# CÁLCULO DE DERIVADAS III

1. $y = 3x^3 - \dfrac{2}{\sqrt[3]{x^2}} + \sqrt[5]{x} + \dfrac{3}{x^2} + \sqrt{2} + \log 7$

→ cte → cte

y ← Todavía non derivo!

$y = 3x^3 - 2\cdot x^{-2/3} + x^{\frac{1}{5}} + 3\cdot x^{-2} + \sqrt{2} + \log 7$

$y' = 9x^2 - 2\cdot\left(-\frac{2}{3}\right) x^{-2/3-1} + \frac{1}{5} x^{\frac{1}{5}-1} + (-2)3x^{-2-1} + 0 + 0$

$y' = 9x^2 + \frac{4}{3} x^{-5/3} + \frac{1}{5} x^{-\frac{4}{5}} - 6x^{-3}$

$y' = 9x^2 + \dfrac{4}{3\sqrt[3]{x^5}} + \dfrac{1}{5\sqrt[5]{x^4}} - \dfrac{6}{x^3}$

2. $y = \operatorname{sen}(3x+1)\cdot\cos(3x+1)$

derivada función composta / derivada función composta

$y' = (\operatorname{sen}(3x+1))'\cdot\cos(3x+1) + \operatorname{sen}(3x+1)\left(\cos(3x+1)\right)'$

$(3x+1)'$ $(3x+1)'$

$y' = +\cos(3x+1)\cdot 3\cdot\cos(3x+1) + \operatorname{sen}(3x+1)\cdot(-\operatorname{sen}(3x+1))\cdot 3$

$y' = +3\cdot\cos^2(3x+1) - 3\cdot\operatorname{sen}^2(3x+1)$

$y' = +3\left(\cos^2(3x+1) - \operatorname{sen}^2(3x+1)\right)$

3. $y = \dfrac{\operatorname{sen} x + \cos x}{\operatorname{sen} x - \cos x}$

$y' = \dfrac{(\operatorname{sen} x + \cos x)'(\operatorname{sen} x - \cos x) - (\operatorname{sen} x + \cos x)\cdot(\operatorname{sen} x - \cos x)'}{(\operatorname{sen} x - \cos x)^2}$

$y' = \dfrac{(\cos x - \operatorname{sen} x)\cdot(\operatorname{sen} x - \cos x) - (\operatorname{sen} x + \cos x)\cdot(\cos x + \operatorname{sen} x)}{(\operatorname{sen} x - \cos x)^2}$

$$y' = \frac{\cos x \operatorname{sen} x - \cos^2 x - \operatorname{sen}^2 x + \operatorname{sen} x \cos x - \operatorname{sen} x \cos x - \operatorname{sen}^2 x - \cos^2 x \ldots}{(\operatorname{sen} x - \cos x)^2}$$

$$y' = \frac{-2\cos^2 x - 2\operatorname{sen}^2 x}{(\operatorname{sen} x - \cos x)^2} = \frac{-2\overbrace{(\operatorname{sen}^2 x + \cos^2 x)}^{1}}{(\operatorname{sen} x - \cos x)^2}$$

$$y' = \frac{-2}{(\operatorname{sen} x - \cos x)^2}$$

4. $y = \frac{2}{x} - \frac{x^2}{2} = 2 \cdot x^{-1} - \frac{1}{2} x^2$

$$y' = -2x^{-2} - \frac{1}{2} \cdot 2x = \frac{-2}{x^2} - x$$

5. $y = \frac{2x^2 - x + 1}{(x^2 + x - 1)^2}$

Reg cade

$$y' = \frac{(2x^2 - x + 1)'(x^2 + x - 1)^2 - (2x^2 - x + 1) \cdot ((x^2 + x - 1)^2)'}{((x^2 + x - 1)^2)^2}$$

$$y' = \frac{(4x - 1) \cdot (x^2 + x - 1)^2 - (2x^2 - x + 1) \cdot 2 \cdot (x^2 + x - 1)^{\overset{1}{2-1}} \cdot (2x + 1)}{(x^2 + x - 1)^4}$$

saco factor común $(x^2 + x - 1)$

$$y' = \frac{\cancel{(x^2 + x - 1)}\,[(4x - 1) \cdot (x^2 + x - 1) - 2 \cdot (2x^2 - x + 1)(2x + 1)]}{(x^2 + x - 1)^{\cancel{4}\,3}}$$

$$y' = \frac{4x^3 + 4x^2 - 4x - x^2 - x + 1 - 8x^3 - 4x^2 + 4x^2 + 2x - 4x - 2}{(x^2 + x - 1)^3}$$

$$y' = \frac{-4x^3 + 3x^2 - 7x - 1}{(x^2 + x - 1)^3}$$

6.- $y = \ln(3x) + e^{\sqrt{x}}$

$$y' = \frac{1}{3x}\cdot \underset{(3x)'}{3} + e^{\sqrt{x}} \cdot \underset{(\sqrt{x})' = (x^{1/2})'}{\frac{1}{2\sqrt{x}}} = \frac{1}{x} + \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

7.- $y = e^{2x} \cdot \tan x$ $\quad x \xrightarrow{2\square} 2x \xrightarrow{e^{\square}} e^{2x}$

Regra cadea

$$y' = \underline{(e^{2x})'} \cdot \tan x + e^{2x} \cdot (\tan x)'$$

$$y' = e^{2x} \cdot 2 \cdot \tan x + e^{2x} \cdot \frac{1}{\cos^2 x}$$

$$y' = 2 \cdot e^{2x} \cdot \tan x + \frac{e^{2x}}{\cos^2 x}$$

$$y' = e^{2x} \left( 2\tan x + \frac{1}{\cos^2 x} \right)$$

8.- $y = (x^2-1) \cdot \ln(x^2-1)$

Regra cadea $\quad x \xrightarrow{\square^2 - 1} x^2-1 \xrightarrow{\ln\square} \ln(x^2-1)$

$$y' = (x^2-1)' \ln(x^2-1) + (x^2-1)\underline{\left(\ln(x^2-1)\right)'}$$

$$y' = 2x \cdot \ln(x^2-1) + \cancel{(x^2-1)} \cdot \frac{1}{\cancel{x^2-1}} \cdot (2x)$$

$$y' = 2x \ln(x^2-1) + 2x$$

$$y' = 2x\left(\ln(x^2-1) + 1\right)$$

9.- $y = \operatorname{sen}^3 x - \operatorname{sen} x^2 + 5 \operatorname{sen} x = \underbrace{(\overset{2^{\circ}}{\operatorname{sen} x})^{\overset{1^{\circ}}{3}}}_{\text{Regra cadea}} - \underbrace{\overset{1^{\circ}}{\operatorname{sen}}(\overset{2^{\circ}}{x^2})}_{\text{Regra cadea}} + 5 \operatorname{sen} x$

$$y' = 3 \cdot (\operatorname{sen} x)^{3-1} \cdot \underline{(\operatorname{sen} x)'} - \cos(x^2) \cdot 2x + 5 \cdot \cos x$$

$$y' = 3 \cdot \cos x \cdot (\operatorname{sen} x)^2 - 2x \cos(x^2) + 5 \cos x$$

$$y' = 3 \cos x \operatorname{sen}^2 x - 2x \cos x^2 + 5 \cos x$$

$y' =$

10.- $y = \ln\left(\frac{x^2-1}{x^2+1}\right) = \ln(x^2-1) - \ln(x^2+1)$

Aplico PROPIEDADES $\underline{\underline{\ln}}$ ANTES DE DERIVAR

AGORA DERIVO $y' = \frac{1}{x^2-1} \cdot 2x - \frac{1}{x^2+1} \cdot 2x$

$$y' = \frac{2x}{x^2-1} - \frac{2x}{x^2+1}$$

11.- $y = \ln\sqrt{\frac{x}{x^2+1}} = \ln\left(\frac{x}{x^2+1}\right)^{1/2}$

Aplico PROPIEDADES $\ln$ ANTES DE DERIVAR

$$y = \frac{1}{2}\left(\ln\frac{x}{x^2+1}\right) = \frac{1}{2}\left(\ln x - \ln(x^2+1)\right)$$

AGORA DERIVO

$$y' = \frac{1}{2} \cdot \left(\ln x - \ln(x^2+1)\right)'$$

$$y' = \frac{1}{2}\left(\frac{1}{x} - \frac{1}{x^2+1} \cdot 2x\right)$$

$$y' = \frac{1}{2x} - \frac{2x}{2(x^2+1)}$$

$$y' = \frac{1}{2x} - \frac{x}{x^2+1}$$

12.- $y = \ln(x \cdot e^{-x})$

Aplico PROPIEDADES $\ln$ ANTES DERIVAR

$$y = \ln x + \ln e^{-x} = \ln x + (-x)$$

AGORA DERIVO

$$y' = \frac{1}{x} - 1$$

13.- $y = \sqrt{\operatorname{sen} x} = \underline{(\operatorname{sen} x)^{1/2}}$ Regra cadea

$$y' = \frac{1}{2} (\operatorname{sen} x)^{1/2 - 1} \cdot (\operatorname{sen} x)'$$

$$y' = \frac{1}{2\sqrt{\operatorname{sen} x}} \cdot \cos x$$

$$y' = \frac{\cos x}{2\sqrt{\operatorname{sen} x}}$$

14.- $y = \underbrace{\cos^2 x}_{\text{Regra cadea}} + \underbrace{e^{\operatorname{sen} x}}_{\text{Regra cadea}} = (\cos x)^2 + e^{\operatorname{sen} x}$

$$y' = 2 \cdot (\cos x)^{2-1} \cdot (\cos x)' + e^{\operatorname{sen} x} \cdot (\operatorname{sen} x)'$$

$$y' = 2 \cdot \cos x \, (-\operatorname{sen} x) + e^{\operatorname{sen} x} \cdot \cos x$$

$$y' = -2 \cos x \operatorname{sen} x + \cos x \cdot e^{\operatorname{sen} x}$$

15.- $y = \underbrace{\arccos}_{1^a} \underbrace{e}_{2^a}^{-x \leftarrow 3^a}$ $\quad x \xrightarrow[3^a]{-\square} -x \xrightarrow[2^a]{e^{\square}} e^{-x} \xrightarrow[1^a]{\arccos \square} \arccos e^{-x}$

$$y' = \frac{-1}{\sqrt{1-(e^{-x})^2}} \cdot \underline{(e^{-x})'}$$

$$y' = \frac{-1}{\sqrt{1 - e^{-2x}}} \cdot e^{-x} \cdot (-1)$$

$$y' = \frac{e^{-x}}{\sqrt{1 - e^{-2x}}}$$

19.- $y = 5^{\frac{x^2}{(2x-1)^2}}$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \left(\frac{x^2}{(2x-1)^2}\right)'$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{(x^2)' \cdot (2x-1)^2 - x^2 \cdot ((2x-1)^2)'}{(2x-1)^4}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{2x \cdot (2x-1)^2 - x^2 \cdot 2 \cdot (2x-1) \cdot 2}{(2x-1)^4}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{\cancel{(2x-1)}\,[2x(2x-1) - 4x^2]}{(2x-1)^{\cancel{4}\,3}}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{4x^2 - 2x - 4x^2}{(2x-1)^3}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{(-2x)}{(2x-1)^3}$$

$$y' = \frac{-2x \ln 5}{(2x-1)^3} \cdot 5^{\frac{x^2}{(2x-1)^2}}$$

20.- $y = \operatorname{arcsen}\sqrt{x} + \frac{7}{\tan x} = \operatorname{arcsen}(x^{1/2}) + 7 \cdot (\tan x)^{-1}$

$$y' = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} + 7 \cdot (-1)(\tan x)^{-2} \cdot \frac{1}{\cos^2 x}$$

$$y' = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} - \frac{7}{\cos^2 x \tan^2 x}$$

$\tan^2 x = (\tan x)^2$

$$y' = \frac{1}{2\sqrt{x}\sqrt{1-x}} - \frac{7}{\operatorname{sen}^2 x}$$

$\cos^2 x \cdot \tan^2 x = \cancel{\cos^2 x} \cdot \frac{\operatorname{sen}^2 x}{\cancel{\cos^2 x}}$

$\cos^2 x = (\cos x)^2$