

■ Hallar las derivadas simplificadas de las siguientes funciones:

1. $y=3$	$(y'=0)$	23. $y = \frac{x+1}{x-1}$	$\left(y' = \frac{-2}{(x-1)^2}\right)$
2. $y=x$	$(y'=1)$	24. $y = \frac{1}{x^2+1}$	$\left(y' = \frac{-2x}{(x^2+1)^2}\right)$
3. $y=5x$	$(y'=5)$	25. $y = 3 \frac{2x^2-1}{x^3+1}$	$\left(y' = 3 \frac{-2x^4+3x^2+4x}{(x^3+1)^2}\right)$
4. $y=-x$	$(y'=-1)$	26. $y = \left(\frac{2x-3}{x+4}\right)^4$	$\left(y' = \frac{44(2x-3)^3}{(x+4)^5}\right)$
5. $y=x^4+x^3+x^2+x+1$	$(y'=4x^3+3x^2+2x+1)$	27. $y = \sqrt{x^2+1}$	$\left(y' = \frac{x}{\sqrt{x^2+1}}\right)$
6. $y = 4x^4 - x^3 + 3x^2 - 7$	$(y'=16x^3-3x^2+6x)$	28. $y = 2\sqrt{x^3-x^2+1} (2x^2+3)$	$\left(y' = \frac{14x^4-12x^3+9x^2+2x}{\sqrt{x^3-x^2+1}}\right)$
7. $y = -\frac{1}{5}x^5 + 4x^4 - \frac{1}{6}x^3 + \frac{1}{2}x^2 - 3$	$\left(y' = -x^4 + 16x^3 - \frac{1}{2}x^2 + x\right)$	29. $y = \log x$	$\left(y' = \frac{1}{x} \log_{10} e = \frac{1}{x \ln 10}\right)$
8. $y=3(x^2+x+1)$	$(y'=3(2x+1))$	30. $y = \ln x$	$(y'=1/x)$
9. $y=4(3x^3-2x^2+5)+x^2+1$	$(y'=36x^2-14x)$	31. $y=3\log_2 x - 4\ln x$	$\left(y' = \frac{-4+3\log_2 e}{x}\right)$
10. $y = \frac{2x^3-3x^2+4x-5}{2}$	$(y'=3x^2-3x+2)$	32. $y = \ln(3x^2+4x+5)$	$\left(y' = \frac{6x+4}{3x^2+4x+5}\right)$
11. $y=(x^2+1)(2x^3-4)$	$(y'=10x^4+6x^2-8x)$	33. $y = \ln \sqrt{x^2-1}$	$\left(y' = \frac{x}{x^2-1}\right)$
12. $y=1/x$	$(y' = -1/x^2)$	34. $y = \sqrt{\ln(x^2-1)}$	$\left(y' = \frac{x}{(x^2-1)\sqrt{\ln(x^2-1)}}\right)$
13. $y=1/x^3$	$(y' = -3/x^4)$	35. $y=2^x$	
14. $y=1/x^5$	$(y' = -5/x^6)$	36. $y = 2^{x^2+x+1}$	
15. $y = \frac{2}{x^3} + \frac{1}{x^2} - \frac{3}{x}$	$\left(y' = \frac{3x^2-2x-6}{x^4}\right)$	37. $y = e^{2x^2-3x+5}$	
16. $y = \sqrt{x}$	$\left(y' = \frac{1}{2\sqrt{x}}\right)$	38. $y=e^{-x}$	$(y' = -1/e^x)$
17. $y = \sqrt[3]{x^2}$	$\left(y' = \frac{2}{3\sqrt[3]{x}}\right)$	39. $y = e^{1/x}$	
18. $y = \sqrt[5]{x^3}$	$\left(y' = \frac{3}{5\sqrt[5]{x^2}}\right)$	40. $y = 10^{\sqrt{x}}$	$\left(y' = \frac{10^{\sqrt{x}} \cdot \ln 10}{2\sqrt{x}}\right)$
19. $y = 2\sqrt[3]{x^2} - 3x^2 + \frac{1}{5}$	$\left(y' = \frac{4}{3\sqrt[3]{x}} - 6x\right)$	41. $y = \sin 2x$	
20. $y=(x+1)^5$	$(y'=5(x+1)^4)$	42. $y = \sin x^2$	
21. $y=(2x^2-3x+1)^3$	$(y'=(2x^2-3x+1)^2(12x-6))$	43. $y = \sin^2 x$	$(y' = \sin 2x)$
22. $y=(x^2+1)^{100}$	$(y'=200x(x^2+1)^{99})$	44. $y=2 \sin x$	
		45. $y = \sin(x^2-2x+1)$	
		46. $y = \cos \sqrt{x}$	$\left(y' = -\frac{\sin \sqrt{x}}{2\sqrt{x}}\right)$

$$47. y = 400x - \frac{400000}{x^2} \quad \left(y' = 400 + \frac{800000}{x^3} \right)$$

$$48. y = \sin^3(x^2+1) \quad (y' = 6x \sin^2(x^2+1) \cos(x^2+1))$$

$$49. y = \operatorname{tg} \frac{1}{x} \quad \left(y' = -\frac{1 + \operatorname{tg}^2(1/x)}{x^2} \right)$$

$$50. y = \operatorname{ctg}(x^2+1) \quad \left(y' = -\frac{2x}{\sin^2(x^2+1)} \right)$$

$$51. y = \frac{1}{3}x^3 - \frac{3}{4}x^4 + \frac{1}{2}x^2 - \frac{1}{x} \quad (y' = -3x^3 + x^2 + x + 1/x^2)$$

$$52. y = 2/x \quad (y' = -2/x^2)$$

$$53. y = 2 \sin(x^2+1) \quad (y' = 4x \cos(x^2+1))$$

$$54. y = 3(x^2-x+1)(x^2+x-1) \quad (y' = 3(4x^3-2x+2))$$

$$55. y = \frac{1}{2} \cos(\sqrt{x}+1) \quad \left(y' = -\frac{\sin(\sqrt{x}+1)}{4\sqrt{x}} \right)$$

$$56. y = \frac{x^2-1}{x^2+1} \quad \left(y' = \frac{4x}{(x^2+1)^2} \right)$$

$$57. y = x/2 \quad (y' = 1/2)$$

$$58. y = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \ln x \quad \left(y' = -\frac{1}{x^2} - \frac{4}{x^3} - \frac{9}{x^4} + \frac{1}{x} \right)$$

$$59. y = \ln^3(x+1) \quad \left(y' = \frac{3\ln^2(x+1)}{x+1} \right)$$

$$60. y = (2x^2-1)(x^2-2)(x^3+1) \quad (y' = 14x^6 - 25x^4 + 8x^3 + 6x^2 - 10x)$$

$$61. y = \sqrt{\frac{1-x^3}{x^2+1}} \quad \left(y' = -\frac{x^4+3x^2+2x}{2\sqrt{(x^2+1)^3}\sqrt{1-x^3}} \right)$$

$$62. y = \ln^2 x \quad \left(y' = \frac{2\ln x}{x} \right)$$

$$63. y = \ln x^2 \quad (y' = 2/x)$$

$$64. y = (x^2+1)(x+2)^3 \quad (y' = 5x^4 + 24x^3 + 39x^2 + 28x + 12)$$

$$65. y = \frac{\ln x}{\sqrt{x}} \quad \left(y' = \frac{2 - \ln x}{2x\sqrt{x}} \right)$$

$$66. y = \frac{1}{3x^5 - x^3 + 2} \quad \left(y' = \frac{-15x^4 + 3x^2}{(3x^5 - x^3 + 2)^2} \right)$$

$$67. y = \ln \sin x$$

$$68. y = \sin \ln x \quad \left(y' = \frac{\cos \ln x}{x} \right)$$

$$69. y = \sqrt{x^4 - 2x^2 + 3} \quad \left(y' = \frac{2x^3 - 2x}{\sqrt{x^4 - 2x^2 + 3}} \right)$$

$$70. y = e^{\sin x}$$

$$71. y = 2 \operatorname{tg}^3 x \quad (y' = 6 \operatorname{tg}^2 x (1 + \operatorname{tg}^2 x))$$

$$72. y = \sqrt{\ln x} \quad \left(y' = \frac{1}{2x\sqrt{\ln x}} \right)$$

$$73. y = 2^{\operatorname{tg} x}$$

$$74. y = \sqrt{\frac{x^2+1}{x^2-1}} \quad \left(y' = \frac{-2x\sqrt{x^2-1}}{(x^2-1)^2 \cdot \sqrt{x^2+1}} \right)$$

$$75. y = \cos(e^x+1)$$

$$76. y = \sqrt[5]{x^2+1} \quad \left(y' = \frac{2}{5\sqrt[5]{x^3}} \right)$$

$$77. y = \operatorname{tg}(1 + \ln^2 x) \quad \left(y' = \frac{2\ln x}{x \cos^2(1 + \ln^2 x)} \right)$$

$$78. y = \log(2^x+5) \quad \left(y' = \frac{2^x \ln 2}{(2^x+5) \ln 10} \right)$$

$$79. y = \frac{x^4 - 2x^2 + 1}{4} \quad (y' = x^3 - x)$$

$$80. y = \frac{5}{x^4 - 2x^2 + 1} \quad \left(y' = \frac{20x - 20x^3}{(x^4 - 2x^2 + 1)^2} \right)$$

$$81. y = 3(x+1)^3 \sqrt[3]{x+1} \quad (y' = 10\sqrt[3]{(x+1)^7})$$

$$82. y = \ln(x-3) \quad \left(y' = \frac{1}{x-3} \right)$$

$$83. y = 4\ln \sqrt{x} \quad (y' = 2/x)$$

$$84. y = \sqrt{4\ln x} \quad \left(y' = \frac{1}{x\sqrt{\ln x}} \right)$$

$$85. y = x^3 \sqrt{x} \quad \left(y' = \frac{7x^2 \sqrt{x}}{2} \right)$$

$$86. y = \sqrt{x} \cdot \ln x \quad \left(y' = \frac{2 + \ln x}{2\sqrt{x}} \right)$$

$$87. y = \ln \frac{x-1}{x+2} \quad \left(y' = \frac{3}{(x+2)(x-1)} \right)$$

$$88. y = \ln(x+1) \cdot \log(x-1) \quad \left(y' = \frac{\log(x-1)}{x+1} + \frac{\ln(x+1) \log e}{x-1} \right)$$

$$89. y = \ln(\ln x) \quad \left(y' = \frac{1}{x \ln x} \right)$$

$$90. y = \frac{3}{\ln(x^2+1)} \quad \left(y' = -\frac{6x}{(x^2+1) \ln^2(x^2+1)} \right)$$

$$91. y = \sqrt[3]{\frac{1}{x+2}} \quad \left(y' = -\frac{1}{3\sqrt[3]{(x+2)^4}} \right)$$

$$92. y = 3 \frac{(x-1)^2(x+2)}{x+1} \quad \left(y' = 3 \frac{2x^3 + 3x^2 - 5}{(x+1)^2} \right)$$

$$93. y = 7 \frac{3x^2 - 5}{\ln(3x^2 - 5)} \quad \left(y' = \frac{42x[-1 + \ln(3x^2 - 5)]}{\ln^2(3x^2 - 5)} \right)$$

$$94. y = e^{x^2} \quad (y' = e^{x^2} \cdot 2x)$$

$$95. y = x \cdot e^x \quad (y' = (x+1) \cdot e^x)$$

$$96. y = \frac{t^2 + 2t}{e^t} \quad \left(y' = \frac{2 - t^2}{e^t} \right)$$

97. $y = \frac{e^x}{x}$	$\left(y' = \frac{e^x(x-1)}{x^2} \right)$	114. $y = \arcsen \frac{2}{\sqrt{x}}$	$\left(y' = -\frac{1}{x\sqrt{x-4}} \right)$
98. $y = \frac{\sqrt{x}}{\ln x}$	$\left(y' = \frac{\ln x - 2}{2\sqrt{x} \ln^2 x} \right)$	115. $y = \sqrt{x^2+1} (x^2-1)^2$	$\left(y' = \frac{5x^5 - 2x^3 - 3x}{\sqrt{x^2+1}} \right)$
99. $y = \frac{2x+4}{\sqrt{x+3}}$	$\left(y' = \frac{x+4}{(x+3)\sqrt{x+3}} \right)$	116. $y = \frac{1}{3} \arcsen e^x$	$\left(y' = \frac{e^x}{3(1+e^{2x})} \right)$
100. $y = \arcsen (x^2-4)$	$\left(y' = \frac{2x}{\sqrt{-x^4+8x^2-15}} \right)$	117. $y = \frac{x^2+5}{x^2-4}$	$\left(y' = \frac{-18x}{(x^2-4)^2} \right)$
101. $y = \arcsen \frac{1}{x}$	NOTA: Recordar que $\sqrt{x^2} = x $ $\left(y' = \frac{1}{ x \sqrt{x^2-1}} \right)$	118. $y = \arcsen (x^2+1)$	$\left(y' = \frac{2}{\sqrt{-x^2-2}} \right)$
102. $y = \frac{-6x^2+72x+4}{(6-x)^2}$	$\left(y' = \frac{440}{(6-x)^3} \right)$	119. $y = \arcsen \sqrt{x}$	$\left(y' = \frac{1}{2\sqrt{x}\sqrt{1-x^2}} \right)$
103. $y = 2(\sqrt{x} - \arcsen \sqrt{x})$	$\left(y' = \frac{\sqrt{x}}{x+1} \right)$	120. $y = \frac{1}{3x^3} + \frac{2}{x^2} - \frac{3}{x} + 5$	$\left(y' = -\frac{1}{x^4} - \frac{4}{x^3} + \frac{3}{x^2} \right)$
104. $y = \arcsen \frac{2x^3-1}{x^2-2}$	$\left(y' = \frac{2x^4-12x^2+2x}{4x^6+x^4-4x^3-4x^2+5} \right)$	121. $y = \arcsen \frac{x^2+1}{x^2-1}$	$\left(y' = \frac{-2x}{x^4+1} \right)$
105. $y = (x^3-4x^2+7x-6)e^x$	$\left(y' = (x^3-x^2-x+1)e^x \right)$	122. $y = \sqrt[3]{(x^3+1)^4}$	$\left(y' = 4x^2 \sqrt[3]{x^3+1} \right)$
106. $y = \arcsen \sqrt{1-x^2}$	$\left(y' = \frac{-1}{\sqrt{1-x^2}} \right)$	123. $y = (x+2) \ln(x+2)$	$(y' = 1 + \ln(x+2))$
107. $y = \frac{1}{2} \arcsen e^{x^2}$	$\left(y' = \frac{x e^{x^2}}{1+e^{2x^2}} \right)$	124. $y = \sqrt{x^2+1} (x^2+1)^2$	$\left(y' = 5x \sqrt{(x^2+1)^3} \right)$
108. $y = \frac{1}{2} \operatorname{tg} e^{x^2}$	$\left(y' = x e^{x^2} (1 + \operatorname{tg}^2 e^{x^2}) \right)$	125. $y = (2x+1)^3 \sqrt[3]{3x-1}$	
109. $y = \arcsen \frac{1+x}{1-x}$	$\left(y' = \frac{1}{1+x^2} \right)$	126. $y = \sqrt{\frac{x+1}{x-1}}$	$\left(y' = -\frac{\sqrt{x-1}}{\sqrt{x+1}(x-1)^2} \right)$
110. $y = \ln \cos (\arcsen x)$	$\left(y' = -\frac{x}{1+x^2} \right)$	127. $y = \sqrt{1+\frac{1}{x^2}}$	$\left(y' = -\frac{1}{x^2\sqrt{x^2+1}} \right)$
111. $y = -\ln \sqrt{x^2+1}$	$\left(y' = -\frac{x}{1+x^2} \right)$	128. $y = \frac{x}{2} \sqrt{24-x^2}$	$\left(y' = \frac{12-x^2}{\sqrt{24-x^2}} \right)$
112. $y = \frac{\ln x}{x^3}$	$\left(y' = \frac{1-3\ln x}{x^4} \right)$	129. $y = \frac{4}{(2x+2)^2}$	$\left(y' = -\frac{16}{(2x+2)^3} \right)$
113. $y = \ln \sqrt{\frac{x+1}{x-1}}$	$\left(y' = \frac{1}{1-x^2} \right)$	130. $y = 1+x^2 e^{-x^2}$	$\left(y' = \frac{2x-2x^3}{e^{x^2}} \right)$

131. Dada $y = \frac{x^2-1}{x^3}$, hallar y' , y'' e y''' $\left(y' = \frac{3-x^2}{x^4}; y'' = \frac{2x^2-12}{x^5}; y''' = \frac{60-6x^2}{x^6} \right)$

■ Derivación implícita:

Hallar, por derivación implícita, la derivada de las siguientes funciones:

132. $y^2+2xy+5=0$	$\left(y' = \frac{-y}{x+y} \right)$	133. $x^2y+xy^2=y+1$	$\left(y' = \frac{y^2+2xy}{-x^2-2xy+1} \right)$
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134. $x^2 + y^2 - xy = 3$	$\left(y' = \frac{2x - y}{x - 2y} \right)$	136. $xy^2 + x^2y - \frac{1}{x} + y = 5$	$\left(y' = -\frac{x^2y^2 + 2x^3y + 1}{2x^3y + x^4 + x^2} \right)$
135. $xy^2 = x^2 + y$	$\left(y' = \frac{2x - y^2}{2xy - 1} \right)$	137. $x^2 + 2xy + y^2 = 4$	$(y' = -1)$

Hallar, por derivación implícita, la derivada de las siguientes funciones, en los puntos que se indican:

138. $x^3 - y^3 = y$ en $P(1,0)$	$\left(y' = \frac{3x^2}{3y^2 + 1}; y'(P) = 3 \right)$
139. $x^2 + y^2 + x + y = 16$ en $Q(-1, -1/2)$	$\left(y' = -\frac{2x + 1}{2y + 1}; y'(Q) = 2 \right)$
140. $xy^2 + \frac{y}{2} = x + 1$ en el origen	$\left(y' = \frac{2 - 2y^2}{4xy + 1}; y'(O) = 2 \right)$
141. $2y - x + 3xy^2 = 5$ en el punto de ordenada $y=0$ ¿De qué punto se trata?	$\left(y' = \frac{1 - 3y^2}{2 + 6xy}; y'(P) = \frac{1}{2}; P(5,0) \right)$

■ Derivación logarítmica:

Hallar, por derivación logarítmica, la derivada de las siguientes funciones:

142. $y = x^x$	$(y' = (1 + \ln x) x^x)$	148. $y = x^{x^2}$	$(y' = (1 + 2 \ln x) x^{x^2 + 1})$
143. $y = x^{1/x}$	$(y' = (1 - \ln x) x^{1 - 2x/x})$	149. $y = (x + 1)^{x-1}$	$\left(y' = (x + 1)^{x-1} \left[\ln(x + 1) + \frac{x - 1}{x + 1} \right] \right)$
144. $y = (\sin x)^{\sin x}$	$(y' = [\cos x \ln(\sin x) + \cos x](\sin x)^{\sin x})$	150. $y = (\sin x)^{1/x}$	$\left(y' = (\sin x)^{1/x} \left[\frac{-\ln \sin x}{x^2} + \frac{\cotg x}{x} \right] \right)$
145. $y = (\sin x)^{\cos x}$	$(y' = [-\sin x \ln(\sin x) + \cotg x \cos x](\sin x)^{\cos x})$	151. $y = x^{\sin x}$	$\left(y' = \left(\cos x \cdot \ln x + \frac{\sin x}{x} \right) \cdot x^{\sin x} \right)$
146. $y = (\sin x)^x$	$(y' = (\ln \sin x + x \cotg x) (\sin x)^x)$		
147. $y = (e^x)^{\sin x}$	$(y' = (\sin x + x \cos x) e^{x \cdot \sin x})$		

■ Ejercicios varios:

152. (S) Dada la función $f(x) = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}}$

se pide: **a)** Determinar los valores de x para los que está definida.

b) Hallar su derivada.

(Soluc: $\forall x \neq \pi/2 + n\pi$ con $n \in \mathbb{Z}$; $f'(x) = 1/\cos x$)

153. (S) Un observador se encuentra a 2000 metros de la torre de lanzamiento de un cohete. Cuando éste despegue verticalmente mide la variación del ángulo $\phi(t)$ que forma la línea visual que le une con el cohete y la del suelo horizontal en función del tiempo transcurrido. Sabiendo que $\phi'(t) = 1/20$ radianes por segundo cuando $\phi = \pi/3$, se pide:

a) ¿Cuál es la altura del cohete cuando $\phi = \pi/3$ radianes?

b) ¿Cuál es la velocidad del cohete cuando $\phi = \pi/3$ radianes?

(Soluc: $2000\sqrt{3}$ m.; 400 m/s)

154. (S) Hallar la derivada vigésimo cuarta de $y = a \sin bx$ para a y b constantes. (Soluc: $y^{(24)} = ab^{24} \sin bx$)