24.-

a) $y = 3x^3 - \dfrac{2}{\sqrt[3]{x^2}} + \sqrt[5]{x} + \dfrac{3}{x^2} + \sqrt{2} + \log 7$

$y = 3x^3 - 2\cdot x^{-\frac{2}{3}} + x^{\frac{1}{5}} + 3x^{-2} + \sqrt{2} + \log 7$

$y' = 3\cdot 3\cdot x^2 - 2\cdot\left(-\frac{2}{3}\right)\cdot x^{-\frac{2}{3}-1} + \frac{1}{5}x^{\frac{1}{5}-1} + 3(-2)\cdot x^{-2-1} + 0 + 0$

$y' = 9x^2 + \dfrac{4}{3\sqrt[3]{x^5}} + \dfrac{1}{5\sqrt[5]{x^4}} - \dfrac{6}{x^3}$

b) $y = \dfrac{\operatorname{sen}(x^2+1)}{\sqrt{1-x^2}} = \dfrac{\operatorname{sen}(x^2+1)}{(1-x^2)^{\frac{1}{2}}}$

$$y' = \frac{(\operatorname{sen}(x^2+1))'\cdot\sqrt{1-x^2} - \operatorname{sen}(x^2+1)\cdot\left((1-x^2)^{\frac{1}{2}}\right)'}{\left(\sqrt{1-x^2}\right)^2}$$

$$y' = \frac{\cos(x^2+1)\cdot 2x\cdot\sqrt{1-x^2} - \operatorname{sen}(x^2+1)\cdot\frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}\cdot(-\cancel{2}x)}{1-x^2}$$

$$y' = \frac{\dfrac{2x\sqrt{1-x^2}\cos(x^2+1)}{1} + \dfrac{x\operatorname{sen}(x^2+1)}{\sqrt{1-x^2}}}{1-x^2}$$

$$y' = \frac{\dfrac{2x\sqrt{1-x^2}\cos(x^2+1)\cdot\sqrt{1-x^2} + x\operatorname{sen}(x^2+1)}{\sqrt{1-x^2}}}{\dfrac{1-x^2}{1}} =$$

$$y' = \frac{2x\cdot(1-x^2)\cos(x^2+1) + x\operatorname{sen}(x^2+1)}{(1-x^2)\cdot\sqrt{1-x^2}}$$

$$y' = \frac{2x\cos(x^2+1) - 2x^3\cos(x^2+1) + x\operatorname{sen}(x^2+1)}{(1-x^2)\cdot\sqrt{1-x^2}}$$

c) $y = \dfrac{\operatorname{sen} x + \cos x}{\operatorname{sen} x - \cos x}$

$$y' = \frac{(\operatorname{sen} x + \cos x)' (\operatorname{sen} x - \cos x) - (\operatorname{sen} x + \cos x) \cdot (\operatorname{sen} x - \cos x)'}{(\operatorname{sen} x - \cos x)^2}$$

$$y' = \frac{(\cos x - \operatorname{sen} x)(\operatorname{sen} x - \cos x) - (\operatorname{sen} x + \cos x) \cdot (\cos x + \operatorname{sen} x)}{(\operatorname{sen} x - \cos x)^2}$$

$$y' = \frac{\cancel{\cos x \operatorname{sen} x} - \cos^2 x - \operatorname{sen}^2 x + \cancel{\operatorname{sen} x \cos x} - \cancel{\operatorname{sen} x \cos x} - \operatorname{sen}^2 x - \cos^2 x - \cancel{\cos x \operatorname{sen} x}}{(\operatorname{sen} x - \cos x)^2} =$$

$$y' = \frac{-(\cos^2 x + \operatorname{sen}^2 x) - (\operatorname{sen}^2 x + \cos^2 x)}{(\operatorname{sen} x - \cos x)^2} = \frac{-1 - 1}{(\operatorname{sen} x - \cos x)^2}$$

$$y' = \frac{-2}{(\operatorname{sen} x - \cos x)^2}$$

d) $y = \operatorname{sen}(3x+1) \cdot \cos(3x+1)$

$$y' = (\operatorname{sen}(3x+1))' \cos(3x+1) + \operatorname{sen}(3x+1) \cdot (\cos(3x+1))'$$

$$y' = \cos(3x+1) \cdot 3 \cdot \cos(3x+1) + \operatorname{sen}(3x+1) \cdot (-\operatorname{sen}(3x+1)) \cdot 3$$

$$y' = 3\cos^2(3x+1) - 3\operatorname{sen}^2(3x+1)$$

$$y' = 3(\cos^2(3x+1) - \operatorname{sen}^2(3x+1))$$

e) $$y = \frac{2x^2 - x + 1}{(x^2 + x - 1)^2}$$

$$y' = \frac{(2x^2-x+1)' (x^2+x-1)^2 - (2x^2-x+1)\cdot\left[(x^2+x-1)^2\right]'}{\left((x^2+x-1)^2\right)^2}$$

$$y' = \frac{(4x-1)\cdot(x^2+x-1)^2 - (2x^2-x+1)\cdot 2\cdot(x^2+x-1)\cdot(2x+1)}{(x^2+x-1)^4}$$

SACO FACTOR COMÚN ANTES FACER OPERACIÓNS

$$y' = \frac{\cancel{(x^2+x-1)}\cdot\left[(4x-1)\cdot(x^2+x-1) - 2\cdot(2x^2-x+1)\cdot(2x+1)\right]}{(x^2+x-1)^{\cancel{4}\,3}}$$

$$y' = \frac{4x^3 + 4x^2 - 4x - x^2 - x + 1 - 8x^3 - 4x^2 + 4x^2 + 2x - 4x - 2}{(x^2+x-1)^3}$$

$$y' = \frac{-4x^3 + 3x^2 - 7x - 1}{(x^2+x-1)^3}$$

f) $$y = \ln(3x) + e^{\sqrt{x}}$$

$$y' = (\ln(3x))' + (e^{\sqrt{x}})' = \frac{1}{3x}\cdot(3x)' + e^{\sqrt{x}}\cdot(\sqrt{x})'$$

$$y' = \frac{3}{3x} + e^{\sqrt{x}}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}$$

$$y' = \frac{1}{x} + \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

g) $y = e^{2x+1} \cdot \tan x$

$y' = (e^{2x+1})' \cdot \tan x + e^{2x+1} \cdot (\tan x)'$

$y' = e^{2x+1} \cdot (2x+1)' \cdot \tan x + e^{2x+1} \cdot \frac{1}{\cos^2 x}$

$y' = e^{2x+1} \cdot 2 \cdot \tan x + \frac{e^{2x+1}}{\cos^2 x}$

$y' = e^{2x+1} \cdot \left(2 \tan x + \frac{1}{\cos^2 x} \right)$

$y' = e^{2x+1} \left(\frac{2 \operatorname{sen} x}{\cos x} + \frac{1}{\cos^2 x} \right)$

$y' = e^{2x+1} \quad \frac{\overset{\operatorname{sen}(2x)}{2 \operatorname{sen} x \cdot \cos x} + 1}{\cos^2 x}$

$y' = \frac{(1 + \operatorname{sen}(2x)) \cdot e^{2x+1}}{\cos^2 x}$

h) $y = (x^2 - 1) \cdot \ln(x^2 - 1)$

$y' = (x^2-1)' \cdot \ln(x^2-1) + (x^2-1) \cdot \ln(x^2-1)$

$y' = 2x \ln(x^2-1) + (x^2-1) \cdot \frac{1}{x^2-1} \cdot (x^2-1)'$

$y' = 2x \ln(x^2-1) + \frac{\cancel{(x^2-1)} \cdot 2x}{\cancel{x^2-1}}$

$y' = 2x \left(\ln(x^2-1) + 1 \right)$

i) $y = \operatorname{sen}^3 x - \operatorname{sen} x^2 + 5 \operatorname{sen} x = (\operatorname{sen} x)^3 - \operatorname{sen}(x^2) + 5 \operatorname{sen} x$

$y' = 3 \cdot (\operatorname{sen} x)^2 \cdot (\operatorname{sen} x)' - \cos(x^2) \cdot (x^2)' + 5 (\operatorname{sen} x)'$

$y' = 3 \operatorname{sen}^2 x \cdot \cos x - 2x \cos x^2 + 5 \cos x$

$y' = 3 \cos x \cdot \operatorname{sen}^2 x - 2x \cos x^2 + 5 \cos x$

j) $y = \dfrac{x^2 - 5x}{(x-1)^3}$

$y' = \dfrac{(x^2-5x)' \cdot (x-1)^3 - (x^2-5x) \cdot ((x-1)^3)'}{((x-1)^3)^2}$

$y' = \dfrac{(2x-5) \cdot (x-1)^3 - (x^2-5x) \cdot 3 \cdot (x-1)^2 \cdot (x-1)'}{(x-1)^6}$

$y' = \dfrac{(2x-5) \cdot (x-1)^3 - (x^2-5x) \cdot 3 \cdot (x-1)^2 \cdot 1}{(x-1)^6}$

SACO FACTOR COMÚN

$y' = \dfrac{\cancel{(x-1)^2} \left[(2x-5) \cdot (x-1) - (x^2-5x) \cdot 3 \right]}{(x-1)^{\cancel{6}\,4}}$

$y' = \dfrac{2x^2 - 2x - 5x + 5 - 3x^2 + 15x}{(x-1)^4}$

$y' = \dfrac{-x^2 + 8x + 5}{(x-1)^4}$

k) $y = 5^{\frac{x^2}{(2x-1)^2}}$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \left(\frac{x^2}{(2x-1)^2} \right)' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{(x^2)'(2x-1)^2 - x^2((2x-1)^2)'}{((2x-1)^2)^2}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{2x\cdot(2x-1)^2 - x^2\cdot 2\cdot(2x-1)^1\cdot 2}{(2x-1)^4}$$

SACO FACTOR COMÚN $(2x-1)$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{\cancel{(2x-1)}\cdot\left[2x\cdot(2x-1) - x^2\cdot 2\cdot 2\right]}{(2x-1)^{\cancel{4}\,3}}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{4x^2 - 2x - 4x^2}{(2x-1)^3}$$

$$y' = 5^{\frac{x^2}{(2x-1)^2}} \cdot \ln 5 \cdot \frac{-2x}{(2x-1)^3} = -\frac{2x\ln 5\cdot 5^{\frac{x^2}{(2x-1)^2}}}{(2x-1)^3}$$

l) $y = \sqrt{\operatorname{sen} x} = (\operatorname{sen} x)^{\frac{1}{2}}$

$$y' = \frac{1}{2}(\operatorname{sen} x)^{\frac{1}{2}-1} \cdot (\operatorname{sen} x)'$$

$$y' = \frac{1}{2}(\operatorname{sen} x)^{-\frac{1}{2}} \cdot \cos x$$

$$y' = \frac{\cos x}{2\sqrt{\operatorname{sen} x}}$$

m) $y = \cos^2 x + e^{\operatorname{sen} x} = (\cos x)^2 + e^{\operatorname{sen} x}$

$y' = 2 \cdot (\cos x)^1 \cdot (\cos x)' + e^{\operatorname{sen} x} \cdot (\operatorname{sen} x)'$

$y' = 2 \cos x \cdot (-\operatorname{sen} x) + e^{\operatorname{sen} x} \cdot \cos x$

$y' = \cos x \cdot (-2 \operatorname{sen} x + e^{\operatorname{sen} x})$

$y' = -2 \operatorname{sen} x \cos x + \cos x \cdot e^{\operatorname{sen} x}$

n) $y = \dfrac{\log x^2}{x} = \dfrac{2 \log x}{x}$

$y' = \dfrac{(2 \log x)' \cdot x - 2 \log x \cdot (x)'}{x^2} = \dfrac{2 \cdot \frac{1}{\cancel{x} \cdot \ln 10} \cdot \cancel{x} - 2 \log x \cdot 1}{x^2}$

$y' = \dfrac{\frac{2}{\ln 10} - 2 \log x}{x^2} = \dfrac{\frac{2 - 2 \ln 10 \log x}{\ln 10}}{\frac{x^2}{1}}$

$y' = \dfrac{2(1 - \ln 10 \cdot \log x)}{x^2 \cdot \ln 10} = \dfrac{2\left(1 - \ln 10 \cdot \frac{\ln x}{\ln 10}\right)}{x^2 \ln 10}$

$y' = \dfrac{2 \cdot (1 - \ln x)}{x^2 \cdot \ln 10}$

o) $y = \arccos e^{-x}$

$$y' = \frac{-1}{\sqrt{1-(e^{-x})^2}} \cdot (e^{-x})' = \frac{-1}{\sqrt{1-e^{-2x}}} \cdot e^{-x} \cdot (-x)'$$

$$y' = \frac{-1}{\sqrt{1-e^{-2x}}} \cdot e^{-x} \cdot (-1)$$

$$y' = \frac{1 \cdot e^{-x}}{\sqrt{1-e^{-2x}}} = \frac{e^{-x}}{\sqrt{1-e^{-2x}}}$$

p) $y = (x^2-3)^3$

$$y' = 3 \cdot (x^2-3)^2 \cdot (x^2-3)' = 3 \cdot (x^2-3)^2 \cdot 2x$$

$$y' = 6x(x^2-3)^2$$

q) $y = \arctan \frac{\sqrt{x}}{2} = \arctan\left(\frac{1}{2}x^{\frac{1}{2}}\right)$

$$y' = \frac{1}{1+\left(\frac{\sqrt{x}}{2}\right)^2} \cdot \left(\frac{1}{2}x^{\frac{1}{2}}\right)' = \frac{1}{1+\frac{x}{4}} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1}$$

$$y' = \frac{1}{1+\frac{x}{4}} \cdot \frac{1}{4} \cdot x^{-\frac{1}{2}} = \frac{\frac{1}{1}}{\frac{4+x}{4}} \cdot \frac{1}{4} \cdot \frac{1}{\sqrt{x}}$$

$$y' = \frac{\cancel{4}}{x+4} \cdot \frac{1}{\cancel{4}} \cdot \frac{1}{\sqrt{x}} = \frac{1}{(x+4)\cdot\sqrt{x}} = \frac{\sqrt{x}}{x\cdot(x+4)}$$

r) $y = \left(\frac{x}{1+x^2}\right)^2$

$y' = 2 \cdot \left(\frac{x}{1+x^2}\right)^{2-1} \cdot \left(\frac{x}{1+x^2}\right)'$

$y' = 2 \cdot \frac{x}{1+x^2} \cdot \frac{(x)' \cdot (1+x^2) - x \cdot (1+x^2)'}{(1+x^2)^2}$

$y' = \frac{2x}{1+x^2} \cdot \frac{1 \cdot (1+x^2) - x \cdot 2x}{(1+x^2)^2}$

$y' = \frac{2x}{1+x^2} \cdot \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{2x \cdot (-x^2+1)}{(1+x^2)^3}$

$y' = \frac{2x(1-x^2)}{(1+x^2)^3}$

s) $y = \arcsen\sqrt{x} + \frac{7}{\tan x} = \arcsen(x^{\frac{1}{2}}) + 7 \cdot (\tan x)^{-1}$

$y' = \frac{1}{\sqrt{1-(x^{\frac{1}{2}})^2}} \cdot (x^{\frac{1}{2}})' + 7 \cdot (-1) \cdot (\tan x)^{-1-1} \cdot (\tan x)'$

$y' = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - 7 \cdot (\tan x)^{-2} \cdot \frac{1}{\cos^2 x}$

$\tan^2 x = \frac{\sen^2 x}{\cos^2 x}$

$y' = \frac{1}{2\sqrt{1-x} \cdot \sqrt{x}} - \frac{7}{\tan^2 x \cos^2 x}$

$y' = \frac{1}{2\sqrt{x}\sqrt{1-x}} - \frac{7}{\sen^2 x}$

t) $y = x \cdot e^{2x+1}$

$y' = (x)' \cdot e^{2x+1} + x \cdot (e^{2x+1})'$

$y' = 1 \cdot e^{2x+1} + x \cdot e^{2x+1} \cdot (2x+1)'$

$y' = e^{2x+1} + x \cdot e^{2x+1} \cdot 2$

$y' = e^{2x+1} \cdot (1 + 2x)$

u) $y = \operatorname{sen} \frac{x}{2} + \cos \frac{x}{2}$

$y' = \cos \frac{x}{2} \cdot \left(\frac{x}{2}\right)' + \left(-\operatorname{sen} \frac{x}{2}\right) \cdot \left(\frac{x}{2}\right)'$

$y' = \frac{1}{2} \cdot \cos \frac{x}{2} - \frac{1}{2} \cdot \operatorname{sen} \frac{x}{2}$

$y' = \frac{1}{2} \left(\cos \frac{x}{2} - \operatorname{sen} \frac{x}{2}\right)$

v) $y = x \cdot \operatorname{sen}(\pi - x)$

$y' = (x)' \cdot \operatorname{sen}(\pi - x) + x \cdot (\operatorname{sen}(\pi - x))'$

$y' = 1 \cdot \operatorname{sen}(\pi - x) + x \cdot \cos(\pi - x) \cdot (\pi - x)'$

$y' = \operatorname{sen}(\pi - x) + x \cdot \cos(\pi - x) \cdot (-1)$

$y' = \operatorname{sen}(\pi - x) - x \cdot \cos(\pi - x)$

w) $y=\dfrac{2}{(x+3)^3}=2\cdot(x+3)^{-3}$

$y'=2\cdot(-3)\cdot(x+3)^{-3-1}\cdot(x+3)'$

$y'=-6\cdot(x+3)^{-4}\cdot 1=\dfrac{-6}{(x+3)^4}$

APLICO PROPIEDADES DOS LOGARITMOS ANTES DE DERIVAR

x) $y=\ln\left(\dfrac{x^2-1}{x^2+1}\right)=\ln(x^2-1)-\ln(x^2+1)$

$y'=\dfrac{1}{x^2-1}\cdot(x^2-1)'-\dfrac{1}{x^2+1}\cdot(x^2+1)'$

$y'=\dfrac{2x}{x^2-1}-\dfrac{2x}{x^2+1}=\dfrac{2x(x^2+1)-2x(x^2-1)}{(x+1)(x-1)(x^2+1)}$

$y'=\dfrac{\cancel{2x^3}+2x\cancel{-2x^3}+2x}{(x^2-1)(x^2+1)}=\dfrac{4x}{(x^2-1)(x^2+1)}$

y) $y=\ln\sqrt{\dfrac{x}{x^2+1}}=\ln\left(\dfrac{x}{x^2+1}\right)^{1/2}$

APLICO PROPIEDADES LOGARITMOS

$y=\dfrac{1}{2}\left(\ln x-\ln(x^2+1)\right)$

$y'=\dfrac{1}{2}\cdot\left(\dfrac{1}{x}-\dfrac{1}{x^2+1}\cdot(x^2+1)'\right)$

$y'=\dfrac{1}{2}\left(\dfrac{1}{x}-\dfrac{2x}{x^2+1}\right)=\dfrac{1}{2x}-\dfrac{\cancel{2}x}{\cancel{2}(x^2+1)}$

$y'=\dfrac{1}{2x}-\dfrac{x}{x^2+1}=\dfrac{x^2+1-2x^2}{2x(x^2+1)}=\dfrac{1-x^2}{2x(x^2+1)}$

APLICO PROPIED. LOGARITMOS APLICO PROPIEDADES LOGARITMOS

z) $y = \ln(x e^{-x}) = \ln x + \ln e^{-x} = \ln x - x$

$$y' = \frac{1}{x} - 1 = \frac{1-x}{x}$$

APLICO PROPIEDADES LOGARITMOS

a') $y = \log \frac{(3x-5)^3}{x} = \log (3x-5)^3 - \log x$

APLICO PROPIEDADES LOGARITMOS

$y = 3 \cdot \log (3x-5) - \log x$

$(3x-5)'$

$$y' = 3 \cdot \frac{1}{3x-5} \cdot \frac{1}{\ln 10} \cdot 3 - \frac{1}{x} \cdot \frac{1}{\ln 10}$$

$$y' = \frac{9}{\ln 10 \cdot (3x-5)} - \frac{1}{\ln 10 \cdot x} = \frac{1}{\ln 10}\left(\frac{9}{3x-5} - \frac{1}{x}\right)$$

b') $y = \log (\tan x)^2$

$$y' = \frac{1}{(\tan x)^2} \cdot \frac{1}{\ln 10} \cdot ((\tan x)^2)'$$

$$y' = \frac{1}{(\tan x)^2} \cdot \frac{1}{\ln 10} \cdot 2 \cdot (\tan x)^1 \cdot (\tan x)'$$

$$y' = \frac{1}{\tan^2 x} \cdot \frac{1}{\ln 10} \cdot 2 \cdot \cancel{\tan x} \cdot \frac{1}{\cos^2 x}$$

$$y' = \frac{2}{\ln 10 \cdot \tan x \cdot \cos^2 x} = \frac{2}{\ln 10 \cdot \frac{\sin x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}} x}$$

$$y' = \frac{2}{\ln 10 \cdot \sin x \cdot \cos x}$$

b') Outra forma !

APLICO PROPIEDADES LOGARITMOS

$y = \log(\tan x)^2 = 2 \cdot \log(\tan x)$

$y' = 2 \cdot (\log(\tan x))' = 2 \cdot \frac{1}{\tan x} \cdot \frac{1}{\ln 10} \cdot (\tan x)'$

$y' = 2 \cdot \frac{1}{\tan x} \cdot \frac{1}{\ln 10} \cdot \frac{1}{\cos^2 x} = \frac{2}{\ln 10 \cdot \operatorname{sen} x \cdot \cos x}$

APLICO PROPIEDADES LOGARITMOS

c') $y = \ln x^x = x \cdot \ln x$

$y' = (x)' \cdot \ln x + x \cdot (\ln x)' = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$y' = \ln x + 1$