

FICHAS nº 6.- (2ª PARTE)

nº 8.-

$$a) \begin{cases} x - y - z = -10 \\ x + 2y + z = 11 \\ 2x - y + z = -8 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & -1 & -10 \\ 1 & 2 & 1 & 11 \\ 2 & -1 & 1 & -8 \end{array} \right) \Rightarrow$$

 $2^{\text{da}} F - 1^{\text{da}} F$
 $3^{\text{da}} F - 2 \cdot 1^{\text{da}} F.$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -10 \\ 0 & 3 & 2 & 21 \\ 0 & 1 & 3 & 12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -1 & -10 \\ 0 & 1 & 3 & 12 \\ 0 & 3 & 2 & 21 \end{array} \right) \Rightarrow$$

 $2^{\text{da}} F \leftrightarrow 3^{\text{da}} F$
 $3^{\text{da}} F - 3 \cdot 2^{\text{da}} F$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -10 \\ 0 & 1 & 3 & 12 \\ 0 & 0 & -7 & -15 \end{array} \right) \Rightarrow \begin{cases} x - y - z = -10 \\ y + 3z = 12 \\ -7z = -15 \end{cases} \Rightarrow z = \frac{15}{7};$$

$$y + 3 \cdot \frac{15}{7} = 12; y = \frac{39}{7}$$

$$x - \frac{39}{7} - \frac{15}{7} = -10; x = -\frac{15}{7}$$

SIST. COMP. DET.

SE CORTAN EN UN PUNTO

b)
$$\begin{cases} x + y + z = 3 \\ 2x - y + z = 2 \\ x - y + z = 1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & -1 & 1 & 2 \\ 1 & -1 & 1 & 1 \end{array} \right) \Rightarrow$$

 $2^{\text{da}} F - 2 \cdot 1^{\text{da}} F$
 $3^{\text{da}} F - 1^{\text{da}} F$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & 0 & -2 \end{array} \right) \Rightarrow \begin{cases} x + y + z = 3 \\ -3y - z = -4 \\ -2y = -2 \end{cases} \Rightarrow \begin{cases} y = 1 \\ z = 1 \\ x = 1 \end{cases}$$

SIST. COMP. DET.

nº 9.-
$$\begin{cases} x^2 + x \cdot y + y^2 = 21 \\ x + y = 1 \end{cases} \Rightarrow$$

$$y = 1 - x$$

$$x^2 + x(1-x) + (1-x)^2 - 21 = 0$$

$$x^2 + x - x^2 + 1 - 2x + x^2 - 21 = 0$$

$$x^2 - x - 20 = 0$$

$$x = \frac{1 \pm \sqrt{1+80}}{2} = \begin{cases} x=5; y=-4 \\ x=-4; y=5 \end{cases}$$

$$b) \begin{cases} x-y=27 \\ \log x - 1 = \log y \end{cases} \Rightarrow \begin{cases} x-y=27 \\ \log x - \log y = \log 10 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x-y=27 \\ \frac{x}{y}=10 \end{cases} \Rightarrow x=10y ; 10y-y=27 \Rightarrow \begin{matrix} y=3 \\ x=30 \end{matrix}$$

$$c) \begin{cases} \log(x^2+y) - \log(x-2y)=1 \\ 5^{x+1}=25^{y+1} \end{cases} \Rightarrow \begin{cases} \log \frac{x^2+y}{x-2y} = \log 10 \\ 5^{x+1}=5^{2(y+1)} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{x^2+y}{x-2y}=10 \\ x+1=2y+2 \end{cases} \Rightarrow \begin{cases} x^2+y-10x+20y=0 \\ x=2y+1 \end{cases}$$

$$(2y+1)^2 + y - 10(2y+1) + 20y = 0$$

$$4y^2 + 4y + 1 + y - 20y - 10 + 20y = 0$$

$$4y^2 + 5y - 9 = 0$$

$$y = \frac{-5 \pm \sqrt{25+144}}{8} = \frac{-5 \pm 13}{8} = \begin{matrix} -\frac{9}{4} \\ 1 \end{matrix}$$

Posibles Soluciones:

$$y = -\frac{9}{4} ; x = -\frac{7}{2} \quad \text{carpetas:}$$

$$\log\left(\frac{49}{4} - \frac{9}{4}\right) - \log\left(-\frac{7}{2} + \frac{9}{2}\right) = 1 \quad \checkmark$$

$$y = 1 ; x = 3 \quad \log(9+1) - \log(3-2) = 1 \quad \checkmark$$

nº 10.- a)
$$\begin{cases} \log x + \log y = 3 \\ \log x - \log y = -1 \end{cases} \Rightarrow \begin{cases} \log x \cdot y = \log 10^3 \\ \log \frac{x}{y} = \log 10^{-1} \Rightarrow \end{cases}$$

$$\Rightarrow \begin{cases} x \cdot y = 10^3 \\ \frac{x}{y} = 10^{-1} \end{cases} \Rightarrow \begin{aligned} x \cdot y &= 10^3 \\ x &= 10^{-1} \cdot y \Rightarrow 10^{-1} \cdot y \cdot y = 10^3 \\ y^2 &= 10^4 \quad y = \pm 10^2 \end{aligned}$$

Solo vale $y = 100$; $x = 10$

OTRA FORMA: POR REDUCCIÓN

Sumando $2 \log x = 2 \Rightarrow \log x = 1 \Rightarrow x = 10$

Restando $2 \log y = 4 \Rightarrow \log y = 2 \Rightarrow y = 100$

b)
$$\begin{cases} \log_2 x + 3 \log_2 y = 5 \\ \log_2 \frac{x^2}{y} = 3 \end{cases} \Rightarrow \begin{cases} \log_2 x + \log_2 y^3 = \log_2 2^5 \\ \log_2 \frac{x^2}{y} = \log_2 2^3 \end{cases}$$

$$\Rightarrow \begin{cases} x \cdot y^3 = 32 \\ \frac{x^2}{y} = 8 \end{cases} \Rightarrow \begin{aligned} y &= \frac{x^2}{8} \\ x \cdot \left(\frac{x^2}{8} \right)^3 &= 32 \end{aligned} \Rightarrow$$

$$x \cdot \frac{x^6}{2^9} = 2^5 \Rightarrow x^7 = 2^{14} \Rightarrow x = 2^2 ; x = 4$$

$$y = \frac{16}{8} ; y = 2$$

c)
$$\begin{cases} \log (x^2 y) = 2 \\ \log x = 6 + \log y^2 \end{cases} \Rightarrow \begin{cases} \log x^2 \cdot y = \log 10^2 \\ \log x = \log 10^6 \cdot y^2 \end{cases}$$

$$\begin{cases} x^2 y = 10^2 \\ x = 10^6 \cdot y^2 \end{cases} \Rightarrow \begin{aligned} 10^{12} \cdot y^4 \cdot y &= 10^2 ; y^5 = 10^{-10} ; y = 10^{-2} \\ x &= 10^6 \cdot 10^{-4} ; x = 10^2 \end{aligned}$$

$$d) \begin{cases} x^2 - y^2 = 11 \\ \log x - \log y = 1 \end{cases} \Rightarrow \begin{cases} x^2 - y^2 = 11 \\ \log \frac{x}{y} = \log 10 \end{cases} \Rightarrow \begin{cases} x^2 - y^2 = 11 \\ x = 10y \end{cases}$$

$$100y^2 - y^2 = 11 \Rightarrow y^2 = \frac{1}{9} \Rightarrow y = \pm \frac{1}{3} \text{ solo es válida}$$

$$y = +\frac{1}{3} \quad x = \frac{10}{3}$$

$$e) \begin{cases} x - y = 25 \\ \log y = \log x - 1 \end{cases} \Rightarrow \begin{cases} x - y = 25 \\ \log y = \log \frac{x}{10} \end{cases} \Rightarrow y = \frac{x}{10}$$

$$x - \frac{x}{10} = 25 \quad ; \quad 9x = 250 \quad ; \quad x = \frac{250}{9} \quad y = \frac{25}{9}$$

$$f) \begin{cases} \ln x - \ln y = 2 \\ \ln x + \ln y = 4 \end{cases}$$

$$\text{Sumando: } 2 \ln x = 6 \quad ; \quad \ln x = 3 \Rightarrow e^3 = x$$

$$\text{Restando: } 2 \ln y = 2 \quad ; \quad \ln y = 1 \Rightarrow e = y$$

no !! a)
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 1 & 0 & -1 & 6 \\ 1 & -2 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 0 & -1 & -2 & -12 \\ 0 & -3 & 0 & -18 \end{array} \right)$$

2° F - 1° F
3° F - 1° F

$$\Rightarrow \begin{cases} x + y + z = 18 \\ -y - 2z = -12 \\ -3y = -18 \end{cases} \Rightarrow \begin{aligned} y &= 6 \\ -6 - 2z &= -12 \quad ; \quad z = 3 \\ x + 6 + 3 &= 18 \quad ; \quad x = 9 \end{aligned}$$

$$\text{Sol: } (9, 6, 3) \quad \text{S. Comp. Det.}$$

$$b) \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 5 & 11 \\ 1 & -5 & 6 & 29 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 7 \\ 0 & -6 & 5 & 27 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 23 & 69 \end{array} \right)$$

$2^{\text{da}} F - 2 \cdot 1^{\text{da}} F$ $3^{\text{da}} F + 6 \cdot 2^{\text{da}} F$

$3^{\text{da}} F - 1^{\text{da}} F$

$$\Rightarrow \begin{cases} x+y+z=2 \\ y+3z=7 \\ 23z=69 \end{cases} \Rightarrow \begin{cases} z=3 \\ y+9=7 \Rightarrow y=-2 \\ x-2+3=2 \Rightarrow x=1 \end{cases} \quad \text{Sol: } (1, -2, 3)$$

nº 12.- a) $\left(\begin{array}{ccc|c} 5 & -4 & 3 & 9 \\ 2 & 1 & -2 & 1 \\ 4 & 3 & 4 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 2 & 1 & -2 & 1 \\ 5 & -4 & 3 & 9 \\ 4 & 3 & 4 & 1 \end{array} \right) \sim$

Voy a usar filas y columnas

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ -4 & 5 & 3 & 9 \\ 3 & 4 & 4 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 13 & -5 & 13 \\ 0 & -2 & 10 & -2 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & -1 & 5 & -1 \\ 0 & 13 & -5 & 13 \end{array} \right)$$

$2^{\text{da}} F + 4 \cdot 1^{\text{da}} F$ $3^{\text{da}} F - 3 \cdot 1^{\text{da}} F$ $\frac{3^{\text{da}} F}{2}$ y la nueva

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & -1 & 5 & -1 \\ 0 & 0 & 60 & 0 \end{array} \right) \Rightarrow \begin{cases} y + 2x - 2z = 1 \\ -x + 5z = -1 \\ 60z = 0 \end{cases} \Rightarrow \begin{cases} z=0 \\ y=-1 \\ x=1 \end{cases}$$

¡¡¡ojo!!!

$$3^{\text{da}} F + 13 \cdot 2^{\text{da}} F$$

$$b) \left(\begin{array}{ccc|c} 2 & -5 & 4 & -1 \\ 4 & -5 & 4 & 3 \\ 5 & 0 & -3 & 13 \end{array} \right) \sim \left(\begin{array}{ccc|c} -5 & 4 & 2 & -1 \\ -5 & 4 & 4 & 3 \\ 0 & -3 & 5 & 13 \end{array} \right) \sim \left(\begin{array}{ccc|c} -5 & 4 & 2 & -1 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 5 & 13 \end{array} \right)$$

$2^{\text{da}} - 1^{\text{da}}$

$$\Rightarrow \begin{cases} -5y + 4z + 2x = -1 \\ 2x = 4 \\ -3z + 5x = 13 \end{cases} \Rightarrow \begin{cases} x=2 \\ y=\frac{1}{5} \\ z=-1 \end{cases}$$

nº 13.- a)
$$\begin{pmatrix} 1 & 1 & -2 & | & 9 \\ 2 & -1 & 4 & | & 4 \\ 2 & -1 & 6 & | & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -2 & | & 9 \\ 0 & -3 & 8 & | & -14 \\ 0 & -3 & 10 & | & -19 \end{pmatrix} \sim$$

$2^{\text{da}} F - 2 \cdot 1^{\text{da}} F$ $3^{\text{da}} F - 2 \cdot 1^{\text{da}} F$

$3^{\text{da}} F - 2 \cdot 1^{\text{da}} F$

$$\begin{pmatrix} 1 & 1 & -2 & | & 9 \\ 0 & -3 & 8 & | & -14 \\ 0 & 0 & 2 & | & -5 \end{pmatrix} \sim \begin{cases} x+y-2z=9 \\ -3y+8z=-14 \\ 2z=-5 \end{cases} \Rightarrow \begin{cases} z = -\frac{5}{2} \\ y = -2 \\ x = 6 \end{cases}$$

b)
$$\begin{pmatrix} 2 & -3 & 1 & | & 0 \\ 3 & 6 & -2 & | & 0 \\ 4 & 1 & -1 & | & 0 \end{pmatrix}$$

LO SE LLAMA SISTEMA HOMOGÉNEO, Y SIEMPRE ES COMPATIBLE, PORQUE POR LO MENOS SIEMPRE VALE LA SOLUCIÓN $x=0$ $y=0$ $z=0$ SOLUCIÓN TRIVIAL.

(2) (x) (y) TRIVIAL.

$$\begin{pmatrix} 1 & 2 & -3 & | & 0 \\ -2 & 3 & 6 & | & 0 \\ -1 & 4 & 1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -3 & | & 0 \\ 0 & 7 & 0 & | & 0 \\ 0 & 6 & -2 & | & 0 \end{pmatrix} \Rightarrow \begin{cases} z+2x-3y=0 \\ 7x=0 \\ 6x-2y=0 \end{cases}$$

$2^{\text{da}} F + 2 \cdot 1^{\text{da}} F$
 $3^{\text{da}} F + 1^{\text{da}} F$

$\Rightarrow x=0$ S. Comp. det.
 $y=0$
 $z=0$ la única solución trivial.

nº 14.- a)
$$\begin{pmatrix} 1 & -1 & 0 & | & 1 \\ 2 & 6 & -5 & | & -4 \\ 1 & 1 & -1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & | & 1 \\ 0 & 8 & -5 & | & -6 \\ 0 & 2 & -1 & | & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & | & 1 \\ 0 & 2 & -1 & | & -1 \\ 0 & 8 & -5 & | & -6 \end{pmatrix}$$

$2^{\text{da}} F - 2 \cdot 1^{\text{da}} F$ $3^{\text{da}} F - 4 \cdot 2^{\text{da}} F$

$3^{\text{da}} F - 1^{\text{da}} F$

$$\begin{pmatrix} 1 & -1 & 0 & | & 1 \\ 0 & 2 & -1 & | & -1 \\ 0 & 0 & -1 & | & -2 \end{pmatrix} \Rightarrow \begin{cases} x-y=1 \\ 2y-z=-1 \\ -z=-2 \end{cases} \Rightarrow \begin{cases} z=2 \\ y=\frac{1}{2} \\ x=\frac{3}{2} \end{cases}$$

$$b) \begin{pmatrix} 2 & -1 & -1 & | & 2 \\ 3 & -2 & -2 & | & 2 \\ -5 & 3 & 5 & | & -1 \end{pmatrix} \sim \begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{x} & | & \\ -1 & -1 & 2 & | & 2 \\ -2 & -2 & 3 & | & 2 \\ 3 & 5 & -5 & | & -1 \end{pmatrix} \sim$$

$$2^{\text{a}} F - 2 \cdot 1^{\text{a}} F.$$

$$3^{\text{a}} F + 3 \cdot 1^{\text{a}} F.$$

$$\begin{pmatrix} -1 & -1 & 2 & | & 2 \\ 0 & 0 & -1 & | & -2 \\ 0 & 2 & 1 & | & 5 \end{pmatrix} \Rightarrow \begin{cases} -y - z + 2x = 2 \\ -x = -2 \\ 2z + x = 5 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = \frac{1}{2} \\ z = \frac{9}{2} \end{cases}$$

no 15. a) $\begin{pmatrix} 1 & 1 & 1 & | & 2 \\ 1 & -1 & 1 & | & 6 \\ 1 & -1 & -1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 2 \\ 0 & -2 & 0 & | & 4 \\ 0 & -2 & -2 & | & -2 \end{pmatrix}$

$$\Rightarrow \begin{cases} x + y + z = 2 \\ -2y = 4 \\ -2y - 2z = -2 \end{cases} \Rightarrow \begin{cases} y = -2 \\ z = 3 \\ x = 1 \end{cases}$$

$$b) \begin{pmatrix} 2 & 3 & 0 & | & 14 \\ 1 & -2 & 1 & | & -3 \\ 2 & -1 & -1 & | & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & | & -3 \\ 2 & 3 & 0 & | & 14 \\ 2 & -1 & -1 & | & 9 \end{pmatrix} \sim$$

$$2^{\text{a}} F - 2 \cdot 1^{\text{a}} F$$

$$3^{\text{a}} F - 2 \cdot 1^{\text{a}} F$$

$$\begin{pmatrix} 1 & -2 & 1 & | & -3 \\ 0 & 7 & -2 & | & 20 \\ 0 & 3 & -3 & | & 15 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & | & -3 \\ 0 & 1 & -1 & | & 5 \\ 0 & 7 & -2 & | & 20 \end{pmatrix} \sim$$

$$\frac{3^{\text{a}} F - 7 \cdot 2^{\text{a}} F}{3} \text{ y intercambio}$$

$$3^{\text{a}} F - 7 \cdot 2^{\text{a}} F$$

$$\begin{pmatrix} 1 & -2 & 1 & | & 3 \\ 0 & 1 & -1 & | & 5 \\ 0 & 0 & 5 & | & -15 \end{pmatrix} \sim \begin{cases} x - 2y + z = 3 \\ y - z = 5 \\ 5z = -15 \end{cases} \Rightarrow \begin{cases} z = -3 \\ y = 2 \\ x = 10 \end{cases}$$