=\ln\left(\frac{e^{-x}}{1-e^{-x}}\right)$

SOLUCIÓNS : DERIVADAS
(REGRA DA CADEA)

1.- $y = e^{3x-1}$

$$y' = e^{3x-1} \cdot \overbrace{(3x-1)'}^{3} = 3 \cdot e^{3x-1}$$

2.- $y = \ln(x^2 - 5x + 1)$

$$y' = \frac{1}{x^2 - 5x + 1} \cdot \overbrace{(x^2 - 5x + 1)'}^{2x-5} = \frac{2x - 5}{x^2 - 5x + 1}$$

3.- $y = \text{sen}\,(4x^3 - 5x + 7)$

$$y' = \cos\,(4x^3 - 5x + 7) \cdot \overbrace{(4x^3 - 5x + 7)'}^{12x-5}$$

$$y = (12x - 5) \cdot \cos(4x^3 - 5x + 7)$$

4.- $y = \sqrt[3]{(5x+3)^2} = (5x+3)^{2/3}$

$$y' = \frac{2}{3}(5x+3)^{\frac{2}{3}-1} \cdot \overbrace{(5x+3)'}^{5}$$

$$y' = 5 \cdot \frac{2}{3} \cdot (5x+3)^{-1/3}$$

$$y' = \frac{10}{3\sqrt[3]{5x+3}}$$

5.- $y = \cos\,(3x - \pi)$

$$y' = -\,\text{sen}\,(3x - \pi) \cdot \overbrace{(3x - \pi)'}^{3}$$

$$y' = -3 \cdot \text{sen}\,(3x - \pi)$$

6.- $y = (x^4 - 3x^3 + 2x^2 + \sqrt{2})^7$

$y' = 7 \cdot (x^4 - 3x^3 + 2x^2 + \sqrt{2})^6 \cdot \underbrace{(x^4 - 3x^3 + 2x^2 + \sqrt{2})'}_{\to 4x^3 - 9x^2 + 4x}$

$y' = 7 \cdot (4x^3 - 9x^2 + 4x) \cdot (x^4 - 3x^3 + 2x^2 + \sqrt{2})^6$

7.- $y = 2^{\operatorname{sen} x}$

$y' = 2^{\operatorname{sen} x} \cdot \ln 2 \cdot \underbrace{(\operatorname{sen} x)'}_{\to \cos x}$

$y' = 2^{\operatorname{sen} x} \cdot \ln 2 \cdot \cos x$

$y' = \ln 2 \cdot \cos x \cdot 2^{\operatorname{sen} x}$

8.- $y = \sqrt[3]{1+2x} = (1+2x)^{\frac{1}{3}}$ $\quad x \xrightarrow{f} 1+2x \xrightarrow{g} \sqrt[3]{1+2x}$

$y' = \frac{1}{3} \cdot (1+2x)^{\frac{1}{3}-1} \cdot \underbrace{(1+2x)'}_{\to 2}$

$y' = \frac{1}{3} \cdot 2 \cdot (1+2x)^{-2/3}$

$y' = \dfrac{2}{3\sqrt[3]{(1+2x)^2}}$

9.- $y = 3 \cdot (\operatorname{sen} x - \cos x)^4$

$y' = 3 \cdot ((\operatorname{sen} x - \cos x)^4)' = 3 \cdot 4 \cdot (\operatorname{sen} x - \cos x)^{4-1} \cdot (\operatorname{sen} x - \cos x)'$

$y' = 12 \cdot (\cos x - (-\operatorname{sen} x)) \cdot (\operatorname{sen} x - \cos x)^3$

$y' = 12 \cdot (\cos x + \operatorname{sen} x) \cdot (\operatorname{sen} x - \cos x)^3$

10.- $y = \arctan(x^{-1})$

$y' = \dfrac{1}{1+(x^{-1})^2} \cdot (x^{-1})' = \dfrac{1}{1+\frac{1}{x^2}} \cdot (-1) \cdot x^{-2} = \dfrac{-1}{\frac{x^2+1}{\cancel{x^2}} \cdot \cancel{x^2}}$

$y' = \dfrac{-1}{x^2+1}$

11.- $y = (\ln x + e^x)^4$ $\qquad x \xrightarrow{\ln\square + e^{\square}} \ln x + e^x \xrightarrow{\square^4} (\ln x + e^x)^4$

$\rightarrow \frac{1}{x} + e^x$

$y' = 4 \cdot (\ln x + e^x)^{4-1} \cdot (\ln x + e^x)'$

$y' = 4 \cdot \left(\frac{1}{x} + e^x\right) \cdot (\ln x + e^x)^3$

12.- $y = \sqrt{x^2 - 4x + 1} = (x^2 - 4x + 1)^{1/2}$

$y' = \frac{1}{2} (x^2 - 4x + 1)^{\frac{1}{2} - 1} \cdot (x^2 - 4x + 1)'$

$y' = \frac{1}{2} (x^2 - 4x + 1)^{-1/2} \cdot (2x - 4)$

$y' = \dfrac{2x - 4}{2 \cdot \sqrt{x^2 - 4x + 1}}$

13.- $y = e^{-x^2 + \pi}$ $\qquad x \xrightarrow{-\square^2 + \pi} -x^2 + \pi \xrightarrow{e^{\square}} e^{-x^2 + \pi}$

$\rightarrow -2x$

$y' = e^{-x^2 + \pi} \cdot (-x^2 + \pi)'$

$y' = (-2x) \cdot e^{-x^2 + \pi}$

14.- $y = \sqrt{3e^x - \ln x} = (3e^x - \ln x)^{1/2}$

$y' = \frac{1}{2} (3e^x - \ln x)^{1/2 - 1} \cdot (3e^x - \ln x)'$

$y' = \frac{1}{2} (3e^x - \ln x)^{-1/2} \cdot \left(3e^x - \frac{1}{x}\right)$

$y' = \dfrac{3e^x - \frac{1}{x}}{2 \cdot \sqrt{3e^x - \ln x}}$

15.- $y = \cos(x^3 - 3x^2 + 5x - 1)$

$y' = -\operatorname{sen}(x^3 - 3x^2 + 5x - 1) \cdot (x^3 - 3x^2 + 5x - 1)'$

$y' = -(3x^2 - 6x + 5) \operatorname{sen}(x^3 - 3x^2 + 5x - 1)$

16.- $y = \log\left(3x^4 - \frac{2}{x}\right) = \log\left(3x^4 - 2x^{-1}\right)$

$$y' = \frac{1}{3x^4 - \frac{2}{x}} \cdot \frac{1}{\ln 10} \cdot \left(3x^4 - \frac{2}{x}\right)' \rightarrow 12x - 2 \cdot (-1)x$$

$$y' = \frac{1}{3x^4 - \frac{2}{x}} \cdot \frac{1}{\ln 10} \cdot \left(12x + \frac{2}{x^2}\right)$$

17.- $y = \ln(\ln x)$

$$y' = \frac{1}{\ln x} \cdot (\ln x)' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \cdot \ln x}$$

18.- $y = \cos(\operatorname{sen} x)$

$$y' = -\operatorname{sen}(\operatorname{sen} x) \cdot (\operatorname{sen} x)' \rightarrow \cos x$$

$$y' = -\cos x \cdot \operatorname{sen}(\operatorname{sen} x)$$

19.- $y = \tan(2x - 3)$

$$y' = \frac{1}{\cos^2(2x-3)} \cdot (2x-3)' \rightarrow 2$$

$$y' = \frac{2}{\cos^2(2x-3)}$$

20.- $y = \operatorname{arcsen}(-x+10)$

$$y' = \frac{1}{\sqrt{1-(-x+10)^2}} \cdot (-x+10)' \rightarrow -1$$

$$y' = \frac{-1}{\sqrt{1-(x^2+100-20x)}} = \frac{-1}{\sqrt{-x^2+20x-99}}$$

21. $y = \text{sen}^2\left(3x + \frac{\pi}{2}\right) = \left(\text{sen}\left(3x + \frac{\pi}{2}\right)\right)^2$ $\quad x \xrightarrow{3\square+\frac{\pi}{2}} 3x + \frac{\pi}{2} \xrightarrow{\text{sen}\,\square} \text{sen}\left(3x+\frac{\pi}{2}\right) \xrightarrow{\square^2} \text{sen}^2\left(3x+\frac{\pi}{2}\right)$

$$y' = 2\cdot\left(\text{sen}\left(3x + \frac{\pi}{2}\right)\right)^{2-1}\cdot\left(\text{sen}\left(3x + \frac{\pi}{2}\right)\right)'$$

$$y' = 2\cdot\text{sen}\left(3x + \frac{\pi}{2}\right)\cdot\cos\left(3x + \frac{\pi}{2}\right)\cdot\left(3x + \frac{\pi}{2}\right)' \to 3$$

$$y' = 2\cdot\text{sen}\left(3x + \frac{\pi}{2}\right)\cdot\cos\left(3x + \frac{\pi}{2}\right)\cdot 3$$

$$y' = 6\cdot\text{sen}\left(3x + \frac{\pi}{2}\right)\cdot\cos\left(3x + \frac{\pi}{2}\right)$$

22.- $y = \text{sen}\,x^3 = \text{sen}(x^3)$ $\quad x \xrightarrow{\square^3} x^3 \xrightarrow{\text{sen}\,\square} \text{sen}\,x^3$

$$y' = \cos(x^3)\cdot(x^3)' \to 3x^2$$

$$y' = 3x^2\cdot\cos(x^3)$$

23.- $y = \text{sen}^3 x = (\text{sen}\,x)^3$ $\quad x \xrightarrow{\text{sen}\,\square} \text{sen}\,x \xrightarrow{\square^3} (\text{sen}\,x)^3$

$$y' = 3\cdot(\text{sen}\,x)^{3-1}\cdot(\text{sen}\,x)'$$

$$y' = 3\cdot\cos x\cdot(\text{sen}\,x)^2 = 3\cos x\cdot\text{sen}^2 x$$

24.- $y = \left(\frac{3x}{2} - 5\right)^4$ $\quad x \xrightarrow{\frac{3\square}{2}-5} \frac{3x}{2} - 5 \xrightarrow{\square^4} \left(\frac{3x}{2} - 5\right)^4$

$$y' = 4\cdot\left(\frac{3x}{2} - 5\right)^{4-1}\cdot\left(\frac{3x}{2} - 5\right)'$$

$$y' = 4\cdot\left(\frac{3x}{2} - 5\right)^3\cdot\frac{3}{2}$$

$$y' = \frac{12}{2}\left(\frac{3x}{2} - 5\right)^3 \Rightarrow y' = 6\cdot\left(\frac{3x}{2} - 5\right)^3$$

25.- $y = \ln\left(\frac{x+1}{x}\right)$

$$\begin{cases} \log \frac{a}{b} = \log a - \log b \\ \log(a \cdot b) = \log a + \log b \\ \log a^b = b \cdot \log a \end{cases}$$

CANDO POIDAMOS ¡APLICAMOS PROPIEDADES LOGARITMOS!

VAINOS FACILITAR O CÁLCULO DA DERIVADA

$$y = \ln\left(\frac{x+1}{x}\right) = \ln(x+1) - \ln x$$

AGORA DERIVAMOS!

$$y' = \frac{1}{x+1} \cdot \overbrace{(x+1)'}^{1} - \frac{1}{x} = \frac{1}{x+1} - \frac{1}{x}$$

MOITO MÁIS FÁCIL!

26.- $y = \ln\left(\frac{e^{-x}}{1-e^{-x}}\right) = \ln(e^{-x}) - \ln(1-e^{-x})$

$$y = -x \cdot \overbrace{\ln e}^{1} - \ln(1-e^{-x})$$

$$y = -x - \ln(1-e^{-x})$$

AGORA DERIVAMOS!

$$y' = -1 - \frac{1}{1-e^{-x}} \cdot (1-e^{-x})'$$

$$y' = -1 - \frac{1}{1-e^{-x}}(0 - e^{-x} \cdot (-1))$$

$$y' = -1 - \frac{e^{-x}}{1-e^{-x}} = \frac{-1 \cancel{+e^{-x}} \cancel{-e^{-x}}}{1-e^{-x}} = \frac{-1}{1-e^{-x}}$$

MOITO MÁIS FÁCIL!