

Ejercicios REFUERZO ECUACIONES Y SISTEMAS

1. ECUACIONES CON FRACCIONES

a) $\frac{4x^2-4x}{3} - x = x^2 - \frac{3x+4}{3}$; $\frac{4x^2-4x-3x}{3} = \frac{3x^2-(3x+4)}{3}$
 $4x^2-4x-3x = 3x^2-3x-4$; $x^2-4x+4=0$; $(x-2)^2=0$

SOL: $x=2$ (DOBLE)

b) $\frac{4x}{x+2} + \frac{x}{x-2} = \frac{14}{3}$; $\frac{4x(x-2)+x(x+2)}{(x+2)(x-2)} = \frac{14}{3}$; $3[4x^2-8x+x^2+2x] = 14 \cdot (x^2-4)$

$3(5x^2-6x) = 14x^2-56$; $15x^2-18x = 14x^2-56$; $x^2-18x+56=0$

$x = \frac{18 \pm 10}{2} = \begin{matrix} \rightarrow 14 \\ \rightarrow 4 \end{matrix}$

c) $\frac{x-1}{x^2+2x} - \frac{2}{x^2-2x} + \frac{-x}{x^2-4} = 0$;
 $x(x+2) \quad x(x-2) \quad (x+2)(x-2)$

$\frac{(x-1)(x-2) - 2(x+2) - x \cdot x}{x(x+2)(x-2)} = 0$; $x^2-2x-x+2-2x-4-x^2=0$;

$-x-2=0$; $x=-2$ NO VÁLIDA, ANULA DENOMINADOR.

d) $\frac{1+x}{x-3} - \frac{x}{x+2} = \frac{3x+5x^2}{x^2-x-6}$;
 $(x-3) \cdot (x+2)$

$\frac{(1+x) \cdot (x+2) - x \cdot (x-3)}{(x-3)(x+2)} = \frac{3x+5x^2}{(x-3)(x+2)}$; $x+2+x+2x-x^2+3x = 3x+5x^2$

$2+3x = 5x^2$; $5x^2-3x-2=0$; $x = \frac{3 \pm \sqrt{9+40}}{10} = \frac{3 \pm 7}{10} = \begin{matrix} \rightarrow 1 \\ \rightarrow -\frac{2}{5} \end{matrix}$

e) $\frac{x-2}{x-1} = \frac{x^2}{(x-1)(x-2)} - \frac{x-1}{2-x}$;
 CAMBIO: $(2-x) = -(x-2)$

$\frac{(x-2)^2}{(x-1)(x-2)} = \frac{x^2 + (x-1)(x-1)}{(x-1)(x-2)}$; $x^2-4x+4 = x^2+x^2-2x+1$;

$-4x+4 = x^2-2x+1$; $x^2+2x-3=0$; $x = \frac{-2 \pm \sqrt{4+12}}{2} = \begin{matrix} \rightarrow -3 \\ \rightarrow 1 \end{matrix}$
 NO VÁLIDA, ANULA DENOMINADOR

f) $\frac{(x-1)^2 - (x-2)^2}{x^2-1} + \frac{x+1}{x-1} = \frac{x-1}{x+1}$;

PUEDO SIMPLIFICAR

$\frac{(x-1)^2 - (x-2)^2 + (x+1)^2}{(x-1)(x+1)} = \frac{(x-1)^2}{(x-1)(x+1)}$; $-x^2+4x-4+x^2+2x+1=0$

$6x-3=0$; $6x=3$; $x=\frac{1}{2}$

2. ECUACIONES CON RADICALES

a) $(\sqrt{3x+16})^2 = (2x-1)^2$; $3x+16 = 4x^2 - 4x + 1$
 $4x^2 - 7x - 15 = 0$; $x = \frac{7 \pm \sqrt{49 + 240}}{8} = \frac{7 \pm 17}{8} = \begin{cases} -\frac{5}{4} \\ 3 \end{cases}$ No válido
 Comprobamos: $\sqrt{3 \cdot (-\frac{5}{4}) + 16} = 2 \cdot (-\frac{5}{4}) - 1$; $\sqrt{49/4} = -7/2$

b) $\sqrt{x+5} - x = 3$;
 $(\sqrt{x+5})^2 = (3+x)^2$; $x+5 = 9 + 6x + x^2$; $x^2 + 5x + 4 = 0$; $x = \frac{-5 \pm \sqrt{9}}{2} = \begin{cases} -1 \\ -4 \end{cases}$ No válido
 Comprobamos: $\sqrt{-4+5} - (-4) = 3$; $1 + 4 \neq 3$
 $\sqrt{-1+5} = 3 - 1$; $2 = 2$ ✓

c) $\sqrt{x+9} + \sqrt{x-3} = 6$
 $(\sqrt{x+9})^2 = (6 - \sqrt{x-3})^2$; $x+9 = 36 - 12\sqrt{x-3} + x-3$; Simplificamos
 $12\sqrt{x-3} = 36 + x - 3 - x - 9$; $12\sqrt{x-3} = 24$; $(\sqrt{x-3})^2 = (2)^2$
 $x-3 = 4$; $x = 7$
 Comprobamos: $\sqrt{7+9} + \sqrt{7-3} = 6$; $4 + 2 = 6$ ✓

d) $2x - 5 + 2\sqrt{x^2 - 5x} = 25$; $2\sqrt{x^2 - 5x} = 30 - 2x$ Simplificamos
 $(\sqrt{x^2 - 5x})^2 = (15 - x)^2$; $x^2 - 5x = 225 - 30x + x^2$
 $25x = 225$; $x = 9$
 Comprobamos: $2 \cdot 9 - 5 + 2\sqrt{81 - 45} = 25$
 $18 - 5 + 2 \cdot 6 = 25$ ✓

e) $(\sqrt{2x + \sqrt{4x-3}})^2 = (3)^2$; $2x + \sqrt{4x-3} = 9$;
 $(\sqrt{4x-3})^2 = (9-2x)^2$; $4x-3 = 81 - 36x + 4x^2$
 $4x^2 - 40x + 84 = 0$
Simplificamos $\rightarrow x^2 - 10x + 21 = 0$; $x = \frac{10 \pm \sqrt{100 - 84}}{2} = \frac{10 \pm 4}{2} = \begin{cases} 7 \\ 3 \end{cases}$ No válido
 Comprobamos: $\sqrt{2 \cdot 7 + \sqrt{28-3}} = 3$; $\sqrt{14+5} \neq 3$
 $\sqrt{6 + \sqrt{12-3}} = 3$; $\sqrt{6+3} = 3$

3. ECUACIONES CON LOGARÍTMOS

a) $\log(x-3)^2 + \log 4 = \log x$; ~~$\log(x-3)^2 \cdot 4 = \log x$~~ ;

$$4 \cdot (x^2 - 6x + 9) = x ; 4x^2 - 25x + 36 = 0 ;$$

$$x = \frac{25 \pm \sqrt{49}}{8} = \begin{cases} 4 \\ 9/4 \end{cases} \quad \text{Comprobamos: } \log(4-3)^2 + \log 4 = \log 4 \checkmark$$

ENUNCIADO ORIGINAL. $\log(9/4-3)^2 + \log 4 = \log 9/4 \checkmark$

b) $\log(x+1) - \log(3x-2) = 1$;

~~$\log \frac{x+1}{3x-2} = \log 10$~~ ; $x+1 = 10 \cdot (3x-2)$; $x+1 = 30x-20$;

$$29x = 21 ; x = \frac{21}{29}$$

Comprobamos: $\log\left(\frac{21}{29}+1\right) - \log\left(3 \cdot \frac{21}{29} - 2\right) = 1$;

$$\log \frac{50}{29} - \log \frac{5}{29} = 1 \checkmark$$

4. ECUACIONES EXPONENCIALES

a) $\frac{3^{x^2-x+1}}{3^{x+1}} = \frac{1}{3}$; $3^{x^2-x+x-x-1} = 3^{-1}$; $x^2 - 2x = -1$; $x^2 - 2x + 1 = 0$

$$(x-1)^2 = 0 ; x = 1$$

b) $2^{x+2} + 2^x - 5 = 0$; Cambio $2^x = t$ $2^x \cdot 2^2 + 2^x - 5 = 0$

$$4 \cdot t + t - 5 = 0 ; 5t - 5 = 0 ; t = 1 ; 2^x = 1 ; x = 0$$

c) ~~2^{x-1}~~ $+ 2^{x+1} - 3 \cdot 2^x + 4 = 0$ Cambio $2^x = t$

$$2^x \cdot 2^{-1} + 2^x \cdot 2 - 3 \cdot 2^x + 4 = 0$$

$$\frac{1}{2}t + 2t - 3t + 4 = 0 ; -\frac{1}{2}t = -4 ; t = 8$$

$$2^x = 8 ; 2^x = 2^3 ; x = 3$$

5. SISTEMAS USANDO GAUSS

a)
$$\begin{cases} x - y + 2z = 6 \\ 2x + y - 3z = -5 \\ -3x + 6y + 4z = -1 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 2 & 1 & -3 & -5 \\ -3 & 6 & 4 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 3 & -7 & -17 \\ 0 & 3 & 10 & 17 \end{array} \right) \sim$$

$F_2 - 2 \cdot F_1$ $F_3 - F_2$

$F_3 + 3 \cdot F_1$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 3 & -7 & -17 \\ 0 & 0 & 17 & 34 \end{array} \right) \sim$$

$x + 1 + 4 = 6 ; x = 1$
 $3y - 14 = -17 ; 3y = -3 ; y = -1$
 $17z = 34 ; z = 2$

SISTEMA COMPATIBLE DETERMINADO.

b)
$$\begin{cases} x + y - z = 3 \\ 3x + 4y - z = 5 \\ x + 2y + 3z = -1 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 3 & 4 & -1 & 5 \\ 1 & 2 & 3 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 2 & -4 \\ 0 & 1 & 4 & -4 \end{array} \right) \sim$$

$F_2 - 3 \cdot F_1$ $F_3 - F_2$

$F_3 - F_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 2 & 0 \end{array} \right) \sim$$

$x - 4 = 3 ; x = 7$
 $y + 0 = -4 ; y = -4$
 $2z = 0 ; z = 0$

SIST. COMP. DETERMINADO

c)
$$\begin{cases} x - 2y + 3z = 0 \\ 2x - 3z = 0 \\ 3x - 2y = 0 \end{cases} \quad \text{SISTEMA HOMOGÊNEO} \Rightarrow \text{SEMPRE COMPATIBLE!!}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & 0 & -3 & 0 \\ 3 & -2 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 4 & -9 & 0 \\ 0 & 4 & -9 & 0 \end{array} \right) \sim$$

$F_2 - 2 \cdot F_1$ $F_3 - 3 \cdot F_1$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 4 & -9 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

SIST. COMP. INDETERMINADO

sol: $(\frac{3}{2}\alpha, \frac{9}{4}\alpha, \alpha)$

$z = \alpha$
 $4y - 9\alpha = 0 ; y = \frac{9}{4}\alpha$
 $x = \frac{9}{2}\alpha - 2\alpha$

$$d) \begin{cases} x - 3y - z = 1 \\ 2x - 6y - 2z = 1 \\ 3x - 9y - 3z = 1 \end{cases} \quad \left(\begin{array}{cccc} 1 & -3 & -1 & 1 \\ 2 & -6 & -2 & 1 \\ 3 & -9 & -3 & 1 \end{array} \right) \sim \left(\begin{array}{cccc} 1 & -3 & -1 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -2 \end{array} \right)$$

$F_2 - 2F_1$
 $F_3 - 3F_1$

$$0 = -2 \text{ IMPOSIBLE}$$

SISTEMA INCOMPATIBLE

6. SISTEMAS NO LINEALES

$$a) \begin{cases} 2(x+2y)^2 - (2x+y)^2 = -1 \\ x - y = 5 \end{cases} \quad \begin{cases} 2(5+y+2y)^2 - (10+2y+y)^2 = -1 \\ x = 5+y \end{cases}$$

$$2.(5+3y)^2 - (10+3y)^2 = -1$$

$$2.(25 + 30y + 9y^2) - (100 + 60y + 9y^2) = -1$$

$$50 + 60y + 18y^2 - 100 - 60y - 9y^2 = -1; 9y^2 = 49; y = \pm \frac{7}{3}$$

$$y = \frac{7}{3} \quad x = \frac{22}{3}$$

$$y = -\frac{7}{3} \quad x = \frac{8}{3}$$

$$b) \begin{cases} x^2 + y^2 + 9x + 14 = 0 \\ y^2 = 16 + 4x \end{cases} \quad \begin{cases} x^2 + 16 + 4x + 9x + 14 = 0 \\ y^2 = 16 + 4x \end{cases}$$

$$x^2 + 13x + 30 = 0; \quad x = \frac{-13 \pm \sqrt{49}}{2} = \frac{-13 \pm 7}{2} = \begin{matrix} -10 \\ -3 \end{matrix}$$

$$x = -10; \quad y^2 = 16 + 4(-10) \text{ IMPOSIBLE}$$

$$x = -3; \quad y^2 = 16 + 4(-3) = 16 - 12 = 4 \quad y = \pm 2$$

$$\text{Sol: } x = -3 \quad y = 2$$

$$x = -3 \quad y = -2$$

7. SISTEMAS CON LOGARITMOS Y EXPONENCIALES

a)
$$\begin{cases} \log_2(x) + \log_2(y) = 2 \\ \log_2(2x) - \log_2(y) = 1 \end{cases} \quad \begin{cases} \cancel{\log_2 x \cdot y = \log_2 2^2} \\ \cancel{\log_2 \frac{2x}{y} = \log_2 2} \end{cases} \quad \begin{cases} x \cdot y = 4 \\ \frac{2x}{y} = 2 \end{cases}$$

$$\begin{cases} x \cdot y = 4 \\ x = y \end{cases} \quad \begin{cases} x^2 = 4 \\ x = y \end{cases} \quad \begin{cases} x = \pm 2 \\ y = \pm 2 \end{cases} \quad \text{solo válida } \begin{cases} x = 2 \\ y = 2 \end{cases}$$

b)
$$\begin{cases} \log_3(3x) - \log_3(y-2) = 1 \\ \log_5(y) + \log_5(x-2) = 1 \end{cases} \quad \begin{cases} \cancel{\log_3 \frac{3x}{y-2} = \log_3 3} \\ \cancel{\log_5 y \cdot (x-2) = \log_5 5} \end{cases}$$

$$\begin{cases} 3x = 3(y-2) \\ y(x-2) = 5 \end{cases} \quad \begin{cases} x = y-2 \\ y \cdot (y-2-2) = 5 \end{cases} \quad y^2 - 4y - 5 = 0; y = \frac{4 \pm 6}{2} = 5, -1$$

$\log_5(-1)$ no existe.

sol: $y = 5, x = 3$

c)
$$\begin{cases} 2 \cdot 3^{x+1} - 3^{y-1} = 15 \\ 5 \cdot 3^{x+2} - 3^{y-1} = 108 \end{cases} \quad \text{Cambio: } 3^x = t; 3^y = s \quad \begin{cases} 2 \cdot 3^x \cdot 3 - 3^y \cdot 3^{-1} = 15 \\ 5 \cdot 3^x \cdot 3^2 - 3^y \cdot 3^{-1} = 108 \end{cases}$$

$$\begin{cases} 6t - \frac{1}{3}s = 15 \\ 45t - \frac{1}{3}s = 108 \end{cases} \quad 39t = 93, t = \frac{31}{13}; 3^x = \frac{31}{13}; \log 3^x = \log \frac{31}{13}$$

$x = \frac{\log 31/13}{\log 3} \quad s = -\frac{27}{13}$ IMPOSIBLE!

Restando

d)
$$\begin{cases} 3 \cdot 2^x - 2 \cdot 3^y = -42 \\ 5 \cdot 2^{x+1} - 4 \cdot 3^{y-1} = 4 \end{cases} \quad \text{Cambio: } 2^x = t, 3^y = s$$

$$\begin{cases} 3t - 2s = -42 \\ 5t \cdot 2 - 4s \cdot \frac{1}{3} = 4 \end{cases} \quad \begin{cases} 3t - 2s = -42 \\ 10t - \frac{4}{3}s = 4 \end{cases} \quad \begin{cases} 3t - 2s = -42 \\ 30t - 4s = 12 \end{cases}$$

$-30t + 30s = 420; 26s = 432; s = \frac{216}{13}; 2^x = \frac{216}{13};$

$\begin{cases} 30t - 4s = 12 \\ -30t + 30s = 420 \end{cases} \quad x = \frac{\log 216/13}{\log 2} = 4.05 \quad t = -38/13$ IMPOSIBLE!

e)
$$\begin{cases} \frac{\log x}{\log y} = \frac{1}{2} \\ \log x^2 + \log y = 4 \end{cases} \quad \begin{cases} 2 \log x = \log y \\ \log x^2 \cdot y = \log 10^4 \end{cases} \quad \begin{cases} x^2 = y \\ x^2 \cdot y = 10^4 \end{cases} \quad \begin{cases} x^2 = y \\ y^2 = 10^4; y = \pm 10^2 \end{cases}$$

sol: $y = 10^2; x = 10$