

ECUACIONES EXPONENCIALES

1- Resuelve las siguientes ecuaciones aplicando propiedades.

a)

$$5^{3x-12} = 125 \quad \text{Solución: } x=5.$$

b)

$$8^{x^2+3x+2} = 1 \quad \text{Solución: } x=-1, x=-2.$$

c)

$$\sqrt[3]{a^{7-x}} = a^2 \quad \text{Solución: } x=1.$$

d)

$$3^{x^2+5x-4} \cdot 9^{2x+3} = 27^{x-1} \quad \text{Solución: } x=-1, x=-5.$$

e)

$$6^{12-3x} = 216 \quad \text{Sol: } x = 3$$

2- Resuelve las ecuaciones:

a)

$$2^x - 2^{x+1} + 1 = 0$$

$$2^x - 2 \cdot 2^x + 1 = 0 \quad t = 2^x \implies t - 2t + 1 = 0 \implies t = 1$$

$$2^x = 1 \implies x = 0$$

b)

$$2^x - 2^{x-1} - 1 = 0$$

$$2^x - \frac{2^x}{2} - 1 = 0 \quad t - \frac{t}{2} - 1 = 0 \implies t = 2 \quad 2^x = 2 \implies x = 1$$

c)

$$5^{2x-1} - 5^x + 1 = 0$$

$$\frac{(5^x)^2}{5} - 5^x + 1 = 0 \quad \frac{t^2}{5} - t + 1 = 0 \Rightarrow t^2 - 5t + 5 = 0$$

$$\begin{cases} t = 5^x = 3,618 \Rightarrow x = 0,714 \\ t = 5^x = 1,381 \Rightarrow x = 0,296 \end{cases}$$

d)

$$3^{2x+1} - 3^{x-1} - 1 = 0$$

$$3(3^x)^2 - \frac{3^x}{3} - 1 = 0 \quad 3t^2 - \frac{t}{3} - 1 = 0 \Rightarrow \begin{cases} t = 0,63557 \\ t = -0,524461 \end{cases}$$

$$\begin{cases} t = 0,63557 = 3^x \Rightarrow x = -0,41255 \\ t = -0,524461 = 3^x \Rightarrow \text{No Vale} \end{cases}$$

e)

$$2^x + 4^x = 6$$

$$2^x + (2^2)^x = 6 \quad 2^x + (2^x)^2 = 6 \quad t + t^2 = 6 \quad t = 2, t = -3$$

$$t = 2 = 2^x \quad x = 1 \quad t = -3 \text{ no es solución}$$

f)

$$4^x + 2^{2x-1} = 24 \quad \text{Sol: } x = 2$$

g)

$$3^x + 3^{x-2} = 270 \quad \text{Sol: } x = 5$$

h)

$$3^{x+3} + 9^{x+2} = 4 \quad \text{Sol: } x = -2$$

i)

$$4^x - 5 \cdot 2^x + 4 = 0 \quad \text{Sol: } x = 2; \quad x = 0$$

LOGARITMOS

1- Calcula usando la definición de logaritmo:

1 $\log_{\frac{1}{2}} 0.25 = y$

$$\left(\frac{1}{2}\right)^y = 0.25 \quad \left(\frac{1}{2}\right)^y = \left(\frac{1}{2}\right)^2 \quad y = 2$$

2 $\log_{\sqrt{5}} 125 = y$

$$\sqrt{5}^y = 125 \quad 5^{\frac{1}{2}y} = 5^3 \quad y = 6$$

3 $\log 0.001 = y$

$$10^y = 0.001 \quad 10^y = 10^{-3} \quad y = -3$$

4 $\ln \frac{1}{e^5} = y$

$$e^y = \frac{1}{e^5} \quad e^y = e^{-5} \quad y = -5$$

5 $\log_{\sqrt{3}} \sqrt[5]{\frac{1}{81}} = y$

$$\sqrt{3}^y = \sqrt[5]{\frac{1}{81}} \quad 3^{\frac{1}{2}y} = 3^{-\frac{4}{5}} \quad y = -\frac{8}{5}$$

6 $\log_2 32 = x$

$$2^x = 32 \quad 2^x = 2^5 \quad x = 5$$

7

$$\log_9 \frac{1}{3} = x \quad (9)^x = \frac{1}{3} \quad 3^{2x} = 3^{-1} \quad x = -\frac{1}{2}$$

8 $\log_{\frac{1}{2}} 0.25 = x$

$$\left(\frac{1}{2}\right)^x = \frac{25}{100} \quad \left(\frac{1}{2}\right)^x = \left(\frac{1}{4}\right)$$

$$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^2 \quad x = 2$$

9 $\log_9 \sqrt[4]{3} = x$

$$(9)^x = \sqrt[4]{3} \quad 3^{2x} = 3^{\frac{1}{4}} \quad x = \frac{1}{8}$$

10 $\log_{\sqrt{2}} \frac{1}{4} = x$

$$(\sqrt{2})^x = \frac{1}{4} \quad 2^{\frac{1}{2}x} = 2^{-2} \quad x = -4$$

11 $\log_x 81 = -4$

$$x^{-4} = 81 \quad x^4 = \frac{1}{81} \quad x = \frac{1}{3}$$

12 $\log_2 x^3 = 6$

$$x^3 = 2^6 \quad x = 4$$

2- Sabiendo que $\log 2 = 0.3010$, calcula:

1 $\log 0.02$

$$\log\left(\frac{2}{100}\right) = \log 2 - \log 10^2 = \log 2 - 2 = 0.3010 - 2 = -1.699$$

2 $\log \sqrt[4]{8}$

$$\log \sqrt[4]{2^3} = \frac{3}{4} \log 2 = \frac{3}{4} \cdot 0.3010 = 0.2257$$

3 $\log 5$

$$\log\left(\frac{10}{2}\right) = \log 10 - \log 2 = 1 - 0.3010 = 0.699$$

4 $\log 0.0625$

$$\log\left(\frac{625}{10000}\right) = \log\left(\frac{5^4}{2^4 \cdot 5^4}\right) = \log\left(\frac{1}{2^4}\right) =$$

$$\log 1 - \log 2^4 = 0 - 4 \log 2 = -1.2040$$

3- Si $\log A = 14.4$, calcula:

a) $\log \frac{A}{100}$

$$\log \frac{A}{100} = \log A - \log 100 = 14'4 - 2 = 12'4$$

$$\text{b) } \log 0'01A^2$$

$$\log 0'01A^2 = \log 0'01 + 2\log A = -2 + 28'8 = 26'8$$

$$\text{c) } \log \sqrt[3]{\frac{1}{A}}$$

$$\log \sqrt[3]{\frac{1}{A}} = \frac{1}{3} \log \frac{1}{A} = \frac{1}{3} (\log 1 - \log A) = -\frac{\log A}{3} = -\frac{14'4}{3} = -4'8$$

4- Indica el valor de x en cada caso:

$$\text{a) } 3^x = 0,05$$

$$\log 3^x = \log 0'05 \quad x \cdot \log 3 = \log 0'05 \quad x = \log 0'05 / \log 3 = -2'73$$

$$\text{b) } 7^x = 115$$

$$x = \log 115 / \log 7 = 2'44$$

$$\text{c) } \log_x \frac{1}{4} = 2$$

$$x^2 = 1/4 = (1/2)^2 \quad x = 1/2$$

$$\text{d) } \log_x 0'04 = -2$$

$$x^{-2} = 0'04 = 4/100 \quad x^2 = 100/4 = 25 \quad x = 5$$

$$\text{e) } \log_x 4 = -1/2$$

$$x^{-1/2} = \frac{1}{\sqrt{x}} = 4 \quad \sqrt{x} = \frac{1}{4} \quad x = \frac{1}{2}$$

ECUACIONES CON LOGARITMOS

a)

$$\log x + \log 50 = \log 1000$$

$$\log(50x) = \log 1000 \quad 50x = 1000 \quad x = \frac{1000}{50} = 20$$

b)

$$2 \log x^3 = \log 8 + 3 \log x$$

$$6 \log x = \log 8 + 3 \log x \quad 6 \log x - 3 \log x = \log 8 \quad 3 \log x = \log 8$$

$$\log x^3 = \log 2^3 \quad x^3 = 2^3 \quad x = 2$$

c)

$$3 \log x^2 = 4 + 4 \log x$$

$$6 \log x - 4 \log x = 4 \quad 2 \log x = 4 \quad \log x = 2 \quad \log x = \log 10^2 \quad x = 10^2 = 100$$

d)

$$\log(1 + x^2) - 1 = \log(x - 2)$$

$$\begin{array}{ll} \log(1+x^2) - \log 10 = \log(x-2) & \log\left(\frac{1+x^2}{10}\right) = \log(x-2) \quad \frac{1+x^2}{10} = x-2 \\ x^2 - 10x + 21 = 0 & x = 7, \quad x = 3 \end{array}$$

e)

$$\log 10(x+2) - \log(x^2) = 1$$

$$\log\frac{10(x+2)}{x^2} = \log 10 \quad \frac{10(x+2)}{x^2} = 10 \quad x^2 - x - 2 = 0 \quad x = 2, \quad x = -1$$