

EJEMPLOS DERIVADAS

$$a) f(x) = \arctan \frac{1+x}{x}$$

$$\begin{aligned} f'(x) &= \frac{1}{1 + \left(\frac{1+x}{x}\right)^2} \cdot \frac{x - (1+x)}{x^2} = \frac{1}{\frac{x^2 + (1+x)^2}{x^2}} \cdot \frac{x - 1 - x}{x^2} = \\ &= \frac{x^2}{x^2 + 1 + 2x + x^2} \cdot \frac{-1}{x^2} = \frac{-1}{2x^2 + 2x + 1} \end{aligned}$$

$$b) f(x) = \ln\left(\cos\left(\frac{1}{x^2}\right)\right)$$

$$f'(x) = \frac{1}{\cos\left(\frac{1}{x^2}\right)} \cdot \left(-\sin\left(\frac{1}{x^2}\right)\right) \cdot \left(\frac{-2}{x^3}\right) = \frac{2 \tan\left(\frac{1}{x^2}\right)}{x^3}$$

$$c) f(x) = \frac{4e^x}{\sqrt{x}}$$

$$f'(x) = \frac{4e^x \sqrt{x} - \frac{1}{2\sqrt{x}} \cdot 4e^x}{(\sqrt{x})^2} = \frac{4e^x \sqrt{x} - \frac{1}{2\sqrt{x}} \cdot 4e^x}{x} = \frac{4e^x \left(\frac{2\sqrt{x}\sqrt{x} - 1}{2\sqrt{x}}\right)}{x} = \frac{2e^x(2x - 1)}{x\sqrt{x}}$$

$$d) f(x) = \ln \sqrt{\frac{x^2}{x^2 + 1}}$$

(se puede hacer derivando la raíz o sacando 1/2 por prop. logaritmos, el resultado es idéntico)

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{\frac{x^2}{x^2 + 1}}} \cdot \frac{1}{2\sqrt{\frac{x^2}{x^2 + 1}}} \cdot \frac{2x(x^2 + 1) - 2x(x^2)}{(x^2 + 1)^2} = \frac{2(x^2 + x - x^2)}{2\frac{x^2}{x^2 + 1}(x^2 + 1)^2} = \frac{x}{x^2(x^2 + 1)} = \\ &= \frac{1}{x(x^2 + 1)} \end{aligned}$$