PÁXINA 1.

Ver o valor na gráfica $f(0)=3$
$f(-2)=1$

1) a) $T.V.M.[-2,0] = \dfrac{f(0)-f(-2)}{0-(-2)} = \dfrac{3-1}{2} = \dfrac{2}{2} = 1$

$\downarrow$

Crecemento medio da función en $[-2,0]$
Pendente da recta que une $(0,f(0))$ e $(-2,f(-2))$.

b) $T.V.M.[0,2] = \dfrac{f(2)-f(0)}{2-0} = \dfrac{0-3}{2} = \dfrac{-3}{2}$

c) $TVM\,[2,5] = \dfrac{f(5)-f(2)}{5-2} = \dfrac{1-0}{3} = \dfrac{1}{3}$

2) $y = x^2-8x+12 = f(x)$

$TVM\,[1,2] = \dfrac{f(2)-f(1)}{2-1} = \dfrac{(2^2-8\cdot2+12)-(1^2-8\cdot1+12)}{1} =$

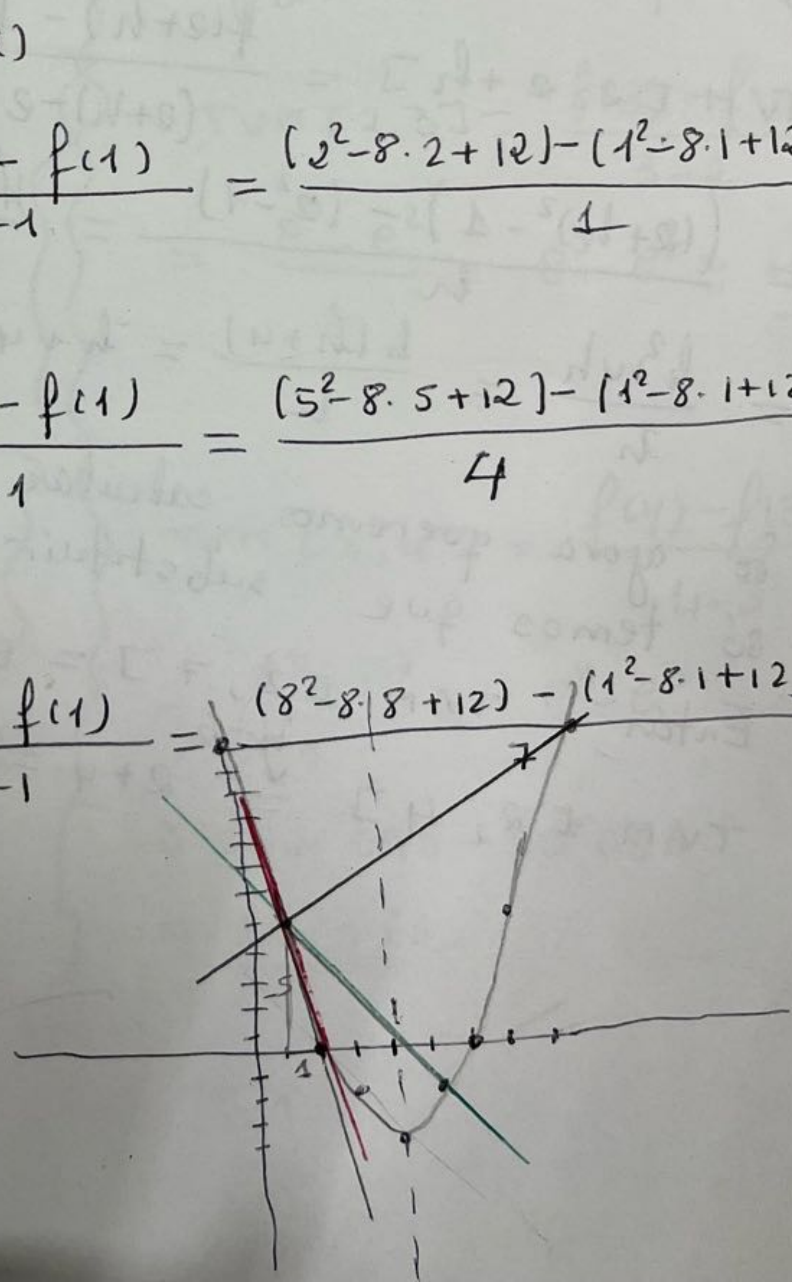
$= \dfrac{0-5}{1} = -5$

$TVM\,[1,5] = \dfrac{f(5)-f(1)}{5-1} = \dfrac{(5^2-8\cdot5+12)-(1^2-8\cdot1+12)}{4} =$

$= \dfrac{-3-5}{4} = \dfrac{-8}{4} = -2$

$TVM\,[1,8] = \dfrac{f(8)-f(1)}{8-1} = \dfrac{(8^2-8\cdot8+12)-(1^2-8\cdot1+12)}{} =$

$= \dfrac{12-5}{7} = \dfrac{7}{7} = 1$



$y = x^2-8x+12$

- $V(4,-4)$
- $\cup$
- $(6,0)\ (2,0)\ (0,12)$

③

$$TVM[1,1+h] = \frac{f(1+h)-f(1)}{(1+h)-1} = \frac{((1+h)^2-8\cdot(1+h)+12)-(1^2-8\cdot 1+12)}{h}$$

$$= \frac{\cancel{1}+h^2+2h\cancel{-8}-8h\cancel{+12}\cancel{-5}}{h} = \frac{h^2-6h}{h} = \frac{\cancel{h}(h-6)}{\cancel{h}} = h-6$$

Como $TVM[1,1+h] = h-6$

$$TVM[1,2] \overset{h=1}{=} 1-6 = \underline{\underline{-5}}$$

$$TVM[1,5] \overset{h=4}{=} 4-6 = \underline{\underline{-2}}$$

$$TVM[1,8] \overset{h=7}{=} 7-6 = \underline{\underline{1}}$$

④ $f(x) = x^2-1$

$$TVM[2,2+h] = \frac{f(2+h)-f(2)}{(2+h)-2} =$$

$$= \frac{((2+h)^2-1)-(2^2-1)}{h} = \frac{\cancel{4}+h^2+4h\cancel{-1}\cancel{-3}}{h} =$$

$$= \frac{h^2+4h}{h} = \frac{h(h+4)}{h} = h+4$$

Se agora queremos calcular $TVM[2,7]$ só temos que substituir $h=5$ pq. $7 = 2 + \overset{h}{5}$

Entón, $TVM[2,7] = 5+4 = 9$

$$TVM[2,4] \overset{h=2}{=} 2+4 = 6$$

(5) $f(x) = -x^2 + 5x - 3$ $\quad f(1) = -1^2 + 5\cdot 1 - 3 = -1 + 5 - 3 = 1$

$$TVM[1, 1+h] = \frac{f(1+h) - f(1)}{h} = \frac{(-(1+h)^2 + 5\cdot(1+h) - 3) - 1}{h} =$$

$$= \frac{\cancel{-1} - h^2 - 2h + \cancel{3} + 5h - \cancel{3} - \cancel{1}}{h} = \frac{-h^2 + 3h}{h} = \frac{\cancel{h}(-h+3)}{\cancel{h}} = -h+3$$

Como $TVM[1, 1+h] = -h + 3$

$$TVM[1,2] \overset{h=1}{=} -1+3 = 2$$

$$TVM[1; 1'5] \overset{h=0'5}{=} -0'5 + 3 = 2'5$$

(6) $f(x) = x^3$

$$TVM[2,3] = \frac{f(3) - f(2)}{3-2} =$$

$$= \frac{3^3 - 2^3}{1} = \frac{27-8}{1} = \underline{\underline{19}}$$

$$TVM[3,4] = \frac{f(4) - f(3)}{4-3} =$$

$$= \frac{4^3 - 3^3}{1} = \frac{64-27}{1} =$$

$$= \underline{\underline{37}}$$

$f(x) = e^x$

$$TVM[2,3] = \frac{f(3) - f(2)}{3-2} =$$

$$= \frac{e^3 - e^2}{1} = e^3 - e^2 \simeq \underline{\underline{12'69}}$$

$$TVM[3,4] = \frac{f(4) - f(3)}{4-3}$$

$$= \frac{e^4 - e^3}{1} = e^4 - e^3$$

$$\simeq 54'598 - 20'086$$

$$= \underline{34'512}$$

7- VER NA GRÁFICA

$f'(a)$ = pendente da recta tanxente a f en $x=a$

Entón $\{f'(-3)=-3\}$ (-3 é a pendente da recta tanxente a f en $x=-3$)

¿Como calculamos esa pendente?

Collemos dous puntos da recta.

Eu collo $\ominus$ $P(-3,3)$ e $Q(-2,0)$ e facemos

$$m=\frac{\text{variación } y}{\text{variación } x}=\frac{3-0}{-3-(-2)}=\frac{3}{-1}=-3=f'(-3)$$

$\{f'(0)=\frac{3}{2}\}$ ($\frac{3}{2}$ é a pendente da recta tanxente a f en $x=0$)

¿Como calculamos esa pendente?

Collo dous puntos da recta: $P(0,4)$ e $Q(2,7)$

Entón $m=\dfrac{\text{variación } y}{\text{variación } x}=\dfrac{7-4}{2-0}=\dfrac{3}{2}=f'(0)$

$\{f'(4)=-2\}$ (-2 é a pendente da recta tanxente a f en $x=4$)

¿Como calculamos esa pendente?

Collo dous puntos da recta: $P(4,4)$ e $Q(6,0)$

Entón $m=\dfrac{\text{variación } y}{\text{variación } x}=\dfrac{0-4}{6-4}=\dfrac{-4}{2}=-2=f'(4)$

⑧

a) <u>En $x=-2$</u> $f'(-2)=0$ (A recta tanxente é horizontal en $x=-2$ é dicir, a pendente é cero)

<u>En $x=2$</u> $f'(2)=0$ "

b) $f'(1)>0$ (A recta tanxente a f en $x=1$ ten pendente ⊕ $\Rightarrow f'(1)>0$)

c) $f'(3)<0$ (A recta tanxente a f en $x=3$ ten pendente ⊖ $\Rightarrow f'(3)<0$)

⑨ Non (As rectas tanxentes en todos os puntos teñen pendente ⊕)

$f'(-2)<f'(0)<f'(2)$ (Debuxa na gráfica as rectas tanxentes en $x=-2$, $x=0$ e $x=2$ para comprobalo)

<u>RECORDA</u>: A pendente mide a INCLINACIÓN da recta.

⑩ $f(x)=5x-x^2$

ⓐ $f'(4)=\lim\limits_{h\to 0}$ T.V.M. $[4, 4+h] = \lim\limits_{h\to 0} \dfrac{f(4+h)-f(4)}{h} =$

$= \lim\limits_{h\to 0} \dfrac{[5\cdot(4+h)-(4+h)^2]-[5\cdot 4-4^2]}{h} =$

$= \lim\limits_{h\to 0} \dfrac{\cancel{20}+5h-\cancel{16}-h^2-8h-\cancel{4}}{h} = \lim\limits_{h\to 0} \dfrac{-h^2-3h}{h} =$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}(-h-3)}{\cancel{h}} = \lim\limits_{h\to 0}(-h-3) = -3$ ← Tamén é a pendente tanxente $x=4$

(6) (10)

$$f'(5) = \lim_{h\to 0} \text{T.V.M.}[5, 5+h] = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h} =$$

$$= \lim_{h\to 0} \frac{[5\cdot(5+h)-(5+h)^2] - [5\cdot 5 - 5^2]}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{25}+5h-\cancel{25}-h^2-10h-0}{h} =$$

$$= \lim_{h\to 0} \frac{-h^2-5h}{h} \overset{\frac{0}{0}}{=} \lim_{h\to 0} \frac{\cancel{h}(-h-5)}{\cancel{h}} =$$

$$= \lim_{h\to 0} (-h-5) = -5 \leftarrow$$ Tamén é a pendente da recta tauxente en $x=5$

(14) ¡RECORDAD!

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} =$$ PENDENTE DA RECTA TANXENTE A f EN $x=a$.

(14)

a) $f(x) = 3x^2 - 1$

$$\underline{\underline{f'(1)}} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} \overset{f(x)=3x^2-1}{=} \lim_{h\to 0} \frac{[3\cdot(1+h)^2-1]-[3\cdot 1^2-1]}{h} =$$

$$= \lim_{h\to 0} \frac{[3(1+h^2+2h)-1]-2}{h} = \lim_{h\to 0} \frac{3+3h^2+6h-1-2}{h} =$$

$$= \lim_{h\to 0} \frac{3h^2+6h}{h} \overset{\frac{0}{0}}{=} \lim_{h\to 0} \frac{\cancel{h}(3h+6)}{\cancel{h}} =$$

$$= \lim_{h\to 0} (3h+6) = \underline{\underline{6}}$$

b) $f(x) = (2x+1)^2$

$$\underline{\underline{f'(1)}} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} \overset{f(x)=3x^2-1}{=} \lim_{h\to 0} \frac{[[2(1+h)+1]^2]-[(2\cdot 1+1)^2]}{h}$$

$$= \lim_{h\to 0} \frac{(2+2h+1)^2-3^2}{h} = \lim_{h\to 0} \frac{(3+2h)^2-9}{h} =$$

$$= \lim_{h\to 0} \frac{9+4h^2+12h-9}{h} = \lim_{h\to 0} \frac{4h^2+12h}{h} \overset{\frac{0}{0}}{=}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4h+12)}{\cancel{h}} = \lim_{h\to 0} (4h+12) = \underline{\underline{12}}$$

c)

$$\underline{f'(1)} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \overset{f(x)=\frac{3}{x}}{=} \lim_{h\to 0} \frac{\frac{3}{1+h} - \frac{3}{1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3 - 3\cdot(1+h)}{1+h}}{h} = \lim_{h\to 0} \frac{\frac{3-3-3h}{1+h}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{\frac{-3h}{1+h}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-3\cancel{h}}{\cancel{h}\cdot(1+h)} =$$

$$= \lim_{h\to 0} \frac{-3}{1+h} = -\frac{3}{1} = \underline{\underline{-3}}$$

d) $f(x) = \frac{1}{x+2}$

$$\underline{\underline{f'(1)}} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \overset{f(x)=\frac{1}{x+2}}{=} \lim_{h\to 0} \frac{\frac{1}{(1+h)+2} - \frac{1}{1+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h\to 0} \frac{\frac{3-(3+h)}{3\cdot(3+h)}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{-h}{3\cdot(3+h)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-\cancel{h}}{3\cdot(3+h)\cdot\cancel{h}} =$$

$$= \lim_{h\to 0} \frac{-1}{3\cdot(3+h)} = \frac{-1}{3\cdot(3+0)} = \underline{\underline{\frac{-1}{9}}}$$

11. $f(x) = \frac{3}{x-2}$

$$f'(-1) = \lim_{h\to 0} \text{T.V.M.}[-1, -1+h] =$$

$$= \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \frac{\frac{3}{-1+h-2} - \frac{3}{-1-2}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{3}{-3+h} - \frac{3}{-3}}{h} = \lim_{h\to 0} \frac{\frac{3}{-3+h} + 1}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3+(-3+h)}{-3+h}}{h} = \lim_{h\to 0} \frac{\frac{h}{-3+h}}{\frac{h}{1}} =$$

$$= \lim_{h\to 0} \frac{h}{h\cdot(-3+h)} \overset{\frac{0}{0}}{=} \lim_{h\to 0} \frac{1}{-3+h} = -\frac{1}{3}$$

Agora tócavos facer a vós!!!...

$$f'(5) = \ldots$$

$$f'(5) = \lim_{h\to 0} \text{T.V.M.}[5, 5+h] = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{3}{(5+h)-2} - \frac{3}{5-2}}{h} = \lim_{h\to 0} \frac{\frac{3}{3+h} - 1}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{3-(3+h)}{3+h}}{\frac{h}{1}} = \lim_{h\to 0} \frac{\frac{-h}{3+h}}{\frac{h}{1}} =$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(3+h)} = \lim_{h\to 0} \frac{-1}{3+h} = -\frac{1}{3}$$

(12.) $f(x) = \frac{1}{x}$

$$f'(2) = \lim_{h\to 0} TVM[2, 2+h] = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2\cdot(2+h)}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{-h}{2\cdot(2+h)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-\cancel{h}}{2\cancel{h}\cdot(2+h)} =$$

$$= \lim_{h\to 0} \frac{-1}{2\cdot(2+h)} = \frac{-1}{4}$$

Agora tócavos a vos facer $f'(-1)$!!!

$f'(-1) = \ldots$

DEFINICIÓN

DEFINICIÓN T.V.M.

$$f'(-1) = \lim_{h\to 0} T.V.M.[-1, -1+h] = \lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{1}{-1+h} - \frac{1}{(-1)}}{h} = \lim_{h\to 0} \frac{\frac{1}{-1+h} + 1}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{1+(-1+h)}{-1+h}}{h} = \lim_{h\to 0} \frac{\frac{h}{-1+h}}{\frac{h}{1}} =$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\cdot(-1+h)} = \lim_{h\to 0} \frac{1}{-1+h} = \frac{1}{-1+0} = -1$$

FAGO O LÍMITE

15,16

$f(x) = x^2 - 5x + 1$

A pendente da recta tanxente en $x=-2$ é $f'(-2)$.

$$f'(-2) = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} = \quad f(x) = x^2 - 5x + 1$$

$$= \lim_{h \to 0} \frac{[(-2+h)^2 - 5\cdot(-2+h) + 1] - [(-2)^2 - 5\cdot(-2) + 1]}{h} =$$

$$= \lim_{h \to 0} \frac{[4 + h^2 - 4h + 10 - 5h + 1] - [4 + 10 + 1]}{h} =$$

$$= \lim_{h \to 0} \frac{h^2 - 9h + \cancel{15} - \cancel{15}}{h} =$$

$$= \lim_{h \to 0} \frac{h^2 - 9h}{h} \overset{\frac{0}{0}}{=} \lim_{h \to 0} \frac{\cancel{h}(h-9)}{\cancel{h}} =$$

$$= \lim_{h \to 0} (h-9) = \underline{\underline{-9}}$$

PENDENTE DA RECTA TANXENTE EN $X = -2$ $= f'(-2) = \underline{\underline{-9}}$

(16) $f(x) = 4x - x^2$

PENDENTE DA RECTA TANXENTE A f EN $x=2$

É $f'(2)$

$f(x) = 4x - x^2$

$$\underline{f'(2)} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} =$$

$$= \lim_{h\to 0} \frac{[4\cdot(2+h) - (2+h)^2] - [4\cdot 2 - 2^2]}{h} =$$

$$= \lim_{h\to 0} \frac{[8 \cancel{+4h} - 4 - h^2 \cancel{-4h}] - 4}{h} =$$

$$= \lim_{h\to 0} \frac{-h^2 + 4 - 4}{h} =$$

$$= \lim_{h\to 0} \frac{-h^2}{h} \overset{\frac{0}{0}}{=} \lim_{h\to 0} \frac{-h\cdot \cancel{h}}{\cancel{h}} =$$

$$= \lim_{h\to 0} (-h) = \underline{\underline{0}}$$

PENDENTE DA RECTA TANXENTE EN $x=2$ $= f'(2) = \underline{\underline{0}}$