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a) $y = \dfrac{e^x + e^{-x}}{2} = \dfrac{1}{2} e^x + \dfrac{1}{2} e^{-x}$

$y' = \dfrac{1}{2} \cdot e^x + \dfrac{1}{2} \cdot e^{-x} \cdot (-x)'$

$y' = \dfrac{1}{2} \cdot e^x + \dfrac{1}{2} e^{-x} \cdot (-1)$

$y' = \dfrac{1}{2} e^x - \dfrac{1}{2} e^{-x} = \dfrac{e^x - e^{-x}}{2}$

b) $y = \dfrac{x^3 - x^2}{x^2} = \dfrac{\cancel{x^2}(x-1)}{\cancel{x^2}} \underset{x \neq 0}{=} x - 1$

$y' = 1 \quad x \neq 0$

c) $y = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}}$

$y' = \dfrac{1}{2} \cdot (x^2+1)^{\frac{1}{2}-1} \cdot (x^2+1)'$

$y' = \dfrac{1}{\cancel{2}} (x^2+1)^{-\frac{1}{2}} \cdot \cancel{2} x$

$y' = \dfrac{x}{\sqrt{x^2+1}}$

d) $y = \sqrt[3]{(x+6)^2} = (x+6)^{\frac{2}{3}}$

$y' = \dfrac{2}{3} \cdot (x+6)^{\frac{2}{3}-1} \cdot (x+6)' = \dfrac{2}{3} (x+6)^{-\frac{1}{3}} \cdot 1 = \dfrac{2}{3\sqrt[3]{x+6}}$

e) $y = \frac{1}{7x+1} = (7x+1)^{-1}$

$y' = -1 \cdot (7x+1)^{-1-1} \cdot (7x+1)'$

$y' = -(7x+1)^{-2} \cdot 7 = \frac{-7}{(7x+1)^2}$

f) $y = \frac{-3}{\sqrt{1-x^2}} = -3 \cdot (1-x^2)^{-\frac{1}{2}}$

$y' = -3 \cdot \left(-\frac{1}{2}\right) \cdot (1-x^2)^{-\frac{1}{2}-1} \cdot (1-x^2)'$

$y' = \frac{3}{\cancel{2}} \cdot (1-x^2)^{-\frac{3}{2}} \cdot (-\cancel{2}x)$

$y' = \frac{-3x}{\sqrt{(1-x^2)^3}}$

g) $y = 7^{x+1} \cdot e^{-x}$

$y' = (7^{x+1})' \cdot e^{-x} + 7^{x+1} \cdot (e^{-x})'$

$y' = 7^{x+1} \cdot \ln 7 \, (x+1)' \cdot e^{-x} + 7^{x+1} \cdot e^{-x} \cdot (-x)'$

$y' = 7^{x+1} \cdot \ln 7 \cdot 1 \cdot e^{-x} + 7^{x+1} \cdot e^{-x} \cdot (-1)$

$y' = 7^{x+1} \cdot e^{-x} \, (\ln 7 - 1)$

h) $y = \frac{1}{3x} + \frac{x}{3} = \frac{1}{3} \cdot x^{-1} + \frac{1}{3} \cdot x$

$y' = \frac{1}{3} \cdot (-1) \cdot x^{-1-1} + \frac{1}{3} \cdot 1 = \frac{-1}{3x^2} + \frac{1}{3}$

$y' = \frac{-1 + x^2}{3x^2} = \frac{x^2 - 1}{3x^2}$

i) $y = \frac{1}{\sqrt{x+4}} = (x+4)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(x+4)^{-\frac{1}{2}-1} \cdot (x+4)' = -\frac{1}{2} \cdot (x+4)^{-\frac{3}{2}} \cdot 1$

$y' = -\frac{1}{2\sqrt{(x+4)^3}}$

j) $y = \frac{x^3}{(x-1)^2}$

$y' = \frac{(x^3)' \cdot (x-1)^2 - x^3 \cdot ((x-1)^2)'}{((x-1)^2)^2}$

SACO FACTOR COMÚN x-1

$y' = \frac{3x^2(x-1)^2 - x^3 \cdot 2 \cdot (x-1)^1 \cdot 1}{(x-1)^4} = \frac{\cancel{(x-1)}[3x^2 \cdot (x-1) - 2x^3]}{(x-1)^{\cancel{4}\,3}}$

$y' = \frac{3x^3 - 3x^2 - 2x^3}{(x-1)^3}$

$y' = \frac{x^3 - 3x^2}{(x-1)^3}$

k) $y = \frac{x+3}{x-3}$

$$y' = \frac{(x+3)'(x-3) - (x+3)(x-3)'}{(x-3)^2} = \frac{1\cdot(x-3) - (x+3)\cdot 1}{(x-3)^2}$$

$$y' = \frac{\cancel{x}-3-\cancel{x}-3}{(x-3)^2} = \frac{-6}{(x-3)^2}$$

l) $y = \sqrt{\frac{x^3}{x^2-4}} = \left(\frac{x^3}{x^2-4}\right)^{\frac{1}{2}}$

$$y' = \frac{1}{2}\left(\frac{x^3}{x^2-4}\right)^{\frac{1}{2}-1} \cdot \left(\frac{x^3}{x^2-4}\right)'$$

$$y' = \frac{1}{2}\left(\frac{x^3}{x^2-4}\right)^{-\frac{1}{2}} \cdot \frac{(x^3)'(x^2-4) - x^3\cdot(x^2-4)'}{(x^2-4)^2}$$

$$y' = \frac{1}{2\cdot\sqrt{\frac{x^3}{x^2-4}}} \cdot \frac{3x^2\cdot(x^2-4) - x^3\cdot 2x}{(x^2-4)^2} -$$

$$y' = \frac{1}{2\cdot\frac{\sqrt{x^3}}{\sqrt{x^2-4}}} \cdot \frac{3x^4 - 12x^2 - 2x^4}{(x^2-4)^2}$$

$$y' = \frac{\sqrt{x^2-4}}{2\sqrt{x^3}} \cdot \frac{x^4-12x^2}{(x^2-4)^2} = \frac{x^2(x^2-12)}{2\sqrt{x^3}\cdot(x^2-4)^{3/2}}$$

$$y' = \frac{\sqrt{x}\cdot(x^2-12)}{2\cdot\sqrt{(x^2-4)^3}}$$

m) $y = \left(\frac{x}{2}\right)^3 \cdot e^{1-x}$

$y' = \left(\left(\frac{x}{2}\right)^3\right)' \cdot e^{1-x} + \left(\frac{x}{2}\right)^3 \cdot (e^{1-x})'$

$y' = 3 \cdot \left(\frac{x}{2}\right)^2 \cdot \frac{1}{2} \cdot e^{1-x} + \left(\frac{x}{2}\right)^3 \cdot e^{1-x} \cdot (-1)$

$y' = \left(\frac{x}{2}\right)^2 \cdot e^{1-x} \left(\frac{3}{2} + \frac{x}{2} \cdot (-1)\right)$

$y' = \left(\frac{x}{2}\right)^2 \cdot e^{1-x} \left(\frac{3-x}{2}\right) = \frac{3-x}{2} \cdot \left(\frac{x}{2}\right)^2 \cdot e^{1-x}$

$y' = \frac{x^2 \cdot (3-x)}{8} \cdot e^{1-x}$

n) $y = \ln 5$

$y' = 0$ (DERIVADA DE UNA CONSTANTE ES CERO)

o) $y = \log \frac{x^2}{3-x} = \log x^2 - \log(3-x) = 2\log x - \log(3-x)$

$y' = 2 \cdot \frac{1}{x} \cdot \frac{1}{\ln 10} - \frac{1}{3-x} \cdot \frac{1}{\ln 10} \cdot (3-x)'$

$y' = \frac{2}{x \ln 10} - \frac{1}{3-x} \cdot \frac{1}{\ln 10} \cdot (-1)$

$y' = \frac{2}{x \ln 10} + \frac{1}{(3-x) \cdot \ln 10}$

$y' = \frac{1}{\ln 10}\left(\frac{2}{x} + \frac{1}{3-x}\right) = \frac{1}{\ln 10}\left(\frac{6-2x+x}{x \cdot (3-x)}\right) = \frac{1}{\ln 10} \cdot \frac{6-x}{x \cdot (3-x)}$

$y' = \frac{6-x}{\ln 10 \cdot x \cdot (3-x)}$

p) $y = \tan^3 x^2 = (\tan(x^2))^3$

$y' = 3 \cdot (\tan(x^2))^2 \cdot (\tan(x^2))'$

$y' = 3 \cdot (\tan(x^2))^2 \cdot \frac{1}{\cos^2 x^2} \cdot (x^2)'$

$y' = 3 \tan^2(x^2) \cdot \frac{1}{\cos^2 x^2} \cdot 2x$

$y' = 3 \cdot \frac{\operatorname{sen}^2(x^2)}{\cos^2(x^2)} \cdot \frac{1}{\cos^2(x^2)} \cdot 2x$

$y' = \frac{6x \cdot \operatorname{sen}^2(x^2)}{\cos^4(x^2)}$

q) $y = \sqrt{\ln x} = (\ln x)^{\frac{1}{2}}$

$y' = \frac{1}{2} \cdot (\ln x)^{\frac{1}{2}-1} \cdot (\ln x)'$

$y' = \frac{1}{2} (\ln x)^{-\frac{1}{2}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$

r) $y = \operatorname{arcsen} \frac{x^2}{3}$

$y' = \frac{1}{\sqrt{1-\left(\frac{x^2}{3}\right)^2}} \cdot \left(\frac{x^2}{3}\right)' = \frac{1}{\sqrt{1-\frac{x^4}{9}}} \cdot \frac{2x}{3}$

$y' = \frac{1}{\sqrt{\frac{9-x^4}{9}}} \cdot \frac{2x}{3} = \frac{\frac{1}{1}}{\frac{\sqrt{9-x^4}}{3}} \cdot \frac{2x}{3} = \frac{\cancel{3}}{\sqrt{9-x^4}} \cdot \frac{2x}{\cancel{3}} = \frac{2x}{\sqrt{9-x^4}}$

s) $y = \arctan(x^2+1)$

$$y' = \frac{1}{1+(x^2+1)^2} \cdot (x^2+1)' = \frac{1}{1+(x^2+1)^2} \cdot 2x$$

$$y' = \frac{2x}{1+(x^2+1)^2} = \frac{2x}{1+x^4+2x^2+1} = \frac{2x}{x^4+2x^2+2}$$

t) $y = \dfrac{x^3-2x}{(2x-1)^2}$

$$y' = \frac{(x^3-2x)' \cdot (2x-1)^2 - (x^3-2x)\cdot\left((2x-1)^2\right)'}{\left((2x-1)^2\right)^2}$$

$(2x-1)'$

$$y' = \frac{(3x^2-2)\cdot(2x-1)^2 - (x^3-2x)\cdot 2\cdot(2x-1)\cdot 2}{(2x-1)^4}$$

SACO FACTOR COMÚN $(2x-1)$

$$y' = \frac{\cancel{(2x-1)}\cdot[(3x^2-2)\cdot(2x-1)-(x^3-2x)\cdot 4]}{(2x-1)^{\cancel{4}\,3}}$$

$$y' = \frac{6x^3-3x^2-4x+2-4x^3+8x}{(2x-1)^3} = \frac{2x^3-3x^2+4x+2}{(2x-1)^3}$$

u) $y = \dfrac{\sqrt{\arctan x}}{3} = \dfrac{1}{3}(\arctan x)^{\frac{1}{2}}$

$(\arctan x)'$

$$y' = \frac{1}{3}\cdot\frac{1}{2}\cdot(\arctan x)^{\frac{1}{2}-1}\cdot\frac{1}{1+x^2}$$

$$y' = \frac{1}{6\cdot\sqrt{\arctan x}\cdot(1+x^2)} = \frac{1}{6\cdot(1+x^2)\cdot\sqrt{\arctan x}}$$

v) $y = \sqrt{x + \sqrt{x}} = (x + x^{\frac{1}{2}})^{\frac{1}{2}}$

$$y' = \frac{1}{2} \cdot (x + x^{\frac{1}{2}})^{\frac{1}{2} - 1} \cdot (x + x^{\frac{1}{2}})'$$

$$y' = \frac{1}{2} (x + x^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 + \frac{1}{2} x^{\frac{1}{2} - 1}\right)$$

$$y' = \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$$

w) $y = \arctan\left(\frac{1-x}{1+x}\right)$

$$y' = \frac{1}{1 + \left(\frac{1-x}{1+x}\right)^2} \cdot \left(\frac{1-x}{1+x}\right)'$$

$$y' = \frac{1}{1 + \frac{(1-x)^2}{(1+x)^2}} \cdot \frac{(1-x)' \cdot (1+x) - (1-x) \cdot (1+x)'}{(1+x)^2}$$

$$y' = \frac{\frac{1}{1}}{\frac{(1+x)^2 + (1-x)^2}{(1+x)^2}} \cdot \frac{-1 \cdot (1+x) - (1-x) \cdot 1}{(1+x)^2}$$

$$y' = \frac{\cancel{(1+x)^2}}{(1+x)^2 + (1-x)^2} \cdot \frac{-1 \cancel{-x} - 1 \cancel{+x}}{\cancel{(1+x)^2}}$$

$$y' = \frac{1}{1 + x^2 \cancel{+2x} + 1 + x^2 \cancel{-2x}} \cdot (-2) = \frac{-2}{2x^2 + 2}$$

$$y' = \frac{\cancel{-2}}{\cancel{2}(x^2 + 1)} = \frac{-1}{x^2 + 1}$$