FICHA LÍMITES

1a) $\lim\limits_{x\to 1} \dfrac{x^4-1}{x^3-x^2+x-1} = \dfrac{1^4-1}{1^3-1^2+1-1} = \dfrac{0}{0}$ Ind.

$\lim\limits_{x\to 1} \dfrac{(x+1)\cancel{(x-1)}\cancel{(x^2+1)}}{\cancel{(x-1)}\cancel{(x^2+1)}} = \lim\limits_{x\to 1} (x+1) = 1+1 = \boxed{2}$

$$\begin{array}{r|rrrrr} & 1 & 0 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 \\ -1 & & -1 & 0 & -1 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -1 & 1 & -1 \\ 1 & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

2a) $\lim\limits_{x\to -2} \dfrac{x^2+4}{x^3+8} = \dfrac{(-2)^2+4}{(-2)^3+8} = \dfrac{8}{0} \to$ límites laterales

$\lim\limits_{x\to -2^-} \dfrac{x^2+4}{x^3+8} = -\infty$ $\left[x=-2{,}01 \quad \dfrac{(-2{,}01)^2+4}{(-2{,}01)^3+8} = \dfrac{+}{-} \right]$

$\lim\limits_{x\to -2^+} \dfrac{x^2+4}{x^3+8} = +\infty$ $\left[x=-1{,}99 \quad \dfrac{(-1{,}99)^2+4}{(-1{,}99)^3+8} = \dfrac{+}{+} \right]$

3a) $\lim\limits_{x\to 0} \dfrac{x^4+5x^3-x^2}{3x^4-7x^2+x} = \dfrac{0}{0}$ Ind.

$\lim\limits_{x\to 0} \dfrac{x^{\cancel{2}}(x^2+5x-1)}{\cancel{x}(3x^3-7x+1)} = \dfrac{0\cdot(0+0-1)}{3\cdot 0^3-7\cdot 0+1} = \dfrac{0}{1} = \boxed{0}$

4a) $\lim\limits_{x\to 2} \sqrt{\dfrac{x^2+2x-8}{x^2-x-2}} = \sqrt{\dfrac{2^2+2\cdot 2-8}{2^2-2-2}} = \sqrt{\dfrac{0}{0}}$ Ind.

$x = \dfrac{-2\pm\sqrt{4+32}}{2} \begin{cases} \dfrac{-2+6}{2} = 2 \\ \dfrac{-2-6}{2} = -4 \end{cases}$

$\lim\limits_{x\to 2} \sqrt{\dfrac{\cancel{(x-2)}(x+4)}{\cancel{(x-2)}(x+1)}} = \sqrt{\dfrac{2+4}{2+1}} = \boxed{\sqrt{2}}$

$x = \dfrac{1\pm\sqrt{1+8}}{2} = \begin{cases} \dfrac{1+3}{2} = 2 \\ -1 \end{cases}$

5a) $\lim\limits_{x\to 0} \frac{1-\sqrt{x^2+4}}{x} = \frac{1-\sqrt{0^2+4}}{0} = \frac{-1}{0} \Rightarrow$ límites laterales

$\lim\limits_{x\to 0^-} \frac{1-\sqrt{x^2+4}}{x} = +\infty \quad \left[x=-0{,}01 \quad \frac{1-\sqrt{(-0{,}01)^2+4}}{-0{,}01} = \frac{-}{-} \right]$

$\lim\limits_{x\to 0^+} \frac{1-\sqrt{x^2+4}}{x} = -\infty \quad \left[x=+0{,}01 \quad \frac{1-\sqrt{0{,}01^2+4}}{0{,}01} = \frac{-}{+} \right]$

6a) $\lim\limits_{x\to 1} \frac{x^2-1}{\sqrt{2x-1}-\sqrt{3x-2}} = \frac{1^2-1}{\sqrt{2-1}-\sqrt{3-2}} = \frac{0}{0}$ Ind.

$\lim\limits_{x\to 1} \frac{(x^2-1)(\sqrt{2x-1}+\sqrt{3x-2})}{(\sqrt{2x-1}-\sqrt{3x-2})(\sqrt{2x-1}+\sqrt{3x-2})} =$

$= \lim\limits_{x\to 1} \frac{(x^2-1)(\sqrt{2x-1}+\sqrt{3x-2})}{(\cancel{\sqrt{2x-1}})^{\cancel{2}}-(\cancel{\sqrt{3x-2}})^{\cancel{2}}}$

$= \lim\limits_{x\to 1} \frac{(x+1)\cancel{(x-1)}(\sqrt{2x-1}+\sqrt{3x-2})}{\underset{\cancel{-(x-1)}}{-x+1}} =$

$= -(1+1)(\sqrt{2-1}+\sqrt{3-2}) = -2\cdot 2 = \boxed{-4}$

$$* 7a)\ \lim_{x\to 0} \frac{\sqrt{x+4}-2}{5-\sqrt{x+25}} = \frac{\sqrt{0+4}-2}{5-\sqrt{0+25}} = \frac{0}{0}\ \text{Ind.}$$

$$\lim_{x\to 0} \frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{(5-\sqrt{x+25})(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{(\cancel{\sqrt{x+4}})^{\cancel{2}}-2^2}{(5-\sqrt{x+25})(\sqrt{x+4}+2)} =$$

$$= \lim_{x\to 0} \frac{x\quad(5+\sqrt{x+25})}{(5-\sqrt{x+25})(\sqrt{x+4}+2)(5+\sqrt{x+25})} =$$

$$= \lim_{x\to 0} \frac{x(5+\sqrt{x+25})}{(5^2-\cancel{\sqrt{x+25}}^{\,2})(\sqrt{x+4}+2)} =$$

$$= \lim_{x\to 0} \frac{\cancel{x}\cdot(5+\sqrt{x+25})}{-\cancel{x}(\sqrt{x+4}+2)} = \frac{5+\sqrt{0+25}}{-(\sqrt{0+4}+2)} = \frac{10}{-4} = \boxed{-\frac{5}{2}}$$

$$8a)\ \lim_{x\to 2} \left(\frac{x^2+1}{x+3}\right)^{\frac{x^4}{x^2-4}} = \left(\frac{2^2+1}{2+3}\right)^{\frac{2^4}{2^2-4}} = 1^{\infty}\ \text{Ind.}$$

$$\lim_{x\to 2} \left(1+\frac{x^2+1}{x+3}-1\right)^{\frac{x^4}{x^2-4}} = \lim_{x\to 2} \left(1+\frac{x^2+1}{x+3}-\frac{x+3}{x+3}\right)^{\frac{x^4}{x^2-4}} =$$

$$= \lim_{x\to 2} \left(1+\frac{x^2-x-2}{x+3}\right)^{\frac{x^4}{x^2-4}} = \lim_{x\to 2} \left(1+\frac{1}{\frac{x+3}{x^2-x-2}}\right)^{\frac{x+3}{x^2-x-2}\cdot\frac{x^2-x-2}{x+3}\cdot\frac{x^4}{x^2-4}} =$$

$$= e^{\lim_{x\to 2} \frac{(x^2-x-2)x^4}{(x+3)(x^2-4)}} = e^{\lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)x^4}{(x+3)\cancel{(x-2)}(x+2)}} = e^{\frac{(2+1)\cdot 2^4}{(2+3)(2+2)}} = \boxed{e^{12/5}}$$

9a) $\lim_{x\to -3}\left(\frac{x^2-5}{x^2+1}\right)^{\frac{x^2}{x+4}} = \left(\frac{(-3)^2-5}{(-3)^2+1}\right)^{\frac{(-3)^2}{-3+4}} = \left(\frac{4}{10}\right)^9 = \boxed{\left(\frac{2}{5}\right)^9}$

* 10a) $\lim_{x\to 5}\left(\frac{x+10}{x^2-10}\right)^{\frac{x+2}{2x-10}} = \left(\frac{5+10}{5^2-10}\right)^{\frac{5+2}{10-10}} = \left(\frac{15}{15}\right)^{\frac{7}{0}} = 1^{\infty}$ Ind.

$$\lim_{x\to 5}\left(1+\frac{x+10}{x^2-10}-1\right)^{\frac{x+2}{2x-10}} = \lim_{x\to 5}\left(1+\frac{x+10}{x^2-10}-\frac{x^2-10}{x^2-10}\right)^{\frac{x+2}{2x-10}} =$$

$$= \lim_{x\to 5}\left(1+\frac{-x^2+x+20}{x^2-10}\right)^{\frac{x+2}{2x-10}} = \lim_{x\to 5}\left(1+\frac{1}{\frac{x^2-10}{-x^2+x+20}}\right)^{\frac{x^2-10}{-x^2+x+20}\cdot\frac{-x^2+x+20}{x^2-10}\cdot\frac{x+2}{2x-10}} =$$

$$= \left[\lim_{x\to 5}\left(1+\frac{1}{\frac{x^2-10}{-x^2+x+20}}\right)^{\frac{x^2-10}{-x^2+x+20}}\right]^{\lim_{x\to 5}\frac{-x^2+x+20}{x^2-10}\cdot\frac{x+2}{2x-10}} =$$

$$= e^{\lim_{x\to 5}\frac{-(x+4)\cancel{(x-5)}\cdot(x+2)}{(x^2-10)\cdot 2\cdot\cancel{(x-5)}}} = e^{\frac{-(5+4)\cdot(5+2)}{(5^2-10)\cdot 2}} = e^{-\frac{9\cdot 7}{15\cdot 2}} = \boxed{e^{\frac{-21}{10}}}$$

$$-x^2+x+20=0 \rightarrow x=\frac{-1\pm\sqrt{1+80}}{-2} = \begin{cases}\frac{-1+9}{-2}=-4\\ \frac{-1-9}{-2}=5\end{cases}$$

$$-x^2+x+20=-(x+4)(x-5)$$

1b) $\lim_{x\to+\infty} \frac{x^4-1}{x^3-x^2+x-1} = \lim_{x\to+\infty} \frac{\cancel{x^4}}{\cancel{x^3}} = \lim_{x\to+\infty} x = \boxed{+\infty}$

2b) $\lim_{x\to-\infty} \frac{x^2+4}{x^3+8} = \lim_{x\to-\infty} \frac{\cancel{x^2}}{\cancel{x^3}} = \lim_{x\to-\infty} \frac{1}{x} = \frac{1}{-\infty} = \boxed{0}$

3b) $\lim_{x\to+\infty} \frac{x^4+5x^3-x^2}{3x^4-7x^2+x} = \lim_{x\to+\infty} \frac{\cancel{x^4}}{3\cancel{x^4}} = \boxed{\frac{1}{3}}$

4b) $\lim_{x\to-\infty} \sqrt{\frac{x^2-6x+8}{x^2-x-2}} = \lim_{x\to-\infty} \sqrt{\frac{\cancel{x^2}}{\cancel{x^2}}} = \boxed{1}$

5b) $\lim_{x\to-\infty} \frac{1-\sqrt{x^2+4}}{x} = \lim_{x\to-\infty} \frac{-\sqrt{\cancel{x^2}}}{\cancel{x}} = \boxed{-1}$

6b) $\lim_{x\to+\infty} \frac{x^2-1}{\sqrt{2x-1}-\sqrt{3x-2}} = \lim_{x\to+\infty} \frac{x^2}{\sqrt{2x}-\sqrt{3x}} =$

$= \lim_{x\to+\infty} \frac{\cancel{x^2}^{3/2}}{(\sqrt{2}-\sqrt{3})\cancel{\sqrt{x}}} = -(+\infty)^{3/2} = \boxed{-\infty}$

Otra opción:

$\lim_{x\to+\infty} \frac{(x^2-1)(\sqrt{2x-1}+\sqrt{3x-2})}{(\sqrt{2x-1}-\sqrt{3x-2})(\sqrt{2x-1}+\sqrt{3x-2})} =$

$= \lim_{x\to+\infty} \frac{(x^2-1)(\sqrt{2x-1}+\sqrt{3x-2})}{(\cancel{\sqrt{2x-1}})^2-(\cancel{\sqrt{3x-2}})^2} = \lim_{x\to+\infty} \frac{(x+1)\cancel{(x-1)}(\sqrt{2x-1}+\sqrt{3x-2})}{\substack{-x+1 \\ -\cancel{(x-1)}}} = \frac{+\infty}{-1} = \boxed{-\infty}$

7b) $$\lim_{x\to+\infty} \frac{\sqrt{x+4}-2}{5-\sqrt{x+25}} = \lim_{x\to+\infty} \frac{\cancel{\sqrt{x}}}{-\cancel{\sqrt{x}}} = \boxed{-1}$$

8b) $$\lim_{x\to+\infty} \left(\frac{x^2+1}{x+3}\right)^{\frac{x^4}{x^2-4}} = (+\infty)^{+\infty} = \boxed{+\infty}$$

9b) $$\lim_{x\to+\infty} \left(\frac{x^2-5}{x^2+1}\right)^{\frac{x^2}{x+4}} = 1^{+\infty} \text{ Ind.}$$

$$\lim_{x\to+\infty}\left(1+\frac{x^2-5}{x^2+1}-1\right)^{\frac{x^2}{x+4}} = \lim_{x\to+\infty}\left(1+\frac{x^2-5}{x^2+1}-\frac{x^2+1}{x^2+1}\right)^{\frac{x^2}{x+4}} =$$

$$= \lim_{x\to+\infty}\left(1+\frac{-6}{x^2+1}\right)^{\frac{x^2}{x+4}} = \lim_{x\to+\infty}\left(1+\frac{1}{\frac{x^2+1}{-6}}\right)^{\frac{x^2+1}{-6}\cdot\frac{-6}{x^2+1}\cdot\frac{x^2}{x+4}}$$

$$= e^{\lim_{x\to+\infty}\frac{-6\cancel{x^2}}{x^{\cancel{3}}+\dots}} = e^{\frac{-6}{+\infty}} = e^0 = \boxed{1}$$

10b) $$\lim_{x\to+\infty}\left(\frac{x+10}{x^2-10}\right)^{\frac{x+2}{2x-10}} = \left(\lim_{x\to+\infty}\frac{x+10}{x^2-10}\right)^{\lim_{x\to+\infty}\frac{x+2}{2x-10}} = 0^{1/2} = \boxed{0}$$

$$\parallel$$

$$\left(\lim_{x\to+\infty}\frac{\cancel{x}}{x^{\cancel{2}}}\right)^{\lim_{x\to+\infty}\frac{\cancel{x}}{2\cancel{x}}} = \left(\frac{1}{+\infty}\right)^{1/2} = 0^{1/2} = \boxed{0}$$