

FICHA nº 6: SISTEMAS

nº 1.- a) $\begin{cases} 2x - y - 1 = 0 \Rightarrow \text{despejamos } y = 2x - 1 \\ x^2 - 7 = y + 2 \end{cases} \xrightarrow{\text{Sustituimos}} x^2 - 7 = 2x - 1 + 2$

$$\begin{cases} y = 2x - 1 \\ x^2 - 2x - 8 = 0 \end{cases} \Rightarrow x = \frac{2 \pm 6}{2} = \begin{cases} x = 4 ; y = 7 \\ x = -2 ; y = -5 \end{cases}$$

b) $\begin{cases} \frac{1}{x} + \frac{1}{y} = 1 - \frac{1}{xy} \\ x \cdot y = 6 \end{cases} \Rightarrow \begin{cases} \frac{y+x}{x \cdot y} = \frac{x \cdot y - 1}{x \cdot y} \\ x \cdot y = 6 \end{cases} \Rightarrow \begin{cases} x + y = 5 \\ x \cdot y = 6 \end{cases}$

$$\begin{cases} y = 5 - x \\ x \cdot (5 - x) = 6 \end{cases} \Rightarrow \begin{cases} y = 5 - x \\ 5x - x^2 - 6 = 0 ; x^2 - 5x + 6 = 0 \end{cases}$$

$$x = \frac{5 \pm 1}{2} = \begin{cases} x = 3 ; y = 2 \\ x = 2 ; y = 3 \end{cases}$$

c) $\begin{cases} x = 2y + 1 \\ \sqrt{x+y} - \sqrt{x-y} = 2 \end{cases} \Rightarrow \begin{cases} x = 2y + 1 \\ \sqrt{3y+1} - \sqrt{y+1} = 2 \end{cases}$

$$(\sqrt{3y+1})^2 = (2 + \sqrt{y+1})^2$$

$$3y+1 = 4 + 4\sqrt{y+1} + y+1 \Rightarrow 2y-4 = 4\sqrt{y+1}$$

$$(y-2)^2 = (2\sqrt{y+1})^2 \Rightarrow y^2 - 4y + 4 = 4y + 4$$

Simplifica por 2.

$$y^2 - 8y = 0 \Rightarrow y = 0 ; x = 1 \text{ No válida.}$$

$$\hookrightarrow \boxed{y = 8 ; x = 17} \checkmark$$

no 2.-

$$a) \begin{cases} \sqrt{x+y} - \sqrt{x-y} = \sqrt{2x} \\ x+y=8 \end{cases} \Rightarrow y=8-x$$

$$\sqrt{x+8-x} - \sqrt{x-8+x} = \sqrt{2x} ; 2\sqrt{2} - \sqrt{2x-8} = \sqrt{2x}$$

$$(2\sqrt{2} - \sqrt{2x})^2 = (\sqrt{2x-8})^2$$

$$8 - 4\sqrt{2} \cdot \sqrt{2x} + 2x = 2x - 8$$

$$8 - 8\sqrt{x} = -8 ; 16 = 8\sqrt{x} ; 2 = \sqrt{x} \Rightarrow \boxed{x=4 ; y=4} \checkmark$$

$$b) \begin{cases} \sqrt{4y+2x} = \sqrt{3y+x} - 1 \\ y+x = -5 \end{cases} \Rightarrow y = -x-5$$

$$\sqrt{-4x-20+2x} = \sqrt{-3x-15+x} - 1 ; (\sqrt{-2x-20})^2 = (\sqrt{-2x-15} - 1)^2$$

$$-2x-20 = -2x-15 - 2\sqrt{-2x-15} + 1$$

$$2\sqrt{-2x-15} = 6 ; (\sqrt{-2x-15})^2 = (3)^2 ; -2x-15 = 9$$

$$-2x = 24 \Rightarrow \boxed{x=-12 ; y=7} \checkmark$$

$$c) \begin{cases} (x+3) \cdot (y-5) = 0 \\ (x-2) \cdot (y-1) = 0 \end{cases} \text{ Equations factorized, NO MULTIPLES!}$$

$$x = -3 ; y = 1$$

Combining
the
solutions.

$$y = 5 ; x = 2 \Rightarrow$$

$$x = 2 ; y = 5$$

$$y = 1 ; x = -3$$

$$\boxed{\begin{array}{l} x = -3 ; y = 1 \\ x = 2 ; y = 5 \end{array}}$$

nº 3.- a) $\begin{cases} 3x^2 - 5y^2 = 7 \\ 2x^2 = 11y^2 - 3 \end{cases} \Rightarrow \begin{cases} 3x^2 - 5y^2 = 7 \\ 2x^2 - 11y^2 = -3 \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} 6x^2 - 10y^2 = 14 \\ -6x^2 + 33y^2 = +9 \end{cases}$$

$$23y^2 = 23 \Rightarrow y^2 = 1$$

$$2x^2 = 11 - 3 ; x^2 = 4$$

$x = 2$	$y = 1$
$x = 2$	$y = -1$
$x = -2$	$y = 1$
$x = -2$	$y = -1$

b) $\begin{cases} (2\sqrt{x})^2 = (3+y)^2 \\ \frac{y+6}{9} + \frac{4(x-9)}{9y} = 3 \end{cases} \Rightarrow \begin{cases} 4x = 9 + 6y + y^2 \\ \frac{y^2 + 6y + 4x - 36}{9y} = \frac{27y}{9y} \end{cases}$

$$y^2 + 6y + 9 + 6y + y^2 - 36 - 27y = 0$$

$$2y^2 - 15y - 27 = 0 : y = \frac{15 \pm 21}{4} = \begin{cases} \rightarrow y = 9 \\ \rightarrow y = -\frac{3}{2} \end{cases}$$

$$\boxed{y = 9} ; 4x = 9 + 54 + 81 ; \boxed{x = 36}$$

$$\boxed{y = -\frac{3}{2}} ; 4x = 9 - 9 + \frac{9}{4} ; \boxed{x = \frac{9}{16}}$$

nº4.- a) $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ 2x + 3y = 2 \end{cases} \Rightarrow \frac{y+x}{xy} = \frac{5}{6} \Rightarrow \begin{cases} 6x + 6y - 5xy = 0 \\ 2x + 3y = 2 \end{cases}$

Despejo "x": $x = \frac{2-3y}{2} = 1 - \frac{3}{2}y$

$$6(1 - \frac{3}{2}y) + 6y - 5(1 - \frac{3}{2}y) \cdot y = 0$$

$$6 - 9y + 6y - 5y + \frac{15}{2}y^2 = 0$$

$$15y^2 - 16y + 12 = 0$$

$$y = \frac{16 \pm \sqrt{16^2 - 4 \cdot 15 \cdot 12}}{30} = \text{NO SOLUCIÓN REAL}$$

b) $\begin{cases} x \cdot y = 15 \\ \frac{x}{y} = \frac{5}{3} \end{cases} \Rightarrow \begin{cases} x \cdot y = 15 \\ x = \frac{5y}{3} \end{cases} \Rightarrow \begin{cases} \frac{5y}{3} \cdot y = 15 \\ x = \frac{5y}{3} \end{cases} \Rightarrow \begin{cases} y^2 = 9 \\ x = \frac{5y}{3} \end{cases}$

$$\Rightarrow \begin{array}{|l|l|} \hline y = +3 & x = 5 \\ \hline y = -3 & x = -5 \\ \hline \end{array} \checkmark$$

c) $\begin{cases} x^2 + y^2 - 5x - 5y + 10 = 0 \\ x^2 - y^2 - 5x + 5y + 2 = 0 \end{cases}$

Sumando: $2x^2 - 10x + 12 = 0$; $x^2 - 5x + 6 = 0$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2} = \begin{array}{l} \rightarrow 3 \\ \rightarrow 2 \end{array}$$

Podemos substituir en una de ellas o ventallas:

$$2y^2 - 10y + 8 = 0$$

$$y^2 - 5y + 4 = 0$$

$$y = \frac{5 \pm \sqrt{25 - 16}}{2} = \begin{array}{l} \rightarrow 4 \\ \rightarrow 1 \end{array}$$

Sol:

$$\begin{array}{|l|l|} \hline x=3 & y=4 \\ x=3 & y=1 \\ x=2 & y=4 \\ x=2 & y=1 \\ \hline \end{array}$$

$$d) \begin{cases} (x+y)(x-y)=7 \\ 3x-4y=0 \end{cases} \Rightarrow \begin{cases} x^2-y^2=7 \\ x=\frac{4y}{3} \end{cases} \Rightarrow \begin{cases} \frac{16y^2}{9}-y^2=7 \\ x=\frac{4y}{3} \end{cases}$$

$$\Rightarrow \begin{cases} 16y^2-9y^2=63 \\ x=\frac{4y}{3} \end{cases} \Rightarrow \begin{cases} 7y^2=63 \\ x=\frac{4y}{3} \end{cases} \Rightarrow y=\pm 3$$

Sol:

$y=3$	$x=4$
$y=-3$	$x=-4$

no 5.- a) $\begin{cases} y^2-2y+1=x \\ \sqrt{x}+y=5 \end{cases} \Rightarrow \sqrt{x}=5-y; x=(5-y)^2$

$$y^2-2y+1=25-10y+y^2 \Rightarrow 8y=24 \Rightarrow \boxed{\begin{matrix} y=3 \\ x=4 \end{matrix}}$$

b) $\begin{cases} 2\sqrt{x+1}=y+1 \\ 2x-3y=1 \end{cases} \Rightarrow \begin{aligned} \sqrt{x+1} &= \frac{1}{2}(y+1) \\ x+1 &= \frac{1}{4}(y+1)^2 \\ x &= \frac{y^2}{4} + \frac{1}{2}y + \frac{1}{4} - 1 \end{aligned}$

Substitution:

$$2\left(\frac{y^2}{4} + \frac{1}{2}y - \frac{3}{4}\right) - 3y = 1; \quad \frac{y^2}{2} + y - \frac{3}{2} - 3y - 1 = 0$$

$$y^2 - 4y - 5 = 0 \quad y = \frac{4 \pm \sqrt{16+20}}{2} = \begin{matrix} 5 \\ -1 \end{matrix}$$

Sol:

$y=5$	$x=8$
$y=-1$	$x=-1$

$$c) \begin{cases} \sqrt{3(x+y)} + x = 12 \\ 2x - y = 6 \end{cases} \Rightarrow y = 2x - 6 \quad y \text{ ahora sustituir}$$

$$\sqrt{3(x+2x-6)} + 2x - 6 = 12 ;$$

$$\sqrt{3(3x-6)} = 18 - 2x \quad \text{elevar al cuadrado}$$

$$3(3x-6) = 324 - 72x + 4x^2 ;$$

$$9x - 18 = 324 - 72x + 4x^2 ; \quad 4x^2 - 81x + 342 = 0$$

$$x = \frac{81 \pm \sqrt{1089}}{8} = \frac{81 \pm 33}{8} = \begin{cases} \frac{57}{4} ; & y = \frac{45}{2} \quad \text{No vale!} \\ \underline{\underline{6}} ; & \underline{\underline{y=6}} \quad \checkmark \end{cases}$$

$$d) \begin{cases} \sqrt{x+y} + 2 = x+1 \\ 2x - y = 5 \end{cases} \Rightarrow y = 2x - 5$$

$$\sqrt{x+2x-5} + 2 = x+1 ; \quad \sqrt{3x-5} = x-1 ;$$

$$3x-5 = x^2 - 2x + 1 ; \quad x^2 - 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 6 ; & y = 7 \quad \text{No vale!} \\ \underline{\underline{-1}} ; & \underline{\underline{y=-7}} \quad \text{No vale!} \end{cases}$$

nº 6.- a) $\begin{cases} 5^{x+y} = 25^3 \\ 3^{x-y} = 9 \end{cases} \Rightarrow \begin{cases} 5^{x+y} = 5^6 \\ 3^{x-y} = 3^2 \end{cases} \Rightarrow \begin{cases} x+y = 6 \\ x-y = 2 \end{cases}$

$$\underline{2x = 8 \Rightarrow x=4 \quad y=2}$$

b) $\begin{cases} 2^{x+2} + 5^y = 141 \\ 2^{x+3} - 5^{y-1} = 7 \end{cases} \Rightarrow \begin{cases} 2^x \cdot 2^2 + 5^y = 141 \\ 2^x \cdot 2^3 - 5^y \cdot 5^{-1} = 7 \end{cases}$ Cambio: $2^x = t$
 $5^y = s$

$\begin{cases} 4t + s = 141 \\ 8t - \frac{1}{5}s = 7 \end{cases} \Rightarrow \begin{cases} 4t + s = 141 \\ 40t - s = 35 \end{cases}$

$$\underline{44t = 176 \Rightarrow t=4} \Rightarrow \underline{x=2; y=3}$$

 $s = 125$

c) $\begin{cases} 2^x + 2^y = 24 \\ 2^x \cdot 2^y = 128 \end{cases} \Rightarrow$ Cambio: $2^x = t$
 $2^y = s$ $\begin{cases} t + s = 24 \\ t \cdot s = 128 \end{cases}$

$\Rightarrow t = 24 - s \Rightarrow (24 - s) \cdot s = 128 \Rightarrow 24s - s^2 = 128$

$s^2 - 24s + 128 = 0$

$s = \frac{24 \pm 8}{2} = \begin{cases} s = 16 \Rightarrow t = 8 \Rightarrow 2^y = 16; 2^x = 8 \\ s = 8 \Rightarrow t = 16 \Rightarrow 2^y = 8; 2^x = 16 \end{cases}$

$\Rightarrow \underline{\underline{\begin{cases} x=3; y=4 \\ x=4; y=3 \end{cases}}}$

d) $\begin{cases} 8^y \cdot 8^{2x} = 128 \\ 3^{2y} \cdot 3^{x-1} = 27 \end{cases} \Rightarrow \begin{cases} 2^{3y} \cdot 2^{6x} = 2^7 \\ 3^{2y+x-1} = 3^3 \end{cases} \Rightarrow \begin{cases} 3y + 6x = 7 \\ 2y + x = 4 \end{cases}$

$\begin{cases} 3y + 6x = 7 \\ -12y - 6x = -24 \end{cases} \Rightarrow -9y = -17 \Rightarrow \underline{\underline{y = \frac{17}{9}; x = \frac{2}{9}}}$

$$e) \begin{cases} 3 \cdot 2^x - 2 \cdot 3^y = -6 \\ 4 \cdot 2^x - 3^{y+1} = -11 \end{cases} \text{ cambio: } \begin{cases} 2^x = t \\ 3^y = s \end{cases} \Rightarrow \begin{cases} 3t - 2s = -6 \\ 4t - 3s = -11 \end{cases}$$

$$\begin{cases} 9t - 6s = -18 \\ -8t + 6s = +22 \end{cases} \Rightarrow \begin{matrix} t=4 \\ s=9 \end{matrix} \Rightarrow \boxed{\begin{matrix} x=2 \\ y=2 \end{matrix}}$$

$t=4$

nº7.- a) $\begin{cases} x+y=70 \\ \log x + \log y = 3 \end{cases} \Rightarrow \begin{cases} x+y=70 \\ \log x \cdot y = \log 10^3 \end{cases}$

$$\begin{cases} x+y=70 \\ x \cdot y=1000 \end{cases} \Rightarrow \begin{matrix} y=70-x \\ x \cdot (70-x)=1000 \\ x^2 - 70x + 1000 = 0 \end{matrix}$$

$$x = \frac{70 \pm 30}{2} = \begin{cases} x=50 ; y=20 \\ x=20 ; y=50 \end{cases}$$

$$b) \begin{cases} \log_x (y+8) = 2 \\ \log_y (x-4) = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} x^2 = y+8 \\ (y^{\frac{1}{2}})^2 = (x-4)^2 \end{cases} \Rightarrow \begin{cases} x^2 = y+8 \\ y = x^2 - 8x + 16 \end{cases}$$

$$x^2 = x^2 - 8x + 16 + 8 ; 8x = 24 ; x = 3 ; y =$$

No es válida!!!

$$c) \begin{cases} \log_2 x + \log_2 y = 5 \\ 2 \cdot \log_2 x - \log_2 y = 1 \end{cases} \Rightarrow \begin{cases} x \cdot y = 2^5 \\ \frac{x^2}{y} = 2 \end{cases} \Rightarrow \frac{x^2}{2} = y$$

$$x \cdot \frac{x^2}{2} = 2^5 ; x^3 = 2^6 ; x = 2^2 ; \boxed{x=4 ; y=8}$$

$$d) \begin{cases} \log x - 2 \log y = 3 \\ 3 \log x + \log y = 2 \end{cases} \Rightarrow \begin{cases} \log \frac{x}{y^2} = \log 10^3 \\ \log x^3 \cdot y = \log 10^2 \end{cases}$$

$$\begin{cases} x = 1000 \cdot y^2 \\ x^3 \cdot y = 100 \end{cases} \Rightarrow (10^3 \cdot y^2)^3 \cdot y = 10^2 ; 10^9 \cdot y^6 \cdot y = 10^2 \\ y^7 = 10^{-7} \Rightarrow \boxed{y = \frac{1}{10} ; x = 10}$$

$$e) \begin{cases} x - y = \frac{999}{10} \\ \log(x \cdot y) = 1 \end{cases} \Rightarrow \begin{cases} x = \frac{999}{10} + y \\ x \cdot y = 10 \end{cases} \Rightarrow \left(\frac{999}{10} + y\right) \cdot y = 10$$

$$y^2 + \frac{999}{10}y - 10 = 0 ; 10y^2 + 999y - 100 = 0 \Rightarrow \boxed{y = \frac{1}{10} ; x = 100} \\ \text{or } \boxed{y = -100 ; x = -\frac{1}{10}}$$

$$f) \begin{cases} \log x - \log y = \log 56 - \log 20 \\ \log x + \log y = 1 + \log 7 \end{cases} \Rightarrow \begin{cases} \frac{x}{y} = \frac{56}{20} \\ x \cdot y = 70 \end{cases} \Rightarrow \begin{cases} x = \frac{14}{5}y \\ \frac{14}{5}y^2 = 70 \end{cases}$$

$$y^2 = 25 \Rightarrow \boxed{y = 5 ; x = 14}$$

$$g) \begin{cases} 2 \log 5 + y \log 25 = x \cdot \log 125 \\ x \cdot \log 4 + 2y \log 8 = \frac{1}{2} \log 64 \end{cases} \Rightarrow \begin{cases} 5^2 \cdot 25^y = 125^x \\ 4^x \cdot 8^{2y} = 8 \end{cases} \Rightarrow \begin{cases} 5^{2+2y} = 5^{3x} \\ 2^{2x+6y} = 2^3 \end{cases}$$

$$\begin{cases} 3x - 2y = 2 \\ 2x + 6y = 3 \end{cases} \Rightarrow \begin{cases} 9x - 6y = 6 \\ 2x + 6y = 3 \end{cases} \Rightarrow 11x = 9 ; \boxed{x = \frac{9}{11} ; y = \frac{5}{22}}$$

$$h) \begin{cases} \log(x+y) + \log(x-y) = \log 33 \\ e^x = \frac{e^{11}}{e^y} \end{cases} \Rightarrow \begin{cases} (x+y) \cdot (x-y) = 33 \\ x = 11 - y \end{cases}$$

$$\begin{cases} x^2 - y^2 = 33 \\ x = 11 - y \end{cases} \Rightarrow 121 - 22y + y^2 - y^2 = 33 ; 88 = 22y \\ \boxed{y = 4 ; x = 7}$$