

Ejercicio 1. LÍMITES $n \rightarrow \infty$ sin L'Hopital: (sin exponenciales)

Polinómicos:

Sol: $-\infty$

1.1. $\lim_{n \rightarrow \infty} (2n - n^3 + 3n^2)$

Racionales:

Sol: $0, -\infty, \frac{-1}{2}, +\infty, 9$

1.2. $\lim_{n \rightarrow \infty} \frac{2n^3 - 3n + 2}{4n^4 - 5}$

1.3. $\lim_{n \rightarrow \infty} \frac{-2n^4 - 3n + 2}{4n^3 - 5}$

1.4. $\lim_{n \rightarrow \infty} \frac{-2n^4 - 3n + 2}{4n^4 - 5}$

1.5. $\lim_{n \rightarrow \infty} \frac{(n^2 + 1)^2 - 3n^2 + 3}{n^3 - 5}$

1.6. $\lim_{n \rightarrow \infty} \frac{(3n^2 + 4n)^2(n^3 - 3)^2(2n - 7)}{(n + 2)^3(n^3 - 3n)^2(2n^2 - 17)}$ (*no hacer todo el cálculo, razonar)

Estrategia similar a racionales:

Sol: $\frac{7\sqrt[3]{25}}{5}, 2, 3$

1.7. $\lim_{n \rightarrow \infty} \frac{7n - 1}{\sqrt[3]{5n^3 + 4n - 2}}$

1.8. $\lim_{n \rightarrow \infty} \frac{\sqrt{4n^4 + n^2 + 1}}{n^2 + 1}$

1.9. $\lim_{n \rightarrow \infty} \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$

Indeterminación $\infty - \infty$

Sol: $-\infty, -1, \frac{-1}{2}, 1$

1.10. $\lim_{n \rightarrow \infty} \frac{3n^2 + 4n - 6}{n + 2}$

1.11. $\lim_{n \rightarrow \infty} \left(\frac{n^2}{n - 1} - \frac{n^2 + 1}{n - 2} \right)$

1.12. $\lim_{n \rightarrow \infty} (\sqrt{n^2 - 2} - \sqrt{n^2 + n})$

1.13. $\lim_{n \rightarrow \infty} (\sqrt{n^2 + 3n} - \sqrt{n^2 + n})$

Indeterminación $\infty \cdot 0$ o $\frac{0}{0}$

Sol: $\frac{1}{2}, \frac{3}{2}, \frac{3}{4}, 0$

1.14. $\lim_{n \rightarrow \infty} (n + 7) \cdot \frac{1}{\sqrt{4n^2 + 3}}$

1.15. $\lim_{n \rightarrow \infty} \frac{\frac{3}{\sqrt{4n^2 + 5}}}{\frac{1}{n - 1}}$

1.16. $\lim_{n \rightarrow \infty} \left(\sqrt{18n^2 + 1} \cdot \frac{1}{\sqrt{32n^2 - 3}} \right)$

1.17. $\lim_{n \rightarrow \infty} \frac{\frac{n^2 + 2}{7}}{\sqrt{n^2 - 1}}$

Ejercicio 2. LÍMITES $n \rightarrow \infty$: (exponenciales)

“Cebos” (no son número e): Sol: $+\infty, 0, 0, 0, +\infty, 1, +\infty, 0$

$$2.1. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n+1} \right)^{\frac{3n^2+2}{5n-3}} \qquad 2.2. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n+1} \right)^{\frac{-3n^2+2}{5n-3}}$$

$$2.3. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n+1} \right)^{\frac{-3n^2+2}{5n^2-3}} \qquad 2.4. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n^3+1} \right)^{\frac{3n^2+2}{5n-3}}$$

$$2.5. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n^3+1} \right)^{\frac{-3n^2+2}{5n-3}} \qquad 2.6. \lim_{x \rightarrow \infty} \left(\frac{2n^2}{3n^2+1} \right)^{\frac{-3n+2}{5n^2-3}}$$

$$2.7. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n^2+1} \right)^{\frac{-3n^2+2}{5n-3}} \qquad 2.8. \lim_{n \rightarrow \infty} \left(\frac{2n^2}{3n^2+1} \right)^{\frac{3n^2+2}{5n-3}}$$

Tipo número e: Sol: $e, e^{-\frac{2}{3}}, e^{-\frac{3}{2}},$

$$2.9. \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+2} \right)^{n-1} \qquad 2.10. \lim_{n \rightarrow \infty} \left(1 - \frac{2}{3n} \right)^n$$

$$2.11. \lim_{n \rightarrow \infty} \left(\frac{2n+1}{2n+4} \right)^{\frac{n^2}{n+1}}$$

Ejercicio 3. LÍMITES $n \rightarrow \infty$: (repaso de todo tipo) sol: a) $1/2$, b) e^3 , c) $\sqrt{e}$, d) -1 , e) $\sqrt{2}$, f) 0 , g) 3

$$3.1. \lim_{n \rightarrow \infty} \left(\sqrt{n^2+1} - \sqrt{n^2-n} \right) \qquad 3.2. \lim_{n \rightarrow \infty} \left(\frac{n+2}{n-1} \right)^{\frac{n^2}{n+1}} \qquad 3.3. \lim_{n \rightarrow \infty} \left(\frac{2n+4}{2n-1} \right)^{\frac{n+1}{5}}$$

$$3.4. \lim_{n \rightarrow \infty} \left(\frac{n^2}{n+1} - \frac{n^2-1}{n} \right) \qquad 3.5. \lim_{n \rightarrow \infty} (2n+5) \sqrt{\frac{n}{2n^3-1}}$$

$$3.6. \lim_{n \rightarrow \infty} \left(\frac{n^3}{3n+1} \right)^{-\frac{2n}{3}} \qquad 3.7. \lim_{n \rightarrow \infty} \frac{\sqrt{9n^2-n+1}}{n+1}$$

Ejercicio 4. INICIACIÓN DERIVADAS.

Del 1 al 20 muy básicas. Se presenta la función como $y = f(x)$ y la solución y'

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|--------------------------------------|---------------------------------------|---|--|
| 1. $y = 3$ | $y' = 0$ | 27. $y = \sqrt{2x-1}$ | $y' = \frac{1}{\sqrt{2x-1}}$ |
| 2. $y = x + 5$ | $y' = 1$ | 28. $y = \sqrt{x^2+1}$ | $y' = \frac{x}{\sqrt{x^2+1}}$ |
| 3. $y = x^7$ | $y' = 7x^6$ | 29. $y = \frac{1-x}{\sqrt{1+x}}$ | $y' = \frac{-1}{(1+x)^2} \sqrt{\frac{1-x}{1+x}}$ |
| 4. $y = x^6 - x^3$ | $y' = 6x^5 - 3x^2$ | 30. $y = \frac{1-x}{\sqrt{1-x^2}}$ | $y' = \frac{1}{(-1+x)\sqrt{1-x^2}}$ |
| 5. $y = 2x^4$ | $y' = 8x^3$ | 31. $y = e^{4x}$ | $y' = 4e^{4x}$ |
| 6. $y = ax + b$ | $y' = a$ | 32. $y = 5^{2x}$ | $y' = 2 \cdot 5^{2x} \ln 5$ |
| 7. $y = 5x - 2$ | $y' = 5$ | 33. $y = e^{3-x^2}$ | $y' = -2xe^{3-x^2}$ |
| 8. $y = a^5$ | $y' = 0$ | 34. $y = \frac{e^x + e^{-x}}{2}$ | $y' = \frac{e^x - e^{-x}}{2}$ |
| 9. $y = ax^2 + bx + c$ | $y' = 2ax + b$ | 35. $y = x^3 \cdot 2^x \cdot e^x$ | $y' = x^2 \cdot 2^x \cdot e^x (3 + x \ln 2 + x)$ |
| 10. $y = x(x-1)$ | $y' = 2x - 1$ | 36. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ | $y' = \frac{4}{(e^x + e^{-x})^2}$ |
| 11. $y = (x+1)(x-1)$ | $y' = 2x$ | 37. $y = a^{x^2+x+1}$ | $y' = (2x+1)a^{x^2+x+1} \ln a$ |
| 12. $y = ax^3 + bx^2 + cx + d$ | $y' = 3ax^2 + 2bx + c$ | 38. $y = \ln(x^2+1)$ | $y' = \frac{2x}{x^2+1}$ |
| 13. $y = x^3 - x^2 + 4x - 5$ | $y' = 3x^2 - 2x + 4$ | 39. $y = \ln(ax^2 + bx + c)$ | $y' = \frac{2ax + b}{ax^2 + bx + c}$ |
| 14. $y = x^4 - 4x^3 + 5x^2$ | $y' = 4x^3 - 12x^2 + 10x$ | 40. $y = \ln 5^{3x}$ | $y' = \frac{5 \ln 43x}{x}$ |
| 15. $y = 2x^3 + 3x^2 - 6x + 5$ | $y' = 6x^2 + 6x - 6$ | 41. $y = x^5 \ln x$ | $y' = x^4(5 \ln x + 1)$ |
| 16. $y = (x+1)(x^2 - x + 3)$ | $y' = 3x^2 + 2$ | 42. $y = x^2 \ln(2-x)$ | $y' = x \left(2 \ln(2-x) - \frac{x}{2-x} \right)$ |
| 17. $y = x(x-1)^2$ | $y' = 3x^2 - 4x + 1$ | 43. $y = \frac{\ln x}{x}$ | $y' = \frac{1 - \ln x}{x^2}$ |
| 18. $y = a(x-1)^2$ | $y' = 2a(x-1)$ | 44. $y = \log_3(1+x^2)$ | $y' = \frac{2x}{1+x^2} \log_3 e$ |
| 19. $y = a(a-1)^2$ | $y' = 0$ | 45. $y = \ln(x-5)$ | $y' = \frac{1}{x-5}$ |
| 20. $y = -x^{-2}$ | $y' = \frac{-2}{x^3}$ | 46. $y = \log_a(3x^2+5)$ | $y' = \frac{6x}{3x^2+5} \log_a e$ |
| 21. $y = \frac{1}{x+1}$ | $y' = \frac{-1}{(x+1)^2}$ | 47. $y = x \cdot \ln x - x$ | $y' = \ln x$ |
| 22. $y = \frac{x^2-3}{x^3+x}$ | $y' = \frac{-x^4+10x^2+3}{(x^3+x)^2}$ | 48. $y = \ln \sqrt{1+x^2}$ | $y' = \frac{x}{1+x^2}$ |
| 23. $y = \frac{x+1}{x}$ | $y' = \frac{-1}{x^2}$ | | |
| 24. $y = \frac{x(x+1)(x-1)}{3x^2-3}$ | $y' = \frac{3x^4-6x^2+3}{(3x^2-3)^2}$ | | |
| 25. $y = \frac{x(x+2)^2}{x^2+4x+4}$ | $y' = 1$ | | |
| 26. $y = \sqrt{3x-2}$ | $y' = \frac{3}{2\sqrt{3x-2}}$ | | |

Ejercicio 5. DERIVADAS.

49. $y = \ln \sqrt{\frac{1-x}{1+x}}$ $y' = \frac{-1}{1-x^2}$
50. $y = \ln \frac{x^2+1}{x^2-1}$ $y' = \frac{-4x}{x^4-1}$
51. $y = \sin 2x$ $y' = 2 \cos x$
52. $y = \cos(2x+1)$ $y' = -2 \sin(2x+1)$
53. $y = \tan(x^2+x+1)$ $y' = (2x+1)\sec^2(x^2+x+1)$
54. $y = \tan \sqrt{x}$ $y' = \frac{1}{2\sqrt{x}} \sec^2 \sqrt{x}$
55. $y = \sqrt{\sin 3x}$ $y' = \frac{3 \cos 3x}{2\sqrt{\sin 3x}}$
56. $y = \sin^{2/3} x$ $y' = \frac{2}{3} \sin^{-1/3} x \cdot \cos x = \frac{2 \cos x}{3\sqrt[3]{\sin x}}$
57. $y = x \cos x$ $y' = \cos x - x \sin x$
58. $y = \ln \sin x$ $y' = \cot x$
59. $y = \sin x \cdot \cos 2x$ $y' = \cos x \cos 2x - 2 \sin x \sin 2x$
60. $y = e^x \tan x$ $y' = e^x(\tan x + \sec^2 x) = e^x(\tan x + \tan x + 1)$
61. $y = \arcsin 2x$ $y' = \frac{2}{\sqrt{1-4x^2}}$
62. $y = \arcsin \sqrt{x}$ $y' = \frac{1}{2\sqrt{1-x}\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}}$
63. $y = \arccos(x^2+1)$ $y' = \frac{-2x}{\sqrt{1-(x^2+1)^2}}$
64. $y = \frac{1}{3} \tan^3 x - \tan x + x$ $y' = \tan^2 x \sec^2 x - \sec^2 x + 1$
65. $y = \arctan \frac{1+x}{1-x}$ $y' = \frac{1}{1+x^2}$
66. $y = \ln \left(\frac{1-\sin x}{1+\sin x} \right)^2$ $y' = \frac{-4(1-\sin x)\cos x}{(1+\sin x)^3}$
67. $y = \ln \tan x$ $y' = \frac{1}{\sin x \cos x}$
68. $y = \ln \sqrt{\frac{1+\cos x}{1-\cos x}}$ $y' = -1/\sin x$
69. $y = \ln \sqrt{\sin x}$ $y' = \cot 2x$

$$\begin{aligned}
 70. \quad y &= \operatorname{arcsec} x^2 & y' &= \frac{2}{x\sqrt{x^4-1}} \\
 71. \quad y &= \arctan(2x+1) & y' &= \frac{-2}{(2x+1)^2+1} \\
 72. \quad y &= \sin(\sin 2x) & y' &= 2 \cos 2x \cdot \cos(\sin 2x) \\
 73. \quad y &= \sin(\ln(3x+5)) & y' &= \frac{3}{3x+5} \cos(\ln(3x+5)) \\
 74. \quad y &= \arcsin(\cos x - x) & y' &= \frac{-\sin x - 1}{\sqrt{1-(\cos x - x)^2}} \\
 75. \quad y &= \ln(x + \ln x) & y' &= \frac{x+1}{x^2+x \ln x} \\
 76. \quad y &= \arcsin\left(\frac{1}{1+x^2}\right) & y' &= \frac{-2}{(1+x^2)^2 \sqrt{x^2+2}} \\
 77. \quad y &= \log\left(\frac{\log x}{x}\right) & y' &= \frac{\log e - \log x}{x \log x} \log e \\
 78. \quad y &= \ln^{1/2} \sin 2x & y' &= \frac{\operatorname{ctan} 2x}{\sqrt{\ln \sin 2x}} \\
 79. \quad y &= x^2 \cos 3x & y' &= 2x \cos 3x - 3x^2 \sin 3x
 \end{aligned}$$

Ejercicio 6. DERIVADAS POTENCIAL - EXPONENCIAL.

$$\begin{aligned}
 80. \quad y &= x^{x+1} & y' &= \left[\ln x + \frac{x+1}{x} \right] x^{x+1} \\
 81. \quad y &= \left(1 + \frac{1}{x}\right)^x & y' &= \left[\ln(x+1) - \frac{1}{x-1} \right] \left(1 + \frac{1}{x}\right)^x \\
 82. \quad y &= (x^5)^{x^2+1} & y' &= \left[\frac{10x \ln x + \frac{5x^2+5}{x}}{x^5} \right] (x^5)^{x^2+1} \\
 83. \quad y &= x^{e^x} & y' &= \left[e^x \ln x + \frac{e^x}{x} \right] x^{e^x} \\
 84. \quad y &= (3x+1)^{2x+3} & y' &= \left[2 \ln(3x+1) + \frac{6x+9}{3x+1} \right] (3x+1)^{2x+3} \\
 85. \quad y &= (\sin x)^x & y' &= [\ln(\sin x) + x \operatorname{ctan} x] (\sin x)^x \\
 86. \quad y &= x^{\sin(2x-9)} & y' &= \left[2 \cos(2x-9) \ln x + \frac{\sin(2x-9)}{x} \right] x^{\sin(2x-9)} \\
 87. \quad y &= (\sin x)^{\cos x} & y' &= [-\sin x \ln(\sin x) + \cos x \operatorname{ctan} x] (\sin x)^{\cos x} \\
 88. \quad y &= (\ln x)^{\ln x} & y' &= \frac{1}{x} [\ln(\ln x) + 1] (\ln x)^{\ln x} \\
 89. \quad y &= (x^2-1)^{\sin x} & y' &= \left[\cos x \ln(x^2-1) + \frac{2x \sin x}{x^2-1} \right] (x^2-1)^{\sin x}
 \end{aligned}$$