

INTEGRALES INMEDIATAS

$$1. \int x \, dx \quad 2. \int x^3 \, dx \quad 3. \int \frac{2}{x^2} \, dx \quad 4. \int \frac{1}{x} \, dx \quad 5. \int \frac{2}{3x} \, dx$$

$$6. \int (3x - 1) \, dx \quad 7. \int (2x^3 - x^2 + \frac{x}{3} - 5) \, dx \quad 8. \int \sqrt[3]{x} \, dx \quad 9. \int \frac{\sqrt[3]{x^2} + \sqrt{x^3}}{\sqrt[4]{x^3}} \, dx$$

$$10. \int \frac{3x^3 + 5x^2 - x + \sqrt{2}}{x} \, dx \quad 11. \int \frac{2x^4 - 6x^3 + 5x}{x - 1} \, dx$$

$$12. \int \frac{5}{(3x - 2)^2} \, dx \quad 13. \int (\sqrt{2}x + 1)^4 \, dx \quad 14. \int x\sqrt{1 + 2x} \, dx$$

$$15. \int \sin 2x \, dx \quad 16. \int \cos(\sqrt{3}x + 1) \, dx \quad 17. \int \tan x \, dx$$

$$18. \int \frac{1}{\sqrt{1 - x^2}} \, dx \quad 19. \int \frac{x}{\sqrt{x^2 - 1}} \, dx \quad 20. \int \frac{1}{1 + x^2} \, dx \quad 20b. \int \frac{x}{1 + x^2} \, dx$$

$$21. \int (e^x + 3x^2) \, dx \quad 22. \int \frac{2^x + 1}{2^x} \, dx \quad 23. \int (2 \sin x + 5^x) \, dx$$

$$24. \int \frac{1}{\sqrt{25 - 4x^2}} \, dx \quad 25. \int \frac{1}{\sqrt{7 - 15x^2}} \, dx \quad 26. \int \sin^5 x \cos x \, dx$$

$$27. \int \cos^3 x \sin x \, dx \quad 28. \int e^{\sqrt{x^2 - 4x}} \cdot \frac{x - 2}{\sqrt{x^2 - 4x}} \, dx \quad 29. \int 2x\sqrt{x - 3} \, dx$$

$$30. \int \frac{dx}{\sqrt{x} + \sqrt[4]{x}} \quad 31^*. \int \sqrt{1 - x^2} \, dx \quad (\text{pistas } x = \sin u) (\cos^2 u = \frac{1 + \cos 2u}{2})$$

*no la pido, resuelta en apuntes

EJERCICIOS DE EXAMENES PASADOS:

$$E1. \int \frac{8x}{\sqrt{1 - 9x^4}} \, dx \quad E2. \int x\sqrt{x - 1} \, dx \quad E3. \int \frac{\sqrt{x} - 3\sqrt[4]{x^3} + 2x^2}{\sqrt[3]{x}} \, dx \quad E4. \int \frac{x}{\sqrt{x^2 + 9}} \, dx$$

$$E5. \int \frac{\sqrt{x} + \sqrt[3]{x^2}}{\sqrt{x^5}} \, dx \quad E6. \int \frac{x}{x^2 - 1} \, dx \quad E7. \int \frac{x^2 + x + 6}{x - 2} \, dx \quad E8. \int \frac{x}{x^2 + 1} \, dx$$

INTEGRALES POR PARTES

$$\begin{array}{llll}
 32. \int x^2 e^x dx & 33. \int x \sin x dx & 34. \int x^2 \ln x dx & 35^*. \int \frac{\ln x}{x} dx \\
 36. \int \arcsin x dx & 37. \int x \arctan x dx & 38^*. \int e^x \sin x dx & 39^*. \int \cos^2 x dx \\
 40. \int \ln(2x-1) dx & 41. \int \frac{x}{e^x} dx & 42. \int (x+1)^2 e^x dx &
 \end{array}$$

*inmediata

* (recursividade, ou cambio de variable ou identidades trigonométricas)

EJERCICIOS DE EXAMENES PASADOS:

$$E9. \int x \sin x dx \quad E10. \int \frac{\ln x}{x^3} dx \quad E11. \int x^2 \ln x dx \quad E12. \int x^2 e^{-x} dx$$

INTEGRALES RACIONALES (no inmediatas)

$$\begin{array}{ll}
 43. \int \frac{x^3 + 4x^2 - 10x + 7}{x^3 - 7x - 6} dx & 44. \int \frac{2x}{x^3 - 3x^2 + 3x - 1} dx \\
 45a. \int \frac{5x - 3}{x^3 - x} dx & 45b. \int \frac{x + 2}{2x^2 + x - 1} dx \quad 45c. \int \frac{x - 1}{4x^2 - 9} dx \quad 45d. \int \frac{x^2 - 8}{x^3 + 2x^2} dx \\
 46. \int \frac{dx}{3x^2 + 3} & 47. \int \frac{dx}{9x^2 + 3} \quad 48. \int \frac{dx}{7x^2 + 11} \\
 49. \int \frac{dx}{x^2 - 4x + 5} & 50. \int \frac{dx}{x^2 - 4x + 10}
 \end{array}$$

EJERCICIOS DE EXAMENES PASADOS:

$$E13. \int \frac{dx}{x^2 + 2x + 2} dx \quad E14. \int \frac{3x - 2}{x^3 - x^2} dx \quad E15. \int \frac{x^2 + x + 6}{x - 2} dx$$

REPASO INMEDIATAS / CAMBIO VARIABLE

$$\begin{array}{llll}
 51. \int \frac{x}{\sqrt[3]{x}} dx & 52. \int \frac{7}{\cos^2 x} dx & 53. \int \frac{x + \sqrt{x}}{x^2} dx & 54. \int \frac{dx}{2 + 4x^2} \\
 55. \int \frac{e^x}{1 + e^{2x}} dx & 56. \int \frac{x^3}{x^2 - 1} dx & 57. \int \frac{1}{\sqrt{1 - 100x^2}} dx & 58. \int \frac{dx}{x\sqrt{1 - (\ln x)^2}} \\
 59. \int \sin x \cdot \cos x dx & 60. \int \sqrt{x^2 - 2x} (x - 1) dx & 61. \int \frac{\tan x}{\cos^2 x} dx & \\
 62^*. \int \frac{\sin^3 x}{\sqrt{\cos x}} dx & (\text{pistas } \sin^3 x = \sin^2 x \sin x & \sin^2 x = 1 - \cos^2 x & u = \cos x)
 \end{array}$$

INTEGRALES ABAU 12-25

$$2025 \text{ ORD} \int e^x \cos 3x \, dx \quad 2024 \text{ ORD} \int x^3 e^{x^2} \, dx. \quad 2022 \text{ XULL a)} \int 2x \sqrt{x^2 + 1} \, dx$$

$$b) \int (\sin x \sin(\cos x)) \, dx \quad c) \int x^2 \sin x \, dx \quad d) \int \frac{1}{(x-1)(x-2)} \, dx$$

$$2020 \text{ XULL} \int x \sqrt{x^2 - 1} \, dx \quad 2020 \text{ XUN} \int x(\ln x - 1) \, dx \quad 2019 \text{ XUN} \int \ln x \, dx$$

$$2019 \text{ XUN} \int x^2 e^{-x} \, dx \quad 2018 \text{ XUN} \int x \sqrt{x+1} \, dx \quad 2018 \text{ SEPT} \int \sqrt{x} \ln x \, dx$$

$$2017 \text{ XUN} \int x \ln(1+x) \, dx \quad 2017 \text{ SEPT} \int \frac{x}{1-x} \, dx \quad 2016 \text{ XUN} \int \frac{1 - \ln(x)}{x^2} \, dx$$

$$2016 \text{ SEPT} \int x \ln(1+x) \, dx \quad 2015 \text{ XUN} \int (2-x)e^{3x} \, dx \quad 2015 \text{ SEPTB} \int x \sin x \, dx$$

$$*2014 \text{ XUN A} \int \frac{e^x}{e^{2x} + 3e^x + 2} \, dx \quad 2014 \text{ SEPT A} \int x \sin(2x + \pi) \, dx$$

$$*2014 \text{ SEPT B} \int \frac{2}{3 + 3e^x} \, dx \quad 2013 \text{ XUN B} \int \frac{x^3 + 3}{x^2 - x} \, dx \quad 2013 \text{ SEPT B} \int \frac{x^3 + 2}{x^2 - 1} \, dx$$

$$2012 \text{ XUN B} \int \frac{5x^3 - 3x + 1}{x^3 - x} \, dx \quad 2012 \text{ SEPT A} \int \frac{(x-1)^2}{x^2 + 1} \, dx$$

$$2012 \text{ SEPT A} \int x e^{2x} \, dx$$

SOLUCIONES INMEDIATAS (sin depurar)

$$1. \frac{x^2}{2} + C. \quad 2. \frac{x^4}{4} + C \quad 3. \frac{-2}{x} + C \quad 4. \ln|x| + C \quad 5. \frac{2}{3} \ln|x| + C$$

$$6. \frac{3x^2}{2} - x + C \quad 7. \frac{x^4}{2} - \frac{x^3}{3} + \frac{x^2}{6} - 5x + C \quad 8. \frac{3x^{\frac{4}{3}}}{4} + C = \frac{3x\sqrt[3]{x}}{4} + C$$

$$9. \frac{12\sqrt[12]{x^{11}}}{11} + \frac{4x\sqrt[4]{x^3}}{7} + C \quad 10. x^3 + \frac{5x^2}{2} - x + \sqrt{2} \ln|x| + C$$

$$11. \frac{x^4}{2} - \frac{4x^3}{3} - 2x^2 + x + \ln|x-1| + C \quad 12. \frac{-5}{3(3x-2)} + C$$

$$13. \frac{(\sqrt{2}x+1)^5}{5\sqrt{2}} + C \quad 14. \frac{\sqrt{(1+2x)^5}}{10} - \frac{\sqrt{(1+2x)^3}}{6} + C$$

$$15. -\frac{1}{2} \cos 2x + C \quad 16. \frac{\sin(\sqrt{3}x+1)}{\sqrt{3}} + C \quad 17. -\ln|\cos x| + C$$

$$18. \arcsin x + C \quad 19. \sqrt{x^2-1} + C \quad 20. \arctan x + C$$

$$20b. \frac{1}{2} \ln|x^2+1| + C \quad 21. e^x + x^3 + C \quad 22. x - \frac{2^{-x}}{\ln 2} + C$$

$$23. -2 \cos x + \frac{5^x}{\ln 5} + C \quad 24. \frac{1}{2} \arcsin\left(\frac{2x}{5}\right) + C \quad 25. \frac{1}{\sqrt{15}} \arcsin\left(\sqrt{\frac{15}{7}}x\right) + C$$

$$26. \frac{\sin^6 x}{6} + C \quad 27. -\frac{\cos^4 x}{4} + C \quad 28. e^{\sqrt{x^2-4x}} + C$$

$$29. \frac{4\sqrt{(x-3)^5}}{5} + 4\sqrt{(x-3)^3} + C \quad 30. 2\sqrt{x} - 4\sqrt[4]{x} + 4\ln|\sqrt[4]{x}+1| + C$$

$$31. \frac{1}{2} \arcsin x + \frac{1}{4} \sin(2 \arcsin x) + C$$

EJERCICIOS DE EXAMENES PASADOS:

$$E1. \frac{4}{3} \arcsin 3x^2 + C \quad E2. \frac{2\sqrt{(x-1)^5}}{5} + \frac{2\sqrt{(x-1)^3}}{3} + C \quad E3. \frac{6x^{\frac{7}{6}}}{7} - \frac{36x^{\frac{17}{12}}}{17} - \frac{3x^{\frac{8}{3}}}{4} + C$$

$$E4. \sqrt{x^2+9} + C \quad E5. -x^{-1} - \frac{6}{5}x^{-\frac{5}{6}} + C \quad E6. \frac{1}{2} \ln|x^2-1| + C \quad E7. \frac{x^2}{2} + 3x + 12 \ln|x-2| + C$$

$$E8. \frac{1}{2} \ln|x^2+1| + C$$

SOLUCIÓN POR PARTES (sin depurar)

$$32. e^x(x^2 - 2x + 2) + C \quad 33. \sin x - x \cos x + C \quad 34. \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C \quad 35^*. \frac{\ln^2 x}{2} + C$$

$$36. x \arcsin x + \sqrt{1-x^2} + C \quad 37. \frac{1}{2} ((x^2 + 1) \arctan x - x) + C$$

$$38^*. \frac{e^x(\sin x - \cos x)}{2} + C \quad 39^*. \frac{\sin x \cos x + x}{2} + C \quad 40. \left(x - \frac{1}{2}\right) \ln(2x - 1) - x + C$$

$$41. -e^{-x}(1+x) + C \quad 42. e^x(x^2 + 1) + C.$$

EJERCICIOS DE EXAMENES PASADOS:

$$E9. \sin x - x \cos x + C \quad E10. -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C \quad E11. \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C \quad E12. -e^{-x}(x^2 + 2x + 2) + C$$

SOLUCIONES INTEGRALES RACIONALES (no inmediatas) (sin depurar)

$$43. x - 5 \ln|x+1| + 7 \ln|x+2| + 2 \ln|x-3| + C. \quad 44. -\frac{2}{x-1} - \frac{1}{(x-1)^2} + C$$

$$45a. 3 \ln|x| - 4 \ln|x+1| + \ln|x-1| + C \quad 45b. \frac{5}{6} \ln|2x-1| - \frac{1}{3} \ln|x+1| + C$$

$$45c. \frac{5}{24} \ln|2x+3| + \frac{1}{24} \ln|2x-3| + C \quad 45d. 2 \ln|x| + \frac{4}{x} - \ln|x+2| + C$$

$$46. \frac{1}{3} \arctan x + C \quad 47. \frac{1}{3\sqrt{3}} \arctan(\sqrt{3}x) + C \quad 48. \frac{\sqrt{11}}{11\sqrt{7}} \arctan\left(\sqrt{\frac{7}{11}}x\right) + C$$

$$49. \arctan(x-2) + C \quad 50. \frac{\sqrt{6}}{6} \arctan\left(\frac{x-2}{\sqrt{6}}\right) + C$$

EJERCICIOS DE EXAMENES PASADOS:

$$E13. \arctan(x+1) + C \quad E14. -\ln|x| - \frac{2}{x} + \ln|x-1| + C \quad E15. \frac{x^2}{2} + 3x + 12 \ln|x-2| + C$$

REPASO INMEDIATAS / CAMBIO VARIABLE (soluciones sin depurar)

$$51. \frac{3\sqrt[3]{x^5}}{5} + C \quad 52. 7 \tan x + C \quad 53. \ln|x| - \frac{2}{\sqrt{x}} + C \quad 54. \frac{1}{2\sqrt{2}} \arctan(\sqrt{2}x) + C$$

$$55. \arctan e^x + C \quad 56. \frac{x^2}{2} + \frac{\ln|x^2+1|}{2} + C \quad 57. \frac{1}{10} \arcsin(10x) + C \quad 58. \arcsin(\ln x) + C$$

$$59. \frac{\sin^2 x}{2} + C \quad 60. \frac{\sqrt{(x^2-2x)^3}}{3} + C \quad 61. \frac{\tan^2 x}{2} + C$$

$$62. -\sqrt{\cos x} + \frac{2\sqrt{\cos^5 x}}{5} + C \quad (\text{pistas } \sin^3 x = \sin^2 x \sin x \quad \sin^2 x = 1 - \cos^2 x \quad u = \cos x)$$

INTEGRALES ABAU 12-25 SOLUCIONES (sin depurar)

$$2025 \text{ ORD } \frac{e^x(\cos 3x + 3 \sin 3x)}{10} + C$$

$$2024 \text{ XUN } \frac{e^{x^2}}{2}(x^2 - 1) + C$$

2022 XULL

$$a) \frac{2\sqrt{(x^2 + 1)^3}}{3} + C$$

$$b) \cos(\cos x) + C$$

$$c) -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$d) \ln|x - 2| - \ln|x - 1| + C = \ln \frac{|x - 2|}{|x - 1|} + C$$

$$2020 \text{ XULL } \frac{\sqrt{(x^2 - 1)^3}}{3} + C$$

$$2020 \text{ XUN } \frac{x^2}{2}(\ln x - \frac{3}{2}) + C$$

$$2019 \text{ XUN } x(\ln x - 1) + C$$

$$2019 \text{ XUN } -e^{-x}(x^2 + 2x + 2) + C$$

$$2018 \text{ XUN } \frac{2\sqrt{(x + 1)^5}}{5} + \frac{2\sqrt{(x + 1)^3}}{3} + C$$

$$2018 \text{ SEPT } \frac{2x\sqrt{x} \ln x}{3} - \frac{4x\sqrt{x}}{9} + C$$

$$2017 \text{ XUN } \frac{x^2 \ln|1 + x|}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\ln|x + 1|}{2} + C$$

$$2017 \text{ SEPT } -x - \ln|1 - x| + C$$

$$2016 \text{ XUN } \frac{\ln x}{x} + C$$

$$2016 \text{ SEPT } = 2017 \text{ XUN}$$

$$2015 \text{ XUN } e^{3x} \left(\frac{7}{9} - \frac{x}{3} \right) + C$$

$$2015 \text{ SEPT } B -x \sin x + \sin x + C$$

$$2014 \text{ XUN } A \ln|e^x + 1| - \ln|e^x + 2| + C$$

$$2014 \text{ SEPT } A - \frac{x \cos(2x + \pi)}{2} + \frac{\sin(2x + \pi)}{4} + C$$

$$2014 \text{ SEPT } B \frac{2}{3}(\ln|e^x| - \ln|1 + e^x|) + C$$

$$2013 \text{ XUN } B \frac{x^2}{2} + x - 3 \ln|x| + 4 \ln|x - 1| + C$$

$$2013 \text{ SEPT } B \frac{x^2}{2} - \frac{1}{2} \ln|x + 1| + \frac{3}{2} \ln|x - 1| + C$$

$$2012 \text{ XUN } B 5x - \ln|x| + \frac{3}{2} \ln|x - 1| - \frac{1}{2} \ln|x + 1| + C$$

$$2012 \text{ SEPT } A x - \ln|x^2 + 1| + C$$

$$2012 \text{ SEPT } A e^{2x} \left(\frac{x}{2} - \frac{1}{4} \right) + C$$