RELACIÓN nº 2:

1.- $$\lim_{n\to\infty} \sqrt[2n]{\left(\frac{n^2+n}{n^2+3}\right)^{4n^2}} = \lim_{n\to\infty} \left(\frac{n^2+n}{n^2+3}\right)^{2n} = [1^\infty] =$$

$$= \lim_{n\to\infty} \left(1+\frac{n^2+n-n^2-3}{n^2+3}\right)^{2n} = \lim_{n\to\infty} \left(1+\frac{1}{\frac{n^2+3}{n-3}}\right)^{\frac{n^2+3}{n-3}\cdot\frac{n-3}{n^2+3}\cdot 2n} =$$

$$= e^{\lim_{n\to\infty} \frac{2n^2-6n}{n^2+3}} = e^2 //$$

2.- $$\lim_{n\to\infty} \frac{\frac{n}{n-1}\cdot\frac{n-1}{n+1}}{1+\frac{2n-3}{n^2+2}} = \frac{1\cdot 1}{1+0} = 1 //$$

3.- $$\lim_{n\to\infty} \sqrt[5]{\frac{27n^3+5n^2}{\frac{n^3}{9}+8}} = \sqrt[5]{\frac{27}{\frac{1}{9}}} = \sqrt[5]{27\cdot 9} = \sqrt[5]{3^3\cdot 3^2} = \sqrt[5]{3^5} = 3 //$$

4.- $$\lim_{n\to\infty} \frac{2+4+6\ldots\ldots+2n}{n^3-1} = \lim_{n\to\infty} \frac{\frac{2n^2+2n}{2}}{n^3-1} = 0 //$$

$$S_n = \frac{(2+2n)\cdot n}{2}$$ Suma de n-términos P.A.

5.- $$\lim_{n\to\infty} \frac{1+2+3\ldots n}{n2} = \lim_{n\to\infty} \frac{n^2+n}{2n^2} = \frac{1}{2} //$$

$$S_n = \frac{(1+n)\cdot n}{2}$$

6.- $$\lim_{x\to\infty} \left(4+\frac{3}{n}\right)^2 = 4^2 = 16$$

7.- $$\lim_{n\to\infty} \left(8-\frac{3}{2^n}\right)^{1/3} = \left(8-\frac{3}{\infty}\right)^{1/3} = \sqrt[3]{8} = 2 //$$

8.- $\lim\limits_{n\to\infty}\left(\dfrac{n^4-3n^2+1}{4n^3+n-3}-\dfrac{n^2-1}{3n^4-2n}\right)=\infty-0=\infty$ //

9.- $\lim\limits_{n\to\infty}\left(\dfrac{2n+5}{n+4}\right)^{-n/2}=2^{-\infty}=\dfrac{1}{2^{\infty}}=\dfrac{1}{\infty}=0$ //

10.- $\lim\limits_{n\to\infty}\sqrt[2n]{\dfrac{1+n}{3+n}}=\lim\limits_{n\to\infty}\left(\dfrac{1+n}{3+n}\right)^{\frac{1}{2n}}=1^0=1$ //

11.- $\lim\limits_{n\to\infty}\left(\dfrac{3-2n}{5-3n}\right)^{\frac{n}{n+1}}=\left(\dfrac{2}{3}\right)^1=\dfrac{2}{3}$ //

12.- $\lim\limits_{n\to\infty}4^{\frac{3n-1}{6n+2}}=4^{1/2}$ //

13.- $\lim\limits_{n\to\infty}\left(\dfrac{3n-1}{5n+2}\right)^{-n-3}=\left(\dfrac{3}{5}\right)^{-\infty}=\left(\dfrac{5}{3}\right)^{\infty}=\infty$ //

14.- $\lim\limits_{n\to\infty}\dfrac{1-\dfrac{n}{n+1}}{3-\dfrac{3n}{n+1}}=\dfrac{1-1}{3-3}=\dfrac{0}{0}$ INDT.

$$\lim_{n\to\infty}\frac{\dfrac{n+1-n}{n+1}}{\dfrac{3n+3-3n}{n+1}}=\lim_{n\to\infty}\frac{1}{3}=\frac{1}{3} \text{ //}$$

15.- $\lim\limits_{n\to\infty}\left(\dfrac{n^4-n}{2n-1}-\dfrac{n^2}{n+1}\right)=(\infty-\infty)=$

$$\lim_{n\to\infty}\frac{(n^4-n).(n+1)-n^2(2n-1)}{(2n-1).(n+1)}=$$

$$=\lim_{n\to\infty}\frac{n^5+n^4-n^2-n-2n^3+n^2}{(2n-1).(n+1)}=$$

$$=\lim_{n\to\infty}\frac{n^5+n^4+\cdots\cdots}{(2n-1).(n+1)}=\infty \text{ //}$$

16.- $\lim\limits_{n\to\infty} \left(n - \sqrt{n^2-n+3}\right) = (\infty - \infty) =$

$$\lim_{n\to\infty} \frac{\left(n-\sqrt{n^2-n+3}\right)\cdot\left(n+\sqrt{n^2-n+3}\right)}{n+\sqrt{n^2-n+3}} =$$

$$= \lim_{n\to\infty} \frac{n-3}{n+\sqrt{n^2-n+3}} = \frac{1}{1+1} = \frac{1}{2} \;//$$

17.- $\lim\limits_{n\to\infty} \left(\sqrt{n^2+5n} - 2n\right) = \lim\limits_{n\to\infty} \dfrac{\left(\sqrt{n^2+5n}-2n\right)\cdot\left(\sqrt{n^2+5n}+2n\right)}{\sqrt{n^2+5n}+2n} =$

$$= \lim_{n\to\infty} \frac{n^2+5n-4n^2}{\sqrt{n^2+5n}+2n} = \lim_{n\to\infty} \frac{-3n^2+5n}{\sqrt{n^2+5n}+2n} = -\infty \;//$$

18.- $\lim\limits_{n\to\infty} \left(\sqrt{n} - \sqrt{n^2+n}\right) = \lim\limits_{n\to\infty} \dfrac{\left(\sqrt{n}-\sqrt{n^2+n}\right)\cdot\left(\sqrt{n}+\sqrt{n^2+n}\right)}{\sqrt{n}+\sqrt{n^2+n}} =$

$$= \lim_{n\to\infty} \frac{n-n^2-n}{\sqrt{n}+\sqrt{n^2+n}} = -\infty \;//$$

19.- $\lim\limits_{n\to\infty} \left(\sqrt{n^3} - \sqrt{n^2+n}\right) = \lim\limits_{n\to\infty} \dfrac{\left(\sqrt{n^3}-\sqrt{n^2+n}\right)\cdot\left(\sqrt{n^3}+\sqrt{n^2+n}\right)}{\sqrt{n^3}+\sqrt{n^2+n}} =$

$$= \lim_{n\to\infty} \frac{n^3-n^2-n}{\sqrt{n^3}+\sqrt{n^2+n}} = \infty \;//$$

20.- $\lim\limits_{x\to-1} \left(\dfrac{x^2-3x}{x-3}\right)^2 = \left[\dfrac{(-1)^2-3(-1)}{-1-3}\right]^2 = \left(\dfrac{1+3}{-4}\right)^2 = (-1)^2 = 1 \;//$

21.- $\lim\limits_{x\to3} \dfrac{x^2-3x}{x-3} = \left(\dfrac{0}{0}\right) = \lim\limits_{x\to3} \dfrac{x(x-3)}{x-3} = 3 \;//$

22.- $\lim\limits_{x \to 0} \dfrac{x^2-3x+5}{2x^2+x} = \dfrac{5}{0}$ Separamos los límites laterales

$$\lim_{x \to 0^+} \frac{x^2-3x+5}{2x^2+x} = +\infty \,//$$

$$\lim_{x \to 0^-} \frac{x^2-3x+5}{2x^2+x} = -\infty \,//$$

23.- $$\lim_{x \to 2} \frac{x^2-4}{(x-2)(x^2+4)} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}.(x^2+4)} = \frac{4}{8} = \frac{1}{2} \,//$$

24.- $$\lim_{x \to 5} \frac{2x-10}{\sqrt{x+4}-3} = \lim_{x \to 5} \frac{2(x-5).(\sqrt{x+4}+3)}{(\sqrt{x+4}-3).(\sqrt{x+4}+3)} =$$

$$= \lim_{x \to 5} \frac{2\cancel{(x-5)}(\sqrt{x+4}+3)}{\cancel{(x-5)}} = 12 \,//$$

25.- $$\lim_{x \to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)(\sqrt{2x+10}+\sqrt{5x+1})}{(\sqrt{2x+10}-\sqrt{5x+1}).(\sqrt{2x+10}+\sqrt{5x+1}).(\sqrt{x+1}+2)} =$$

$$\lim_{x \to 3} \frac{(x-3).(\sqrt{2x+10}+\sqrt{5x+1})}{(-3x+9).(\sqrt{x+1}+2)} = \lim_{x \to 3} \frac{\cancel{(x-3)}.(\sqrt{2x+10}+\sqrt{5x+1})}{-3.\cancel{(x-3)}.(\sqrt{x+1}+2)}$$

$$= \frac{8}{-3.4} = -\frac{2}{3} \,//$$

26.- $$\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}.(\sqrt{x+4}+2)} = \frac{1}{4} \,//$$

27.- $\lim\limits_{x \to 1} \left( \dfrac{3x+4}{2x+5} \right)^{\frac{3}{x-1}} = \left[ 1^{\infty} \right] =$

$$= \lim_{x \to 1} \left( 1 + \frac{3x+4-2x-5}{2x+5} \right)^{\frac{3}{x-1}} = \lim_{x \to 1} \left( 1 + \frac{x-1}{2x+5} \right)^{\frac{3}{x-1}} =$$

$$= \lim_{x \to 1} \left( 1 + \frac{1}{\frac{2x+5}{x-1}} \right)^{\frac{2x+5}{x-1} \cdot \frac{x-1}{2x+5} \cdot \frac{3}{x-1}} =$$

$$= e^{\lim\limits_{x \to 1} \frac{3}{2x+5}} = e^{3/7} \;//$$

28.- $\lim\limits_{x \to 3} \left( \dfrac{3x^2-1}{-x+3} \right)^{\frac{3x+2}{x^2-1}} = \left( \dfrac{27-1}{-3+3} \right)^{\frac{11}{8}} = \left( \dfrac{26}{0} \right)^{\frac{11}{8}}$

$\to \lim\limits_{x \to 3^+} f(x) = (-\infty)^{11/8} = \sqrt[8]{-\infty}$ NO EXISTE

$\to \lim\limits_{x \to 3^-} f(x) = (+\infty)^{11/8} = +\infty$

29.- $\lim\limits_{x \to \infty} \dfrac{x^3-3x+9}{4x^3-6x} = \dfrac{1}{4} \;//$

30.- $\lim\limits_{x \to \infty} \sqrt{\dfrac{3x^2}{5x^2-4x+1}} = \sqrt{3/5} \;//$

31.- $\lim\limits_{x \to \infty} \dfrac{x^3-2x+1}{5x} \cdot \dfrac{x^2+1}{4x^4-3} = \lim\limits_{x \to \infty} \dfrac{x^5+\dots\dots}{20x^5-15x} = \dfrac{1}{20} \;//$

32.- $\lim\limits_{x \to \infty} \dfrac{x^2+5x-3-x^2}{\sqrt{x^2+5x-3}+x} = \dfrac{5}{2} \;//$

33.- $\lim_{x\to\infty}\left(\frac{4-3x}{5-3x}\right)^{x-3} = [1^{\infty}] =$

$$= \lim_{x\to\infty}\left(1+\frac{4-\cancel{3x}-5+\cancel{3x}}{5-3x}\right)^{x-3} = \lim_{x\to\infty}\left(1+\frac{1}{-5+3x}\right)^{(-5+\cancel{3x})\cdot\frac{(x-3)}{(-5+3x)}}$$

$$= e^{\lim_{x\to\infty}\frac{x-3}{-5+3x}} = e^{1/3}$$ //

34.- $\lim_{x\to\infty}\left(\frac{2x^2-6x}{2x^2-x-5}\right)^{\frac{x^2}{2}} = [1^{\infty}]$

$$\lim_{x\to\infty}\left(1+\frac{\cancel{2x^2}-6x-\cancel{2x^2}+x+5}{2x^2-x-5}\right)^{\frac{x^2}{2}} =$$

$$= \lim_{x\to\infty}\left(1+\frac{1}{\frac{2x^2-x-5}{-5x+5}}\right)^{\frac{2x^2-x-5}{-5x+5}\cdot\frac{-5x+5}{2x^2-x-5}\cdot\frac{x^2}{2}} =$$

$$= e^{\lim_{x\to\infty}\frac{-5x^3+5x^2}{4x^2-2x-10}} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$ //

35.- $\lim_{x\to\infty}\left(\frac{2\sqrt{x^2+1}}{x+1}\right) = \frac{2}{1} = 2$ //

36.- $\lim_{x\to\infty}\left(\sqrt{9x^2+2x-3}-3x\right) =$

$$\lim_{x\to\infty}\frac{\cancel{9x^2}+2x-3-\cancel{9x^2}}{\sqrt{9x^2+2x-3}+3x} = \frac{2}{3+3} = \frac{1}{3}$$ //

37.- $\lim_{x\to-\infty}\frac{x^3-3x+9}{4x^2-6x} = -\infty$ //

38.- $\lim_{x \to -\infty} \left( \frac{-2}{1-x^3} \right) = 0$ //

39.- $\lim_{x \to 2} (x-1)^{\frac{1}{x-2}} = [1^\infty] =$

$= \lim_{x \to 2} (1 + x - 2)^{\frac{1}{x-2}} =$

$= \lim_{x \to 2} \left(1 + \frac{1}{\frac{1}{x-2}}\right)^{\frac{1}{x-2}} = e$ //

40.- $\lim_{x \to 1} x^{\frac{2}{1-x}} = [1^\infty] =$

$= \lim_{x \to 1} (1 + x - 1)^{\frac{2}{1-x}} = \lim_{x \to 1} \left(1 + \frac{1}{\frac{1}{x-1}}\right)^{\frac{1}{(x-1)} \cdot (x-1) \cdot \frac{2}{(1-x)}}$

$= e^{\lim_{x \to 1} \frac{2(x-1)}{1-x}} =$

$= e^{\lim_{x \to 1} \frac{2\cancel{(x-1)}}{-\cancel{(x-1)}}} = e^{-2}$ //