FICHA DERIVACIÓN : nº6

61.- $y=(x-2)^3$; $y'=3(x-2)^2$

62.- $y=(1-x^2)^3$; $y'=3(1-x^2)^2\cdot(-2x)$

63.- $y=\dfrac{\sqrt{x}}{(1-x)^5}=x^{1/2}\cdot(1-x)^{-5}$; $y'=\frac{1}{2}x^{-1/2}\cdot(1-x)^{-5}+x^{1/2}\cdot(-5)(1-x)^{-6}\cdot(-1)$

ó directamente como cociente:

$$y'=\frac{\frac{1}{2}x^{-1/2}\cdot(1-x)^5-\sqrt{x}\cdot 5(1-x)^4\cdot(-1)}{(1-x)^{10}}$$

64.- $y=\sqrt{\dfrac{1-x}{1+x}} \neq \dfrac{\sqrt{1-x}}{\sqrt{1+x}}$ no es la misma función!

$$y'=\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{(-1)(1+x)-(1-x)\cdot 1}{(1+x)^2}=$$

$$=\frac{1}{2}\sqrt{\frac{1+x}{1-x}}\cdot\frac{-1-x-1+x}{(1+x)^2}=-\sqrt{\frac{1+x}{1-x}}\cdot\frac{1}{(1+x)^2}$$

65.- $y=\ln\left[\operatorname{sen}^3\sqrt{\dfrac{1-x}{1+x}}\right]=3\cdot\ln\left[\operatorname{sen}\sqrt{\dfrac{1-x}{1+x}}\right]$

$$y'=3\cdot\frac{1}{\operatorname{sen}\sqrt{\frac{1-x}{1+x}}}\cdot\cos\sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}$$

66.- $y=\dfrac{\ln x}{x}$; $y'=\dfrac{\frac{1}{\cancel{x}}\cdot\cancel{x}-\ln x\cdot 1}{x^2}=\dfrac{1-\ln x}{x^2}$

67.- $y=8^{3x^2-1}$; $y'=8^{3x^2-1}\cdot\ln 8\cdot 6x$

68.- $y = 2^{\operatorname{sen}\sqrt{x}}$; $y' = 2^{\operatorname{sen}\sqrt{x}} \cdot \ln 2 \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

69.- $y = \sqrt[x]{\operatorname{sen} x} = (\operatorname{sen} x)^{1/x}$ DERIVACIÓN LOGARÍTMICA

$\ln y = \ln (\operatorname{sen} x)^{1/x}$

$\ln y = \frac{1}{x} \cdot \ln(\operatorname{sen} x)$; $\ln y = \frac{\ln(\operatorname{sen} x)}{x}$ ahora derivamos.

$$\frac{1}{y} \cdot y' = \frac{\frac{1}{\operatorname{sen} x} \cdot \cos x \cdot x - \ln(\operatorname{sen} x) \cdot 1}{x^2}$$

$$y' = \sqrt[x]{\operatorname{sen} x} \cdot \left[\frac{\cos x}{\operatorname{sen} x \cdot x} - \frac{\ln(\operatorname{sen} x)}{x^2} \right]$$

70.- $y = \cos^3 5x$; $y' = 3\cos^2 5x \cdot (-\operatorname{sen} 5x) \cdot 5$

71.- $y = \tan^2(\operatorname{sen} x)$; $y' = 2\tan(\operatorname{sen} x) \cdot \frac{1}{\cos^2(\operatorname{sen} x)} \cdot \cos x$

72.- $y = \frac{1}{\cos x} = (\cos x)^{-1}$; $y' = -1 \cdot \cos^{-2} x \cdot (-\operatorname{sen} x) = \frac{\operatorname{sen} x}{\cos^2 x}$

73.- $y = \sqrt{1 + \operatorname{sen} x}$; $y' = \frac{1}{2}(1 + \operatorname{sen} x)^{-1/2} \cdot \cos x$

74.- $y = \frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \frac{\operatorname{sen}^2 x}{\cos^2 x}}{1 + \frac{\operatorname{sen}^2 x}{\cos^2 x}} = \frac{\cos^2 x - \operatorname{sen}^2 x}{1} =$

$= \cos^2 x - \operatorname{sen}^2 x$ ... más fácil.

$y' = 2\cos x\,(-\operatorname{sen} x) - 2\operatorname{sen} x \cdot \cos x = -4\operatorname{sen} x \cdot \cos x$

75.- $y=\dfrac{1}{\tan\sqrt{x}}=\tan^{-1}\sqrt{x}$; $y'=-1.\tan^{-2}\sqrt{x}.\dfrac{1}{\cos^2\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}}$

76.- $y=x^{\operatorname{sen}x}$; $\ln y=\operatorname{sen}x.\ln x$

$$\frac{1}{y}\cdot y'=\cos x.\ln x+\operatorname{sen}x.\frac{1}{x}$$

$$y'=x^{\operatorname{sen}x}.\left[\cos x.\ln x+\frac{\operatorname{sen}x}{x}\right]$$

77.- $y=(\cos x)^{x^2}$; $\ln y=x^2.\ln\cos x$

$$\frac{1}{y}\cdot y'=2x\ln\cos x+x^2.\frac{1}{\cos x}(-\operatorname{sen}x)$$

$$y'=(\cos x)^{x^2}.\left[2x\ln\cos x-\frac{x^2\operatorname{sen}x}{\cos x}\right]$$

78.- $y=\operatorname{sen}^3x.\cos^2(6x+1)$

$$y'=3\operatorname{sen}^2x.\cos x.\cos^2(6x+1)+\operatorname{sen}^3x.2\cos(6x+1)(-\operatorname{sen}(6x+1)).6$$

79.- $y=\ln\left(x^2.\sqrt{1-2x}\right)=\ln x^2+\ln\sqrt{1-2x}=2\ln x+\frac{1}{2}\ln(1-2x)$

$$y'=\frac{2}{x}+\frac{1}{2}\cdot\frac{1}{1-2x}\cdot(-2)=\frac{2}{x}-\frac{1}{1-2x}$$

80.- $y=(\operatorname{sen}x)^{\operatorname{sen}x}$; $\ln y=\operatorname{sen}x.\ln(\operatorname{sen}x)$

$$\frac{1}{y}\cdot y'=\cos x.\ln(\operatorname{sen}x)+\operatorname{sen}x.\frac{1}{\operatorname{sen}x}.\cos x$$

$$y'=(\operatorname{sen}x)^{\operatorname{sen}x}.\left[\cos x.\ln(\operatorname{sen}x)+\cos x\right]$$

81.- $y = \dfrac{\operatorname{sen} x}{\cos e^x}$; $y' = \dfrac{\cos x . \cos e^x - \operatorname{sen} x . (-\operatorname{sen} e^x) . e^x}{(\cos e^x)^2}$

82.- $y = \frac{3}{4} . x^{2/5} - \frac{1}{2} . x^{-2/3}$

$y' = \frac{3}{4} . \frac{2}{5} . x^{-3/5} - \frac{1}{2} . (-\frac{2}{3}) x^{-5/3} = \frac{3}{10} . x^{-3/5} + \frac{1}{3} x^{-5/3}$

83.- $y = x^{2/5} - 3 . x^{3/4}$

$y' = 2/5 \, x^{-3/5} - 3 . 3/4 . x^{-1/4}$

84.- $y = (3x-5)^4$; $y' = 4(3x-5)^3 . 3$

85.- $y = 9(2x^2-1)^3$; $y' = 27(2x^2-1)^2 . 4x$

86.- $y = (3x-1)^2 . (2x+1)$; $y' = 2(3x-1) . 3(2x+1) + (3x-1)^2 . 2$

87.- $y = \dfrac{4}{(2x-1)^7} = 4 . (2x-1)^{-7}$; $y' = -28(2x-1)^{-8} . 2$

88.- $y = \dfrac{3x^2-2}{5x+4}$; $y' = \dfrac{6x . (5x+4) - (3x^2-2) . 5}{(5x+4)^2}$

89.- $y = \dfrac{10+5x}{10-5x}$; $y' = \dfrac{5(10-5x) - (10+5x) . (-5)}{(10-5x)^2}$

90.- $y = \dfrac{3x^2+2}{x^3-11x^2}$; $y' = \dfrac{6x(x^3-11x^2) - (3x^2+2) . (3x^2-22x)}{(x^3-11x^2)}$

91.- $y = \ln(4x^2-5x)$; $y' = \dfrac{8x-5}{4x^2-5x}$

92.- $y = \ln(3x-5)$; $y' = \dfrac{3}{3x-5}$

93.- $y = \ln\left(\dfrac{4-5x}{2x+3}\right) = \ln(4-5x) - \ln(2x+3)$

$$y' = \frac{-5}{4-5x} - \frac{2}{2x+3}$$

94.- $y = \ln(3x^2-2x)^4 = 4\ln(3x^2-2x)$

$$y' = \frac{4.(6x-2)}{3x^2-2x}$$

95.- $y = \ln(4+\sqrt{x})$; $y' = \dfrac{1}{4+\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}}$

96.- $y = 4x.5^{3x}$; $y' = 4.5^{3x} + 4x.5^{3x}.\ln 5.3$

97.- $y = (3x-2)^{2/5}$; $y' = \dfrac{2}{5}.(3x-2)^{-3/5}.3$

98.- $y = (6x^2-1)e^{3x}$; $y' = 12x.e^{3x} + (6x^2-1).e^{3x}.3$

99.- $y = \dfrac{3^{5x}}{2x-1}$; $y' = \dfrac{3^{5x}.\ln 3.5.(2x-1) - 3^{5x}.2}{(2x-1)^2}$

100.- $y = \ln\dfrac{x}{\sqrt{x^2}-3} = \ln\dfrac{x}{x-3}$

$$y' = \frac{x-3}{x} \cdot \frac{x-3-1.x}{(x-3)^2} = \frac{-3}{x(x-3)}$$

101.- $y = \ln\left(\dfrac{1-e^x}{1-e^x}\right) = \ln 1 = 0$; $y' = 0$