GAUSS.COMPLEXOS.TRIGONOMETRÍA.MATI.

(10 de Novembro de 2023)

Nome:____

1.- Utiliza o método de Gauss para discutir e resolver o seguinte sistema:

(1 PUNTO)

$$\begin{cases} 2x-y+z=-7 \\ x+2y-z=9 \\ -3x+y+2z=-8 \end{cases}$$

2.- Calcula (pasa primeiro o número complexo en forma binómica a forma polar e despois de facer a operación volve a pasalo a forma binómica):

(1,5 PUNTOS)

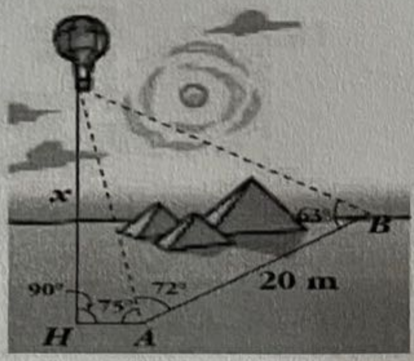
a) $(-\sqrt{3}-i)^4$ b) $\sqrt[3]{-i}$

3.- Calcula sen utilizar a calculadora: **(1 PUNTO)**

$$2\sqrt{2}\,sen\frac{7\pi}{4}+2\sqrt{3}\cos\frac{11\pi}{6}-2\cos\frac{4\pi}{3}+sen\frac{3\pi}{2}-\tan\pi$$

4.- Para calcular a altura dun globo, realizamos as medicións indicadas na figura. Canto dista o globo do punto *A*? Canto do punto *B* ? A que altura está o globo?

(1,5 PUNTOS)



5.- As diagonais dun paralelogramo córtanse nos seus puntos medios. Unha das diagonais mide 10 cm , a outra 8 cm e entre elas forman un ángulo de 70º. Calcula a medida dos lados do paralelogramo. **(1,5 PUNTOS)**

6.- Sabendo que $sen\,\alpha=\frac{-4}{5}$ e $\cos\alpha<0$, calcula sen achar o valor de α. Indica o cuadrante no que está α e xustifica as respostas: **(2 PUNTOS)**

a) $\cos\alpha$ b) $\cos\left(\alpha+\frac{\pi}{4}\right)$ c) $sen\left(\alpha-\frac{\pi}{6}\right)$ d) $\tan\frac{\alpha}{2}$

7.- Resolve: $\cos(2x)=cosx-1$ **(1,5 PUNTOS)**

1. $\begin{pmatrix} 2 & -1 & 1 & -7 \\ 1 & 2 & -1 & 9 \\ -3 & 1 & 2 & -8 \end{pmatrix} \xrightarrow{E_1 \leftrightarrow E_2} \begin{pmatrix} 1 & 2 & -1 & 9 \\ 2 & -1 & 1 & -7 \\ -3 & 1 & 2 & -8 \end{pmatrix} \xrightarrow[E_3+3E_1]{E_1 \;\; E_2-2E_1}$

$\begin{pmatrix} 1 & 2 & -1 & 9 \\ 0 & -5 & 3 & -25 \\ 0 & 7 & -1 & 19 \end{pmatrix} \xrightarrow[5E_3]{E_1 \;\; 7E_2} \begin{pmatrix} 1 & 2 & -1 & 9 \\ 0 & -35 & 21 & -175 \\ 0 & 35 & -5 & 95 \end{pmatrix} \xrightarrow[E_3+E_2]{E_1 \;\; E_2}$

$\begin{pmatrix} 1 & 2 & -1 & 9 \\ 0 & -35 & 21 & -175 \\ 0 & 0 & 16 & -80 \end{pmatrix}$ $\left.\begin{array}{r} x+2y-z=9 \\ -35y+21z=-175 \\ 16z=-80 \end{array}\right\}$

E_3 $16z=-80 \Rightarrow z=-5$

$-35y+21\cdot(-5)=-175 \Rightarrow 35y=-105+175$

$35y=70$

$y=2$

$x+2y-z=9$

$x+2\cdot 2-(-5)=9$

$x+4+5=9 \Rightarrow x=0$

Sol. $(0,2,-5)$

2. a) $(-\sqrt{3}-i)^4$

$z=-\sqrt{3}-i$ pasamos a forma polar $|z|=\sqrt{(-\sqrt{3})^2+(-1)^2}=\sqrt{4}=2$

$\tan\alpha=\frac{-1}{-\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

$\Rightarrow \alpha=30^\circ$

$z=-\sqrt{3}-i=2_{210^\circ}$

$\alpha=210^\circ \leftarrow$ $z=-\sqrt{3}-i$ en 3^{er} cuadrante

$(-\sqrt{3}-i)^4=(2_{210^\circ})^4=16_{4\cdot 210^\circ}=$

$=16_{840^\circ}=$ $\begin{array}{l|l} 840^\circ & 360^\circ \\ \hline 120 & 2 \end{array}$ $16_{120^\circ}=16\,(\cos 120^\circ+i\,\operatorname{sen} 120^\circ)$

$=16\cdot\left(-\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)=-8+8\sqrt{3}\,i$

b) $\sqrt[3]{-i} = \sqrt[3]{1_{270^\circ}} = \sqrt[3]{1}_{\frac{270^\circ+360^\circ k}{3}} = \begin{cases} \xrightarrow{k=0} 1_{90^\circ} \\ \xrightarrow{k=1} 1_{210^\circ} \\ \xrightarrow{k=2} 1_{330^\circ} \end{cases}$

$k = 0, 1, 2$

$z = -i = 1_{270^\circ}$

$k=0 \quad 1_{90^\circ} = 1\,(\cos 90 + i \operatorname{sen} 90) = i$

$k=1 \quad 1_{210^\circ} = 1\,(\cos 210^\circ + i \operatorname{sen} 210^\circ) = -\frac{\sqrt{3}}{2} - \frac{1}{2} i$

$k=2 \quad 1_{330^\circ} = 1\,(\cos 330^\circ + i \operatorname{sen} 330^\circ) = \frac{\sqrt{3}}{2} - \frac{1}{2} i$

3

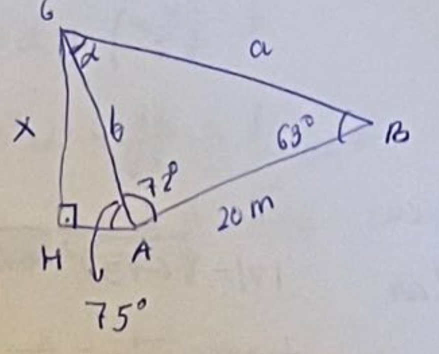
$2\sqrt{2} \cdot \operatorname{sen} \frac{7\pi}{4} + 2\sqrt{3} \cos \frac{11\pi}{6} - 2\cos\frac{4\pi}{3} + \operatorname{sen}\frac{3\pi}{2} - \tan \pi =$

$= 2\sqrt{2} \operatorname{sen} 315^\circ + 2\sqrt{3} \cos 330^\circ - 2\cos 240^\circ + \operatorname{sen} 270^\circ - \tan 180^\circ =$

$= 2\sqrt{2}(-\operatorname{sen} 45) + 2\sqrt{3} \cdot \cos 30^\circ - 2(-\cos 60^\circ) + (-1) - 0 =$

$= -\cancel{2}\sqrt{2}\, \frac{\sqrt{2}}{\cancel{2}} + 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} + 2 \cdot \frac{1}{2} - 1 = -2 + 3 + 1 - 1 = \underline{\underline{1}}$

4



$\alpha = 180^\circ - 72^\circ - 63^\circ = 180 - 135 = 45^\circ$

$\frac{20}{\operatorname{sen} 45^\circ} = \frac{a}{\operatorname{sen} 72} \Rightarrow a = 26'90\ m$

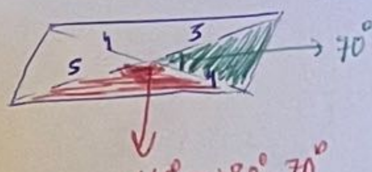
$\frac{20}{\operatorname{sen} 45^\circ} = \frac{b}{\operatorname{sen} 63^\circ} \Rightarrow b = 25'20\ m$

$\operatorname{sen} 75^\circ = \frac{x}{b} = \frac{x}{25'2} \Rightarrow x = 25'2 \cdot \operatorname{sen} 75^\circ = 24'34\ m$

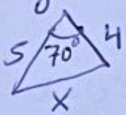
El globo dista 25'2 m de A y 26'9 de B.

La altura del globo es 24'34 m.

(5)



No triángulo verde:

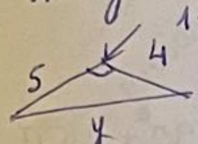


T. COSENO

$x^2 = 5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cdot \cos 70°$

$x = 5'23 \text{ cm}$

No triángulo vermello:



T. COSENO

$y^2 = 5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cdot \cos 110°$

$y = 7'39 \text{ cm}$

Los lados miden $5'23$ cm y $7'39$ cm.

(6) $\left.\begin{array}{l} \operatorname{sen} \alpha = \frac{-4}{5} \\ \cos \alpha < 0 \end{array}\right\} \Rightarrow \alpha \in 3^{er}$ CUADRANTE

a) $\cos^2 \alpha + \operatorname{sen}^2 \alpha = 1$

$\cos^2 \alpha + \left(\frac{-4}{5}\right)^2 = 1$

$\cos^2 \alpha + \frac{16}{25} = 1$

$\cos \alpha = \pm \frac{3}{5} \xRightarrow{\alpha \in 3^{\underline{o}}} \cos \alpha = -\frac{3}{5}$

b) $\cos\left(\alpha + \frac{\pi}{4}\right) =$

$= \cos \alpha \cdot \cos \frac{\pi}{4} - \operatorname{sen} \alpha \operatorname{sen} \frac{\pi}{4} =$

$= -\frac{3}{5} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{4}{5}\right) \cdot \frac{\sqrt{2}}{2} =$

$= \frac{-3\sqrt{2} + 4\sqrt{2}}{10} = \frac{\sqrt{2}}{10}$

c) $\operatorname{sen}\left(\alpha - \frac{\pi}{6}\right) =$

$= \operatorname{sen} \alpha \cdot \cos \frac{\pi}{6} - \cos \alpha \cdot \operatorname{sen} \frac{\pi}{6} =$

$= \frac{-4}{5} \cdot \frac{\sqrt{3}}{2} - \left(-\frac{3}{5}\right) \cdot \frac{1}{2} =$

$= \frac{-4\sqrt{3} + 3}{10}$

d) $\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$

$180° < \alpha < 270°$
$\Downarrow$
$90° < \frac{\alpha}{2} < 135°$

$= \ominus \sqrt{\frac{1 - \left(-\frac{3}{5}\right)}{1 + \left(-\frac{3}{5}\right)}} = -\sqrt{\frac{\frac{8}{5}}{\frac{2}{5}}}$

$= -\sqrt{4} = -2$

⑦

$\cos(2x) = \cos x - 1$

$\cos^2 x - \operatorname{sen}^2 x = \cos x - 1$

$\cos^2 x - (1 - \cos^2 x) = \cos x - 1$

$-1 + 2\cos^2 x = \cos x - 1$

$2\cos^2 x - \cos x = 0$

$\cos x\,(2\cos x - 1) = 0$

$\cos x = 0 \Longrightarrow x = 90^\circ + 2k\pi$, $x_2 = 270^\circ + 2k\pi$ $\quad k \in \mathbb{Z}$

$2\cos x - 1 = 0 \Longrightarrow \cos x = \frac{1}{2}$

$\Longrightarrow x_3 = 60^\circ + 2k\pi \quad k \in \mathbb{Z}$

$x_4 = 300^\circ + 2k\pi \quad k \in \mathbb{Z}$