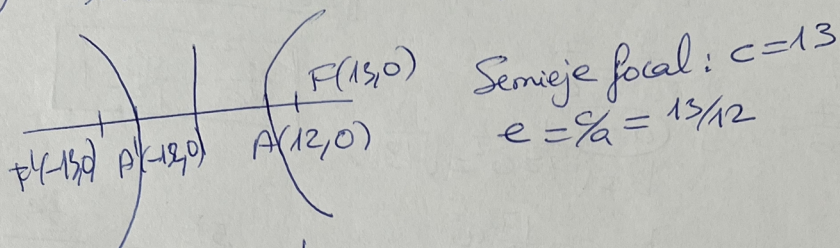


FICHA HIPÉRBOLA

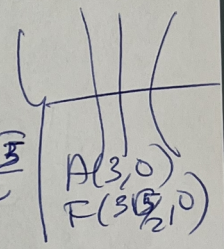
① $a=2$ | $b^2 = c^2 - a^2$
 $c=4$ | $b^2 = 4^2 - 2^2 = 12$
 $b = 2\sqrt{3}$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

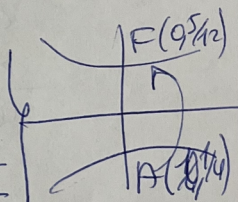
② a) $\frac{x^2}{144} - \frac{y^2}{25} = 1 \rightarrow \begin{cases} c(0,0) \\ a=12 \\ b=5 \end{cases} \quad c = \sqrt{12^2 + 5^2} = 13$



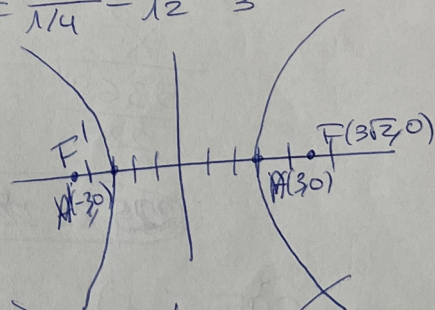
b) $\begin{cases} x^2 - 4y^2 = 9 \\ \frac{x^2}{9} - \frac{y^2}{9/4} = 1 \end{cases} \quad \begin{cases} a=3 \\ b=3/2 \end{cases}$
 $c = \sqrt{9 + 9/4} = \sqrt{27/4} = \frac{3\sqrt{3}}{2}$
 $e = \frac{c}{a} = \frac{3\sqrt{3}/2}{3} = \frac{\sqrt{3}}{2}$



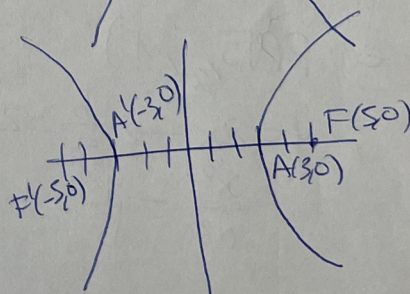
c) $\begin{cases} 16y^2 - 9x^2 = 1 \\ \frac{y^2}{1/16} - \frac{x^2}{1/9} = 1 \end{cases} \quad \begin{cases} a=1/4 \\ b=1/3 \end{cases}$
 $c = \sqrt{1/16 + 1/9} = \sqrt{25/144} = \frac{5}{12}$
 $e = \frac{c}{a} = \frac{5/12}{1/4} = \frac{20}{12} = \frac{5}{3}$



d) $\begin{cases} x^2 - y^2 = 9 \\ \frac{x^2}{9} - \frac{y^2}{9} = 1 \end{cases} \quad \begin{cases} a=3 \\ b=3 \end{cases}$
 $c = \sqrt{3^2 + 3^2} = 3\sqrt{2}$
 $e = \frac{c}{a} = \frac{3\sqrt{2}}{3} = \sqrt{2}$



e) $\begin{cases} 16x^2 - 9y^2 = 144 \\ \frac{x^2}{9} - \frac{y^2}{16} = 1 \end{cases} \quad \begin{cases} a=12/4=3 \\ b=4 \end{cases}$
 $c = \sqrt{3^2 + 4^2} = 5$
 $e = 5/3$



3) a) $F(9,0)$ $F(-9,0)$ $A(9,0)$ $A(-9,0)$
 $2c=30$ $c=15$ $b=\sqrt{18^2-12^2}=9$
 $2a=24$ $a=12$

$$\boxed{\frac{x^2}{144} - \frac{y^2}{81} = 1}$$

b) $2a=12$
 $P(-10,4)$

$$\frac{x^2}{6^2} - \frac{y^2}{b^2} = 1$$

$$\frac{(-10)^2}{6^2} - \frac{4^2}{b^2} = 1$$

$$\frac{4^2}{b^2} = \frac{100}{36} - 1$$

$$\frac{16}{b^2} = \frac{64}{36}$$

$$b = \sqrt{\frac{16 \cdot 36}{64}} = \frac{4 \cdot 6}{8} = 3$$

$$\boxed{\frac{x^2}{36} - \frac{y^2}{9} = 1}$$

c) $c=7$
 $e=\frac{7}{6}=\frac{c}{a}$ $\left\{ \begin{array}{l} \frac{7}{a} = \frac{7}{6} \\ a=6 \end{array} \right.$
 $b=\sqrt{7^2-6^2}=\sqrt{13}$

$$\boxed{\frac{x^2}{36} - \frac{y^2}{13} = 1}$$

d) $P(5,2)$
 $e=\frac{17}{8}=\frac{c}{a}$
 $c^2=a^2+b^2$
 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$c = \frac{17}{8}a$$

$$\frac{17^2}{8^2}a^2 = a^2 + b^2$$

$$\frac{225}{64}a^2 = b^2$$

$$\frac{5^2}{a^2} - \frac{2^2}{\frac{225}{64}a^2} = 1$$

$$\frac{25}{a^2} - \frac{256}{225a^2} = 1$$

$$\frac{5625}{225a^2} - \frac{256}{225a^2} = \frac{225a^2}{225a^2}$$

$$5369 = 225a^2$$

$$\boxed{a^2 = \frac{5369}{225}} \quad \boxed{b^2 = \frac{5369}{64}}$$

$$\frac{x^2}{\frac{5369}{225}} - \frac{y^2}{\frac{5369}{64}} = 1$$

$$\boxed{225x^2 - 64y^2 = 5369}$$

e) $P(\sqrt{5}/2, 1)$
 $y=2x \rightarrow \frac{b}{a}=2$
 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$b=2a$$

$$\frac{x^2}{a^2} - \frac{y^2}{4a^2} = 1$$

$$\frac{(\sqrt{5}/2)^2}{a^2} - \frac{1^2}{4a^2} = 1$$

$$\frac{5}{4a^2} - \frac{1}{4a^2} = 1$$

$$\frac{1}{a^2} = 1 \quad \boxed{a=1} \quad \boxed{b=2}$$

$$\boxed{x^2 - \frac{y^2}{4} = 1}$$

HIPÉRBOLA 2

f) $|d(P,F) - d(P,F')| = 2a = 16 - 12$ $a = 2$
 $F(8,0)$ $c = 8$

$c^2 = a^2 + b^2$ $8^2 = 2^2 + b^2$ $b = \sqrt{60}$

$$\boxed{\frac{x^2}{4} - \frac{y^2}{60} = 1}$$

g) $\left. \begin{array}{l} 2c = 34 \quad c = 17 \\ c - a = 2 \end{array} \right\} \begin{array}{l} a = 17 - 2 = 15 \\ b = \sqrt{17^2 - 15^2} = 8 \end{array}$

$$\boxed{\frac{x^2}{225} - \frac{y^2}{64} = 1}$$

h) $\left. \begin{array}{l} P(-15,4) \\ 2a = 18 \quad a = 9 \\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \end{array} \right\} \begin{array}{l} \frac{(-15)^2}{9^2} - \frac{4^2}{b^2} = 1 \\ \frac{225}{81} - 1 = \frac{16}{b^2} \\ \frac{16}{9} = \frac{16}{b^2} \\ b = 3 \end{array}$

$$\boxed{\frac{x^2}{81} - \frac{y^2}{9} = 1}$$

i) $\left. \begin{array}{l} P(2, \sqrt{3}) \\ e = \sqrt{3} = \frac{c}{a} \\ c^2 = a^2 + b^2 \\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \end{array} \right\} \begin{array}{l} c = \sqrt{3} \cdot a \\ (\sqrt{3}a)^2 = a^2 + b^2 \\ 3a^2 - a^2 = b^2 \\ b^2 = 2a^2 \end{array}$

$$\begin{array}{l} \frac{2^2}{a^2} - \frac{(\sqrt{3})^2}{2a^2} = 1 \\ \frac{4}{a^2} - \frac{3}{2a^2} = 1 \\ \frac{8}{2a^2} - \frac{3}{2a^2} = \frac{2a^2}{2a^2} \\ \boxed{\frac{5}{2} = a^2 \quad 5 = b^2} \end{array}$$

$$\frac{x^2}{5/2} - \frac{y^2}{5} = 1$$

$$\boxed{2x^2 - y^2 = 5}$$

j) $\left. \begin{array}{l} c - a = 2 \\ c + a = 50 \end{array} \right\} \begin{array}{l} 2c = 52 \quad c = 26 \\ a = 50 - 26 = 24 \\ b = \sqrt{26^2 - 24^2} = 10 \end{array}$

$$\boxed{\frac{x^2}{576} - \frac{y^2}{100} = 1}$$

k) $\left. \begin{array}{l} c = 13 \\ 2a = 10 \quad a = 5 \\ b = \sqrt{13^2 - 5^2} = 12 \end{array} \right\} \boxed{\frac{x^2}{25} - \frac{y^2}{144} = 1}$

$$l) \left. \begin{array}{l} a=3 \\ c=5 \\ b=\sqrt{5^2-3^2}=4 \end{array} \right\} \boxed{\frac{x^2}{9} - \frac{y^2}{16} = 1}$$

$$m) \left. \begin{array}{l} b=6 \\ c=10 \\ a=\sqrt{10^2-6^2}=8 \end{array} \right\} \boxed{\frac{x^2}{64} - \frac{y^2}{36} = 1}$$

$$n) \left. \begin{array}{l} a=6 \\ 2c=16 \rightarrow c=8 \\ b=\sqrt{8^2-6^2}=\sqrt{28} \end{array} \right\} \boxed{\frac{x^2}{36} - \frac{y^2}{28} = 1}$$

$$\tilde{n}) \left. \begin{array}{l} b=4 \\ 2c=10 \rightarrow c=5 \\ a=\sqrt{5^2-4^2}=3 \end{array} \right\} \boxed{\frac{x^2}{9} - \frac{y^2}{16} = 1}$$

$$\textcircled{5} a) \left. \begin{array}{l} 4x^2 - 8y^2 = 1 \\ \frac{x^2}{1/4} - \frac{y^2}{1/8} = 1 \\ a=1/2 \\ b=1/\sqrt{2} \\ b/a = \frac{1/\sqrt{2}}{1/2} = \frac{1}{\sqrt{2}} \end{array} \right\} \boxed{\begin{array}{l} y = \frac{1}{\sqrt{2}}x \\ y' = -\frac{1}{\sqrt{2}}x \end{array}} \left. \begin{array}{l} m = \frac{1}{\sqrt{2}} = \operatorname{tg} \alpha \\ \alpha = 35,3^\circ \\ m' = -\frac{1}{\sqrt{2}} = \operatorname{tg} \beta \\ \beta = -35,3^\circ \end{array} \right\}$$

$$b) \left. \begin{array}{l} 4x^2 - y^2 = 8 \\ \frac{x^2}{8/4} - \frac{y^2}{8} = 1 \\ a=\sqrt{2} \quad b=2\sqrt{2} \\ \frac{b}{a} = 2 \end{array} \right\} \boxed{\begin{array}{l} y = 2x \\ y' = -2x \end{array}} \left. \begin{array}{l} m = 2 = \operatorname{tg} \alpha \quad \alpha = 63,4^\circ \\ m' = -2 = \operatorname{tg} \beta \quad \beta = -63,4^\circ \end{array} \right\}$$

$$c) \left. \begin{array}{l} 4x^2 - 5y^2 = 20 \\ \frac{x^2}{20/4} - \frac{y^2}{20/5} = 1 \\ \frac{x^2}{5} - \frac{y^2}{4} = 1 \\ \frac{b}{a} = \frac{5}{4} \end{array} \right\} \boxed{\begin{array}{l} y = \frac{5}{4}x \\ y' = -\frac{5}{4}x \end{array}} \left. \begin{array}{l} m = \frac{5}{4} \quad \alpha = \operatorname{arctg}\left(\frac{5}{4}\right) = 51,3^\circ \\ m' = -\frac{5}{4} \quad \beta = \operatorname{arctg}\left(-\frac{5}{4}\right) = -51,3^\circ \end{array} \right\}$$

HIPÉRBOLA (3)

$$d) \left. \begin{array}{l} x^2 - 4y^2 = 4 \\ \frac{x^2}{4} - y^2 = 1 \\ a=2 \quad b=1 \\ b/a = 1/2 \end{array} \right\} \left\{ \begin{array}{l} y = \frac{1}{2}x \\ y = -\frac{1}{2}x \end{array} \right. \quad \begin{array}{l} \alpha = \arctg(1/2) = 26,6^\circ \\ \beta = \arctg(-1/2) = -26,6^\circ \end{array}$$

(6) $16x^2 - 9y^2 = 144$

$$\frac{x^2}{144/16} - \frac{y^2}{144/9} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$a=3 \quad b/2 = 4/3$$

$$c = \sqrt{3^2 + 4^2} = 5$$

$$F(5,0) \quad F(-5,0)$$

$$A(3,0) \quad A'(-3,0)$$

$$\text{Asintotas: } y = \frac{4}{3}x \quad y = -\frac{4}{3}x$$

$$e = c/a = 5/3$$

$$d(F, \text{asintota}) = \frac{|4x - 3y|}{\sqrt{4^2 + 3^2}} = \frac{20}{5} = 4 = b$$

(7) $\left. \begin{array}{l} 2x - y = 0 \\ 2x + y = 0 \end{array} \right\} y = 2x \quad b/a = 2 \rightarrow \boxed{b = 2a}$

$$c^2 = a^2 + b^2$$

$$5^2 = a^2 + 4a^2$$

$$a = 1 \quad b = 2$$

$$c = 5$$

$$F(0,5) \rightarrow \text{hipérbola vertical} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\boxed{\frac{y^2}{1} - \frac{x^2}{4} = 1}$$

(8) $3x - y = 0 \rightarrow b/a = 3$

$$5^2 = a^2 + 9a^2$$

$$\frac{25}{10} = a^2 \quad \boxed{a^2 = 5/2} \quad \boxed{b = 3 \cdot \sqrt{5/2}}$$

$$F(5,0) \rightarrow \text{hipérbola horizontal} \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\boxed{\frac{x^2}{5/2} - \frac{y^2}{45/2} = 1}$$

(9) $F(1,0)$
 $F'(0,-3)$
 $\uparrow$
 eje oblicuo

$$|d(P,F) - d(P,F')| = 2$$

$$\left| \sqrt{(x-1)^2 + y^2} - \sqrt{x^2 + (y+3)^2} \right| = 2$$

$$\left(\sqrt{(x-1)^2 + y^2} \right)^2 - \left(\sqrt{x^2 + (y+3)^2} \right)^2 - 2\sqrt{\dots} = 4$$

No entra en examen

10) $a=b$
 $P(2,3)$

HORIZONTAL

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

$$\frac{4}{a^2} - \frac{9}{a^2} = 1$$

$$\frac{4}{a^2} - \frac{9}{a^2} = \frac{a^2}{a^2}$$

$$-5 = a^2$$

No tiene solución

VERTICAL

$$\frac{y^2}{a^2} - \frac{x^2}{a^2} = 1$$

$$\frac{9}{a^2} - \frac{4}{a^2} = 1$$

$$\frac{9}{a^2} - \frac{4}{a^2} = \frac{a^2}{a^2}$$

$$a = \sqrt{5}$$

$$\frac{y^2}{5} - \frac{x^2}{5} = 1$$

$$\boxed{y^2 - x^2 = 5}$$

11) $a=b$ $\left\{ \begin{array}{l} c^2 = a^2 + b^2 \\ 2c = 6 \end{array} \right.$
 $9 = a^2 + a^2$
 $a^2 = 9/2 = b^2$

HORIZONTAL

$$\frac{x^2}{9/2} - \frac{y^2}{9/2} = 1$$

$$\boxed{x^2 - y^2 = 9/2}$$

VERTICAL

$$\boxed{y^2 - x^2 = 9/2}$$

12) $a=b$
 $P(4,1)$

HORIZONTAL

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

$$\frac{16}{a^2} - \frac{1}{a^2} = 1$$

$$15 = a^2$$

$$\boxed{x^2 - y^2 = 15}$$

VERTICAL

$$\boxed{y^2 - x^2 = 15}$$

13) $P(2,1)$
 $y = \pm \frac{3}{4}x$

$$\frac{b}{a} = \frac{3}{4}$$

$$b = \frac{3}{4}a$$

HORIZONTAL

$$\frac{x^2}{a^2} - \frac{y^2}{9/16 a^2} = 1$$

$$\frac{4}{a^2} - \frac{1}{9/16 a^2} = 1$$

$$\frac{4}{a^2} - \frac{16}{9a^2} = 1$$

$$\frac{36}{9a^2} - \frac{16}{9a^2} = 1$$

$$20 = 9a^2$$

$$a^2 = 20/9 \quad b = \frac{3}{4} \cdot \frac{\sqrt{20}}{3} = \frac{\sqrt{20}}{4}$$

$$\textcircled{14} \quad e = \frac{4}{3} = \frac{c}{a} \quad \left\{ \begin{array}{l} a = 8 \cdot \frac{3}{4} = 6 \\ c = 8 \end{array} \right. \quad b = \sqrt{c^2 - a^2} = \sqrt{64 - 36} = \sqrt{28}$$

$$F(8,0) \quad \text{horizontal}$$

$$\boxed{\frac{x^2}{36} - \frac{y^2}{28} = 1}$$

Hyperbola #4

$$\textcircled{15} \quad P(2\sqrt{5}, 1)$$

$$a = 2b$$

HORIZONTAL

$$\frac{(2\sqrt{5})^2}{(2b)^2} - \frac{1^2}{b^2} = 1$$

$$\frac{20}{4b^2} - \frac{1}{b^2} = 1$$

$$\frac{4}{b^2} = 1$$

VERTICAL

$$\frac{(2\sqrt{5})^2}{4b^2} - \frac{1^2}{b^2} = 1$$

$$\frac{1}{4b^2} - \frac{80}{4b^2} = 1$$

Not a sol.

$$\boxed{\frac{x^2}{16} - \frac{y^2}{4} = 1}$$

$$\textcircled{b=2}$$

$$\textcircled{a=4}$$

$$\textcircled{16} \quad P(2, 2\sqrt{3})$$

$$Q(\sqrt{2}, 2)$$

HORIZONTAL

$$\frac{2^2}{a^2} - \frac{(2\sqrt{3})^2}{b^2} = 1$$

$$\frac{(\sqrt{2})^2}{a^2} - \frac{2^2}{b^2} = 1$$

$$\frac{4}{a^2} - \frac{12}{b^2} = 1$$

$$\frac{2}{a^2} - \frac{4}{b^2} = 1$$

$$-4/b^2 = -1$$

$$\textcircled{b=2}$$

$$\frac{4}{a^2} - \frac{12}{4} = 1$$

$$\frac{4}{a^2} = 4 \quad \textcircled{a=1}$$

$$\boxed{\frac{x^2}{1} - \frac{y^2}{4} = 1}$$

VERTICAL

$$\left. \begin{aligned} \frac{(2\sqrt{3})^2}{a^2} - \frac{2^2}{b^2} &= 1 \\ \frac{2^2}{a^2} - \frac{(\sqrt{2})^2}{b^2} &= 1 \end{aligned} \right\}$$

$$\frac{12}{a^2} - \frac{4}{b^2} = 1$$

$$\frac{4}{a^2} - \frac{2}{b^2} = 1$$

$$\frac{12}{a^2} - \frac{4}{b^2} = 1$$

$$\left. \begin{aligned} \frac{12}{a^2} - \frac{4}{b^2} &= 1 \\ \frac{-8}{a^2} + \frac{4}{b^2} &= -2 \end{aligned} \right\}$$

$$\frac{4}{a^2} = -1 \text{ Not needed}$$