

ELIPSE

1. a) $a = 9$ $P(6,4)$

HORIZONTAL: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{6^2}{9^2} + \frac{4^2}{b^2} = 1 \rightarrow \frac{16}{b^2} = 1 - \frac{36}{81} \rightarrow b^2 = \frac{16}{5/9}$$

$$b^2 = 144/5$$

$$\boxed{\frac{x^2}{81} + \frac{y^2}{144/5} = 1} \xrightarrow{\cdot 144/5 = 1286} \boxed{16x^2 + 45y^2 = 1296}$$

VERTICAL: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$\frac{6^2}{b^2} + \frac{4^2}{9^2} = 1 \rightarrow \frac{36}{b^2} = 1 - \frac{16}{81} \rightarrow b^2 = \frac{36}{65/81} = \frac{2916}{65}$$

$$\boxed{\frac{x^2}{2916/65} + \frac{y^2}{81} = 1} \xrightarrow{\cdot 2916} \boxed{65x^2 + 36y^2 = 2916}$$

b) $2c = 16$ $c = 8$ $\left\{ \begin{array}{l} \frac{8}{a} = \frac{4}{5} \\ b^2 = a^2 - c^2 = 100 - 64 = 36 \end{array} \right.$ $(a = 10)$ HORIZONTAL

$$\boxed{\frac{x^2}{100} + \frac{y^2}{36} = 1}$$

c) $F(4,0) \Rightarrow$ HORIZONTAL
 $A(5,0)$ $c = 4$ $a = 5$

$$b^2 = 5^2 - 4^2 = 9$$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

d) $P(0,4)$

$$e = \frac{3}{5} = \frac{c}{a} \rightarrow c = \frac{3}{5}a$$

$$a^2 = b^2 + c^2 \left\{ \begin{array}{l} a^2 = b^2 + (\frac{3}{5}a)^2 \\ b^2 = a^2 - \frac{9}{25}a^2 = \frac{16}{25}a^2 \end{array} \right.$$

HORIZONTAL: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{0^2}{a^2} + \frac{4^2}{16/25 a^2} = 1 \rightarrow a^2 = \frac{16}{16/25} = 25$$

$$b^2 = \frac{16}{25} \cdot 25 = 16$$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$

VERTICAL: $\frac{0^2}{16/25 a^2} + \frac{4^2}{b^2} = 1 \rightarrow a^2 = 16$

$$b^2 = \frac{16}{25} \cdot 16 = \frac{256}{25}$$

$$\boxed{\frac{x^2}{256/25} + \frac{y^2}{16} = 1}$$

$$e) \begin{cases} 2a=9 & a=9/2 \\ 2c=4\sqrt{2} & c=2\sqrt{2} \end{cases} \left\{ \begin{aligned} b^2 &= a^2 - c^2 = \frac{81}{4} - 8 = \frac{49}{4} \end{aligned} \right.$$

$$\boxed{\frac{x^2}{81/4} + \frac{y^2}{49/4} = 1}$$

$$\frac{4x^2}{81} + \frac{4y^2}{49} = 1$$

$$f) \begin{cases} e = \frac{4}{5} = \frac{c}{a} \\ b = 3 \end{cases} \left\{ \begin{aligned} c &= \frac{4}{5}a \\ a^2 &= 3^2 + \left(\frac{4}{5}a\right)^2 \\ \left(1 - \frac{16}{25}\right)a^2 &= 9 \\ \frac{9}{25}a^2 &= 9 \\ a^2 &= 25 \end{aligned} \right. \left\{ \begin{aligned} &\text{HORIZONTAL:} \\ &\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1} \\ &\text{VERTICAL:} \\ &\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1} \end{aligned} \right.$$

$$g) \begin{cases} e = \frac{3}{5} = \frac{c}{a} \\ P(0,4) \end{cases} \left\{ \begin{aligned} c &= \\ &\text{Igual al apartado d)} \end{aligned} \right.$$

$$h) \begin{cases} e = \frac{2}{3} = \frac{c}{a} \\ 2a = 18 \quad a = 9 \end{cases} \left\{ \begin{aligned} c &= \frac{2}{3}a = 6 \\ b^2 &= a^2 - c^2 = 81 - 36 = 45 \end{aligned} \right.$$

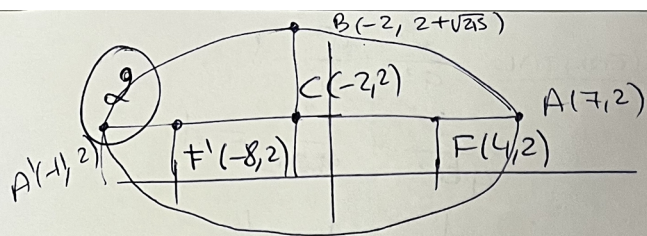
$$\text{HORIZONTAL: } \boxed{\frac{x^2}{81} + \frac{y^2}{45} = 1}$$

$$\text{VERTICAL: } \boxed{\frac{x^2}{45} + \frac{y^2}{81} = 1}$$

$$i) \begin{cases} e = \frac{3}{4} = \frac{c}{a} \\ 2c = 19 \end{cases} \left\{ \begin{aligned} a &= \frac{4}{3}c = \frac{4}{3} \cdot \frac{19}{2} = \frac{38}{3} \\ b^2 &= a^2 - c^2 = \left(\frac{38}{3}\right)^2 - \left(\frac{19}{2}\right)^2 = \frac{2527}{36} \end{aligned} \right.$$

$$\text{HORIZONTAL: } \boxed{\frac{x^2}{(38/3)^2} + \frac{y^2}{2527/36} = 1}$$

$$\text{VERTICAL: } \boxed{\frac{x^2}{2527/36} + \frac{y^2}{(38/3)^2} = 1}$$



$$c\left(\frac{-8+4}{2}, 2\right) = (-2, 2)$$

$$\left. \begin{aligned} c &= 4 - (-2) = 6 \\ e &= \frac{2}{3} = \frac{c}{a} \end{aligned} \right\} \begin{aligned} a &= \frac{2}{\frac{2}{3}} c = \frac{3}{2} \cdot 6 \\ a &= 9 \\ b^2 &= a^2 - c^2 = 45 \end{aligned}$$

HORIZONTAL:

$$\boxed{\frac{(x+2)^2}{9^2} + \frac{(y-2)^2}{45} = 1}$$

③ $C(-1, -1)$ $\left\{ \begin{aligned} a &= 5 - (-1) = 6 \\ e &= \frac{2}{3} = \frac{c}{a} \end{aligned} \right.$ HORIZONTAL

$$\left. \begin{aligned} c &= \frac{2}{3} \cdot 6 = 4 \\ b^2 &= 6^2 - 4^2 = 20 \end{aligned} \right\} \boxed{\frac{(x+1)^2}{36} + \frac{(y+1)^2}{20} = 1}$$

④ $\left\{ \begin{aligned} 2a &= 2 + 8 \rightarrow a = 5 \\ 2c &= 6 \rightarrow c = 3 \end{aligned} \right. \quad b^2 = 25 - 9 = 16$

HORIZONTAL: $\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$ VERTICAL: $\boxed{\frac{x^2}{16} + \frac{y^2}{25} = 1}$

⑤ NO

⑥ $\frac{9x^2}{225} + \frac{25y^2}{225} = 1$

$$\frac{x^2}{225/9} + \frac{y^2}{225/25} = 1$$

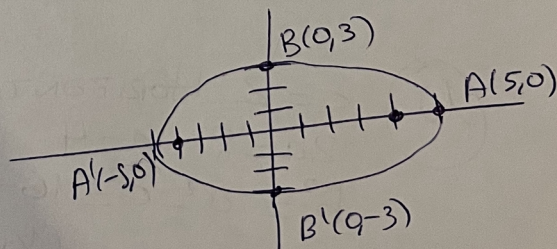
$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

$a = 5$ $\left\{ \begin{aligned} &\text{semiejes} \\ b &= 3 \end{aligned} \right.$

$$c^2 = a^2 - b^2 = 25 - 9 = 16$$

$$c = 4 \text{ semidistancia focal}$$

$$e = \frac{c}{a} = \frac{4}{5}$$



7 $a=2b$ $\left\{ \begin{array}{l} P(2,1) \end{array} \right.$

HORIZONTAL: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 $\frac{2^2}{4b^2} + \frac{1^2}{b^2} = 1$

$\frac{1}{b^2} + \frac{1}{b^2} = 1$

$\frac{2}{b^2} = 1$

$b^2 = 2 \quad a^2 = 4 \cdot b^2 = 4 \cdot 2 = 8$

$\boxed{\frac{x^2}{8} + \frac{y^2}{2} = 1}$

8 $C(3,1)$ HORIZONTAL

$e = 1/2 \left\{ \begin{array}{l} \frac{c}{a} = \frac{1}{2} \\ c = 4 \end{array} \right.$

$a = 2c = 2 \cdot 4 = 8$
 $b^2 = a^2 - c^2 = 8^2 - 4^2 = 48$

$\boxed{\frac{(x-3)^2}{64} + \frac{(y-1)^2}{48} = 1}$

VERTICAL: $\boxed{\frac{(x-3)^2}{48} + \frac{(y-1)^2}{64} = 1}$

9 $\left. \begin{array}{l} x^2 + 4y^2 = 8 \\ y = x - 1 \end{array} \right\} \begin{array}{l} x^2 + 4(x-1)^2 = 8 \\ x^2 + 4x^2 - 8x + 4 = 8 \\ 5x^2 - 8x - 4 = 0 \end{array}$

$x = \frac{8 \pm \sqrt{64 + 80}}{10} = \frac{8 \pm 12}{10} \left(\begin{array}{l} 2 \\ -2/5 \end{array} \right.$

Recta secante a la elipse en los puntos $(2,1)$
 $(-2/5, -7/5)$

10 $c=2$ HORIZONTAL $\left\{ \begin{array}{l} \boxed{\frac{x^2}{16} + \frac{y^2}{12} = 1} \text{ Elipse} \\ 2a=8 \quad a=4 \\ b^2 = a^2 - c^2 = 16 - 4 = 12 \end{array} \right.$

11 HORIZONTAL: $\left. \begin{array}{l} \frac{(1/2)^2}{a^2} + \frac{(-\sqrt{2}/2)^2}{b^2} = 1 \\ \frac{(-1)^2}{a^2} + \frac{0^2}{b^2} = 1 \end{array} \right\}$

$$\begin{aligned}
 & \textcircled{11} \left. \begin{aligned} \frac{1}{4a^2} + \frac{2}{4b^2} &= 1 \\ \frac{1}{a^2} &= 1 \end{aligned} \right\} \begin{aligned} & \textcircled{a^2=1} \\ & \frac{1}{4} + \frac{2}{4b^2} = 1 \\ & \frac{\cancel{b^2}}{4\cancel{b^2}} + \frac{2}{4b^2} = \frac{4b^2}{4b^2} \\ & 2 = 3b^2 \\ & \textcircled{b^2 = 2/3} \end{aligned} \left\{ \boxed{x^2 + \frac{y^2}{2/3} = 1} \right.
 \end{aligned}$$

$$\begin{aligned}
 \text{VERTICAL: } & \left. \begin{aligned} \frac{(1/2)^2}{b^2} + \frac{(-\sqrt{2}/2)^2}{a^2} &= 1 \\ \frac{(-1)^2}{b^2} + \frac{0^2}{a^2} &= 1 \end{aligned} \right\} \begin{aligned} & \textcircled{b^2=1} \\ & \frac{1}{4} + \frac{2}{4a^2} = 1 \\ & \frac{a^2}{4a^2} + \frac{2}{4a^2} = \frac{4a^2}{4a^2} \\ & 2 = 3a^2 \\ & \textcircled{a^2 = 2/3} < b^2 = 1 \\ & \text{No tiene soluci3n} \end{aligned}
 \end{aligned}$$

~~x² + 1 = 1~~