

FICHA 1: LÍMITES DE SUCESIONES PARTE I

$$1.- \lim_{n \rightarrow \infty} \frac{6n^2 + 4n - 6}{3n^2 - 6n + 7} = \frac{6}{3} = 2 \quad ; \text{ mismo grado}$$

$$2.- \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 5n - 1}}{\sqrt{2n^3 + 3n^2 + 2}} = \left[\frac{(\text{grado } 1)}{(\text{grado } 3/2)} \right] = 0$$

$$3.- \lim_{n \rightarrow \infty} \left(\frac{2n+6}{3n-5} \right)^n = \left(\frac{2}{3} \right)^\infty = 0$$

$$4.- \lim_{n \rightarrow \infty} \left(\sqrt{\frac{8n^2 - 2n}{2n^2 - 3}} \cdot \sqrt{\frac{(n+1)^2}{n^2 - 3n + 5}} \right) = \sqrt{4} \cdot \sqrt{1} = 2$$

$$5.- \lim_{n \rightarrow \infty} \left[(2n+4)^{-1} \cdot \sqrt{n^2 - 3} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - 3}}{2n+4} = \frac{1}{2}$$

$$6.- \lim_{n \rightarrow \infty} \left(\frac{n^2 + 2}{n+1} - \frac{3n-1}{3} \right) = [\infty - \infty] = \lim_{n \rightarrow \infty} \frac{3n^2 + 6 - (3n-1)(n+1)}{3(n+1)} = \lim_{n \rightarrow \infty} \frac{-3n + n + 7}{3n+3} = -1$$

$$7.- \lim_{n \rightarrow \infty} (\sqrt{n^2} - \sqrt{n^2 + n}) = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2} - \sqrt{n^2 + n})(\sqrt{n^2} + \sqrt{n^2 + n})}{(\sqrt{n^2} + \sqrt{n^2 + n})} = \lim_{n \rightarrow \infty} \frac{-n}{\sqrt{n^2} + \sqrt{n^2 + n}} = -\frac{1}{2}$$

$$8.- \lim_{n \rightarrow \infty} \left[n \cdot (n+1) - \frac{1}{(\sqrt{n})^{-1}} \right] = \lim_{n \rightarrow \infty} [n^2 + n - \sqrt{n}] = \infty$$

9.-

$$10.- \lim_{n \rightarrow \infty} \left(\frac{n^2 - 3n + 1}{n^2 + n - 1} \right)^{2n} = (1)^\infty = \lim_{n \rightarrow \infty} \left(1 + \frac{n^2 - 3n + 1 - n^2 - n + 1}{n^2 + n - 1} \right)^{2n} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4n + 2}{n^2 + n - 1} \right)^{2n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{n^2 + n - 1}{-4n + 2}} \right)^{-4n + 2} \cdot \frac{-4n + 2}{n^2 + n - 1} \cdot 2n = e^{-8}$$

$$11.- \lim_{n \rightarrow \infty} \left(\frac{2n}{2n+1} \right)^{n+1} = (1)^\infty = \lim_{n \rightarrow \infty} \left(1 + \frac{2n - 2n - 1}{2n+1} \right)^{n+1} = \lim_{n \rightarrow \infty} \left(1 + \frac{-1}{2n+1} \right)^{n+1} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{-2n-1} \right)^{-2n-1} \cdot \frac{1}{-2n-1} \cdot (n+1) = e^{\lim_{n \rightarrow \infty} \frac{n+1}{-2n-1}} = e^{-1/2}$$

$$12.- \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+4} - n}{2n-1} = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2+4} - n)(\sqrt{n^2+4} + \sqrt{n})}{\sqrt{n^2+4} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{4}{\sqrt{n^2+4} + \sqrt{n}} = 0$$

$$13.- \lim_{n \rightarrow \infty} \sqrt[3]{\frac{-8n^4 - 5n + 1}{n^4 + 6n^2 + 3}} = \sqrt[3]{-8} = -2$$

$$14.- \lim_{n \rightarrow \infty} \frac{1+4+7+\dots+(3n-2)}{5n^2+6} = \lim_{n \rightarrow \infty} \frac{\frac{1+(3n-2)}{2} \cdot n}{5n^2+6} = \lim_{n \rightarrow \infty} \frac{3n^2 - n}{10n^2 + 12} = \frac{3}{10}$$

$a_n = (3n-2)$ P. ARITHMETICAL

$$S_n = \frac{(a_1 + a_n) \cdot n}{2}$$

$$15.- \lim_{n \rightarrow \infty} \frac{2 + \frac{5}{n}}{4 - \frac{3}{n^2}} = \frac{2}{4} = \frac{1}{2}$$

$$16.- \lim_{n \rightarrow \infty} \left(1 - \frac{2}{n}\right)^{\frac{1}{n}-2} = (1)^{-2} = 1$$

$$17.- \lim_{n \rightarrow \infty} \left[\left(4^{\frac{1}{n}}\right)^{2n} + \sqrt{3 + \frac{1}{n}} \right] = \lim_{n \rightarrow \infty} \left(4^{\frac{2n}{n}} + \sqrt{3 + \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} \left(4^2 + \sqrt{3 + \frac{1}{n}} \right) = 16 + \sqrt{3}$$

$$18.- \lim_{n \rightarrow \infty} \left(\frac{2n+5}{n+4} \right)^{\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}$$

$$19.- \lim_{n \rightarrow \infty} \frac{4n-6}{2n^2+7n+1} = 0$$

$$20.- \lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{3n^2} = \lim_{n \rightarrow \infty} \frac{\frac{1+2n-1}{2} \cdot n}{3n^2} = \lim_{n \rightarrow \infty} \frac{2n^2}{6n^2} = \frac{1}{3}$$

$$a_n = 2n-1 \text{ P. ARITMÉTICA}$$

$$S_n = \frac{(a_1 + a_n) \cdot n}{2}$$

$$21.- \lim_{n \rightarrow \infty} \left(\frac{3n-5}{2n+6} \right)^n = \left(\frac{3}{2} \right)^\infty = \infty$$

$$22.- \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{4}{n}}{\frac{6}{n}} = \frac{0}{0} \text{ INDET.} = \lim_{n \rightarrow \infty} \frac{\frac{5}{n}}{\frac{6}{n}} = \lim_{n \rightarrow \infty} \frac{5n}{6n} = \frac{5}{6}$$

$$23.- \lim_{n \rightarrow \infty} \frac{\frac{n-1}{n^2+1} - \frac{n}{2n+1}}{\frac{n^2}{n^2} - \frac{n+1}{2n}} = \lim_{n \rightarrow \infty} \frac{\frac{(n-1)(n+1) - n^2}{n(n+1)}}{\frac{2(n^2+1) - (2n+1) \cdot n}{2n^2}} = \lim_{n \rightarrow \infty} \frac{\frac{n^2-1-n^2}{n(n+1)}}{\frac{2-n}{2n^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{-2n^2}{n(n+1)(2-n)} = \lim_{n \rightarrow \infty} \frac{-2n^2}{-n^3 + \dots} = 0$$

$$24.- \lim_{n \rightarrow \infty} (\sqrt{n^2+1} - n) = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2+1} - n)(\sqrt{n^2+1} + n)}{(\sqrt{n^2+1} + n)} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1} + n} = 0$$

$$25.- \lim_{n \rightarrow \infty} (n^3 - 3n^2 + n - 200) = \infty$$

$$26.- \lim_{n \rightarrow \infty} \left(\frac{1-3n}{4-3n} \right)^{n-2} = (1^\infty) = \lim_{n \rightarrow \infty} \left(1 + \frac{1-3n-4+3n}{4-3n} \right)^{n-2} =$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{4-3n} \right)^{n-2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{4-3n}{-3}} \right)^{\frac{4-3n}{-3} \cdot \frac{-3}{4-3n} \cdot (n-2)} =$$

$$= e^{\lim_{n \rightarrow \infty} \frac{-3n+6}{4-3n}} = e^1 = e$$

$$\begin{aligned}
 27.- \quad \lim_{n \rightarrow \infty} \sqrt[2n]{\left(\frac{n^2+n}{n^2+3}\right)^{4n^2}} &= \lim_{n \rightarrow \infty} \left(\frac{n^2+n}{n^2+3}\right)^{\frac{4n^2}{2n}} = \lim_{n \rightarrow \infty} \left(\frac{n^2+n}{n^2+3}\right)^{2n} = \\
 &= \lim_{n \rightarrow \infty} \left(1 + \frac{n^2+n-n^2-3}{n^2+3}\right)^{2n} = \lim_{n \rightarrow \infty} \left(1 + \frac{n-3}{n^2+3}\right)^{2n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{n^2+3}{n-3} \cdot \frac{n-3}{n^2+3}}\right)^{2n} \\
 &= e^{\lim_{n \rightarrow \infty} \frac{2n^2-6n}{n^3+\dots}} = e^0 = 1
 \end{aligned}$$

$$28.- \quad \lim_{n \rightarrow \infty} \frac{\frac{n}{n-1} \cdot \frac{n-1}{n+1}}{1 + \frac{2n-3}{n^2+2}} = \frac{1 \cdot 1}{1+0} = 1$$

$$29.- \quad \lim_{n \rightarrow \infty} \sqrt[5]{\frac{27n^3+5n^2}{\frac{n^3}{9}+8}} = \left(\frac{27}{\frac{1}{9}}\right)^{1/5} = \left(\frac{3^3}{\frac{1}{3^2}}\right)^{1/5} = (3^5)^{1/5} = 3$$

$$30.- \quad \lim_{n \rightarrow \infty} \frac{2+4+6+\dots+2n}{n^3-1} = \lim_{n \rightarrow \infty} \frac{\frac{2+2n}{2} \cdot n}{n^3-1} = \lim_{n \rightarrow \infty} \frac{2n^2+2n}{2(n^3-1)} = 0$$

$$a_n = 2n$$

$$S_n = \frac{(a_1 + a_n)n}{2}$$

$$31.- \quad \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{n^2} = \lim_{n \rightarrow \infty} \frac{\frac{1+n}{2} \cdot n}{n^2} = \lim_{n \rightarrow \infty} \frac{n^2+n}{2n^2} = \frac{1}{2}$$

$$a_n = n$$

$$32.- \quad \lim_{n \rightarrow \infty} \left(4 + \frac{3}{n}\right)^2 = 4^2 = 16$$

$$33.- \quad \lim_{n \rightarrow \infty} \left(8 - \frac{3}{2^n}\right)^{1/3} = \left(8 - \frac{3}{\infty}\right)^{1/3} = 8^{1/3} = 2$$

$$34.- \quad \lim_{n \rightarrow \infty} \left(\frac{n^4-3n^2+1}{4n^3+n-3} - \frac{n^2-1}{3n^4-2n}\right) = (\infty - 0) = \infty$$

$$35.- \lim_{n \rightarrow \infty} \left(\frac{2n+5}{n+4} \right)^{-\frac{1}{2}} = 2^{-\infty} = \frac{1}{2^{\infty}} = \frac{1}{\infty} = 0$$

$$36.- \lim_{n \rightarrow \infty} \sqrt[2n]{\frac{1+n}{3+n}} = \lim_{n \rightarrow \infty} \left(\frac{1+n}{3+n} \right)^{\frac{1}{2n}} = 1^0 = 1$$

$$37.- \lim_{n \rightarrow \infty} \left(\frac{3-2n}{5-3n} \right)^{\frac{n}{n+1}} = \left(\frac{-2}{-3} \right)^1 = \frac{2}{3}$$

$$38.- \lim_{n \rightarrow \infty} 4^{\frac{3n-1}{6n+2}} = 4^{\frac{1}{2}} = 2$$

$$39.- \lim_{n \rightarrow \infty} \left(\frac{3n-1}{5n+2} \right)^{-n-3} = \left(\frac{3}{5} \right)^{-\infty} = \left(\frac{5}{3} \right)^{\infty} = \infty.$$

$$40.- \lim_{n \rightarrow \infty} \frac{1 - \frac{n}{n+1}}{3 - \frac{3n}{n+1}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1-n}{n+1}}{\frac{3n+3-3n}{n+1}} = \lim_{n \rightarrow \infty} \frac{1}{3} = \frac{1}{3}$$

$$41.- \lim_{n \rightarrow \infty} \left[(n+1) \cdot \left(\frac{n^3+3}{n^3} - \frac{2n^2+2}{2n^2} \right) \right] = [\infty \cdot 0] = \lim_{n \rightarrow \infty} (n+1) \left(\frac{2n^3+6-2n^3-2n}{2n^3} \right) =$$

$$= \lim_{n \rightarrow \infty} \frac{(n+1) \cdot (6-2n)}{2n^3} = \lim_{n \rightarrow \infty} \frac{-2n^2 + \dots}{2n^3} = 0$$

$$42.- \lim_{n \rightarrow \infty} \left(\frac{n^4-n}{2n-1} - \frac{n^2}{n+1} \right) = [\infty - \infty] = \lim_{n \rightarrow \infty} \frac{(n^4-n)(n+1) - n^2(2n-1)}{(2n-1)(n+1)} =$$

$$= \lim_{n \rightarrow \infty} \frac{n^5 + n^4 - n^2 - n - 2n^3 + n^2}{2n^2 + \dots} = \infty$$

$$43.- \lim_{n \rightarrow \infty} (n - \sqrt{n^2 - n + 3}) = \lim_{n \rightarrow \infty} \frac{(n - \sqrt{n^2 - n + 3}) \cdot (n + \sqrt{n^2 - n + 3})}{n + \sqrt{n^2 - n + 3}} =$$

$$= \lim_{n \rightarrow \infty} \frac{n - 3}{n + \sqrt{n^2 - n + 3}} = \frac{1}{2}$$

$$44.- \lim_{n \rightarrow \infty} (\sqrt{n^2 + 5n} - 2n) = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + 5n} - 2n)(\sqrt{n^2 + 5n} + 2n)}{\sqrt{n^2 + 5n} + 2n} =$$

$$= \lim_{n \rightarrow \infty} \frac{n^2 + 5n - 4n^2}{\sqrt{n^2 + 5n} + 2n} = \lim_{n \rightarrow \infty} \frac{-3n^2 + 5n}{\sqrt{n^2 + 5n} + 2n} = -\infty$$

$$45.- \lim_{n \rightarrow \infty} (\sqrt{n} - \sqrt{2n+1}) = \lim_{n \rightarrow \infty} \frac{n - 2n - 1}{\sqrt{n} + \sqrt{2n+1}} = \lim_{n \rightarrow \infty} \frac{-n - 1}{\sqrt{n} + \sqrt{2n+1}} = -\infty$$

$$46.- \lim_{n \rightarrow \infty} (\sqrt{n^3} - \sqrt{n^2 + n}) = \lim_{n \rightarrow \infty} \frac{n^3 - n^2 - n}{\sqrt{n^3} + \sqrt{n^2 + n}} = \infty$$