IES RICARDO MELLA 2º Bacharelato Matemáticas II

Nome: ………………………………………………………………………………… Data: ……………………………

TAREFA Nº2 - 1ª AVALIACIÓN: INTEGRAIS – SOLUCIÓN

ESTUDANTE A:

(10 puntos; 1 punto por cada apartado) **Calcula as seguintes integrais:**

1. $$\int (x^2+1)^2\,dx = \int (x^4+2x^2+1)\,dx = \frac{x^5}{5}+\frac{2x^3}{3}+x+k$$

2. $$\int (x - sen\,x)\,dx = \frac{x^2}{2}+cos\,x+k$$

3. $$\int \frac{x+\sqrt{x}}{x^2}\,dx = \int\left(\frac{1}{x}+x^{-3/2}\right)dx = ln\,|x| - \frac{2}{\sqrt{x}}+k$$

4. $$\int \frac{dx}{x\cdot\sqrt{1-(ln\,x)^2}} = arc\,sen\,ln\,x+k,\ \text{ya que}\ D[ln\,x] = \frac{1}{x}$$

5. $$\int \frac{sen\,x\,dx}{cos^5 x} = -\int(-sen\,x)\cdot cos^{-5}x\,dx = \frac{-\ cos^{-4}x}{-4}+k = \frac{1}{4cos^4 x}+k$$

6. $$\int \frac{1}{x^2+x-6}\,dx$$

$$\frac{1}{x^2+x-6} = \frac{A}{x+3}+\frac{B}{x-2} \quad A=-1/5,\quad B=1/5$$

$$\int \frac{1}{x^2+x-6}\,dx = \int\frac{-1/5}{x+3}\,dx+\int\frac{1/5}{x-2}\,dx = -\frac{1}{5}\,ln\,|x+3|+\frac{1}{5}\,ln\,|x-2|+k$$

7. $$\int \frac{dx}{x-\sqrt[4]{x}}$$

Cambio: $x = t^4 \ \rightarrow\ dx = 4t^3\,dt$

$$\int\frac{dx}{x-\sqrt[4]{x}} = \int\frac{4t^3\,dt}{t^4-t} = \int\frac{4t^2\,dt}{t^3-1} = \frac{4}{3}\int\frac{3t^2\,dt}{t^3-1} = \frac{4}{3}\,ln\,|t^3-1|+k = \frac{4}{3}\,ln\,|\sqrt[4]{x^3}-1|+k$$

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8. $\int x^3 \, sen\, x \, dx$

$$\begin{cases} u = x^3 \;\rightarrow\; du = 3x^2\, dx \\ dv = sen\, x\, dx \;\rightarrow\; v = -cos\, x \end{cases}$$

$$\int x^3 \, sen\, x\, dx = -x^3\, cos\, x + 3 \int \underbrace{x^2\, cos\, x\, dx}_{I_1}$$

$$\begin{cases} u_1 = x^2 \;\rightarrow\; du_1 = 2x\, dx \\ dv_1 = cos\, x\, dx \;\rightarrow\; v_1 = sen\, x \end{cases}$$

$$I_1 = x^2\, sen\, x - 2 \int \underbrace{x\, sen\, x\, dx}_{I_2}$$

$$\begin{cases} u_2 = x \;\rightarrow\; du_2 = dx \\ dv_2 = sen\, x\, dx \;\rightarrow\; v_2 = -cos\, x \end{cases}$$

$$I_2 = -x\, cos\, x + \int cos\, x\, dx = -x\, cos\, x + sen\, x$$

Así: $I_1 = x^2\, sen\, x + 2x\, cos\, x - 2sen\, x$

Por tanto:

$$\int x^3\, sen\, x\, dx = -x^3\, cos\, x + 3x^2\, sen\, x + 6x\, cos\, x - 6sen\, x + k$$

9. $$\int \sqrt{x^2-2x}\,(x-1)\, dx = \frac{1}{2}\int \sqrt{x^2-2x}\,(2x-2)\, dx = \frac{1}{2}\int (x^2-2x)^{1/2}(2x-2)\, dx =$$

$$= \frac{1}{2}\,\frac{(x^2-2x)^{3/2}}{\frac{3}{2}} + k = \frac{\sqrt{(x^2-2x)^3}}{3} + k$$

10. $\int x \cdot 2^{-x}\, dx$

$$\begin{cases} u = x \;\rightarrow\; du = dx \\ dv = 2^{-x}\, dx \;\rightarrow\; v = \dfrac{-2^{-x}}{ln\, 2} \end{cases}$$

$$\int x\, 2^{-x}\, dx = \frac{-x\cdot 2^{-x}}{ln\, 2} + \int \frac{2^{-x}}{ln\, 2}\, dx = \frac{-x\cdot 2^{-x}}{ln\, 2} + \frac{1}{ln\, 2}\int 2^{-x}\, dx = \frac{-x\cdot 2^{-x}}{ln\, 2} - \frac{2^{-x}}{(ln\, 2)^2} + k$$

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ESTUDANTE B:

(10 puntos; 1 punto por cada apartado) **Calcula as seguintes integrais:**

1. $$\int (x-5)^3\,dx = \frac{(x-5)^4}{4} + k$$

2. $$\int (e^{2x} + 3e^{-x})\,dx = \int e^{2x}\,dx + \int 3e^{-x}\,dx = \frac{1}{2}\int e^{2x}\,2\,dx - 3\int e^{-x}(-1)\,dx = \frac{1}{2}e^{2x} - 3e^{-x} + k$$

$$= \frac{1}{2}e^{2x} - \frac{3}{e^x} + k$$

3. $$\int \frac{3x}{1+x^2}\,dx = \frac{3}{2}\int \frac{2x}{1+x^2}\,dx = \frac{3}{2}\,ln\,(1+x^2) + k$$

4. $$\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{1/2\,dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}} = arc\ sen\left(\frac{x}{2}\right) + k$$

5. $$\int \frac{2x\,dx}{\sqrt{9-x^2}} = -\int -2x(9-x^2)^{-1/2}\,dx = -\frac{(9-x^{2})^{1/2}}{\frac{1}{2}} + k = -2\sqrt{9-x^2} + k$$

6. $$\int \frac{x^2+1}{x^2+x}\,dx$$

Descomponemos en fracciones simples:

$$\frac{x^2+1}{x^2+x} = 1 + \frac{-x+1}{x^2+x} = 1 + \frac{1}{x} - \frac{2}{x+1}$$

$$\int \frac{x^2+1}{x^2+x}\,dx = x + ln\,|x| - 2ln\,|x+1| + k$$

7. $$\int \frac{1}{x+\sqrt{x}}\,dx$$

Cambio: $x = t^2 \;\rightarrow\; dx = 2t\,dt$

$$\int \frac{1}{x+\sqrt{x}}\,dx = \int \frac{2t\,dt}{t^2+t} = \int \frac{2\,dt}{t+1} = 2\,ln\,|t+1| + k = 2\,ln\,(\sqrt{x}+1) + k$$

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8. $\int e^{2x+1} \cos x \, dx$

Esta es una integral que se resuelve aplicando el método de integración por partes dos veces:

$$I = \int e^{2x+1} \cos x \, dx$$

Integramos por partes:

$$\begin{cases} u = e^{2x+1} \rightarrow du = 2e^{2x+1}\, dx \\ dv = \cos x \, dx \rightarrow v = \operatorname{sen} x \end{cases}$$

$$I = e^{2x+1} \operatorname{sen} x - 2\int e^{2x+1} \operatorname{sen} x \, dx = e^{2x+1} \operatorname{sen} x - 2I_1$$

$$I_1 = \int e^{2x+1} \operatorname{sen} x \, dx$$

Integramos I_1 por partes:

$$\begin{cases} u = e^{2x+1} \rightarrow du = 2e^{2x+1}\, dx \\ dv = \operatorname{sen} x \, dx \rightarrow v = -\cos x \end{cases}$$

$$I_1 = -e^{2x+1} \cos x + 2\int e^{2x+1} \cos x \, dx$$

$$I = e^{2x+1} \operatorname{sen} x - 2\left(-e^{2x+1} \cos x + 2I\right) \Rightarrow 5I = e^{2x+1} \operatorname{sen} x + 2\, e^{2x+1} \cos x$$

$$I = \int e^{2x+1} \cos x \, dx = \frac{1}{5}\, e^{2x+1} \operatorname{sen} x + \frac{2}{5}\, e^{2x+1} \cos x$$

9. $$\int \frac{x+4}{\sqrt{1-x^2}}\, dx = \int \frac{x}{\sqrt{1-x^2}}\, dx + \int \frac{4}{\sqrt{1-x^2}}\, dx = -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\, dx + 4\int \frac{1}{\sqrt{1-x^2}}\, dx =$$

$$= -\frac{1}{2}\,\frac{(1-x^2)^{1/2}}{\frac{1}{2}} + 4 \operatorname{arc\,sen} x + k = -\sqrt{1-x^2} + 4\operatorname{arc\,sen} x + k$$

10. $\int x\, e^{2x}\, dx$

$$\begin{cases} u = x \rightarrow du = dx \\ dv = e^{2x}\, dx \rightarrow v = \frac{1}{2} e^{2x} \end{cases}$$

$$\int x\, e^{2x}\, dx = \frac{x}{2} e^{2x} - \int \frac{1}{2} e^{2x}\, dx = \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} + k$$

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TAREFA Nº2 - 1ª AVALIACIÓN: INTEGRAIS – SOLUCIÓN

ESTUDANTE C:

(10 puntos; 1 punto por cada apartado) **Calcula as seguintes integrais:**

1. $$\int \sqrt{3x+5}\, dx = \frac{1}{3}\int (3x+5)^{1/2}\, 3\, dx = \frac{1}{3}\,\frac{(3x+5)^{3/2}}{3} + k = \frac{2}{9}\sqrt{(3x+5)^3} + k$$

2. $$\int (\cos x + e^x)\, dx = \int \cos x\, dx + \int e^x\, dx = sen\, x + e^x + k$$

3. $$\int \frac{x^2}{2-x^3}\, dx = -\frac{1}{3}\int \frac{-3x^2}{2-x^3}\, dx = -\frac{1}{3}\, ln\left|2-x^3\right| + k$$

4. $$\int \frac{1}{\sqrt{1-100x^2}}\, dx = \frac{1}{10}\int \frac{10}{\sqrt{1-(10x)^2}}\, dx = \frac{1}{10}\, arc\ sen\, 10x + k$$

5. $$\int \frac{x\, dx}{\sqrt{x^2+5}} = \frac{1}{2}\int 2x(x^2+5)^{-1/2}\, dx = \frac{1}{2}\,\frac{(x^2+5)^{1/2}}{\frac{1}{2}} = \sqrt{x^2+5} + k$$

6. $$\int \frac{3x-1}{2x^2+8}\, dx$$

Como el denominador no tiene raíces:

$$I = \frac{3}{4}\int \frac{4x}{2x^2+8}\, dx - \int \frac{dx}{2x^2+8} = \frac{3}{4}\, ln\,(2x^2+8) - \frac{1}{8}\int \frac{dx}{\left(\frac{x}{2}\right)^2+1} =$$

$$= \frac{3}{4}\, ln\,(2x^2+8) - \frac{1}{8}\,\frac{1}{\frac{1}{2}}\, arc\ tg\, \frac{x}{2} + k = \frac{3}{4}\, ln\,(2x^2+8) - \frac{1}{4}\, arc\ tg\, \frac{x}{2} + k$$

7. $$\int \frac{\sqrt{x}}{1+x}\, dx$$

Cambio: $x = t^2 \rightarrow dx = 2t\, dt$

$$\int \frac{\sqrt{x}}{1+x}\, dx = \int \frac{t\cdot 2t\, dt}{1+t^2} = \int \frac{2t^2\, dt}{1+t^2} = \int \left(2 - \frac{2}{1+t^2}\right) dt = 2t - 2\, arc\ tg\, t + k = 2\sqrt{x} - 2\, arc\ tg\, \sqrt{x} + k$$

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8. $$\int 3x \cos x \, dx = 3\int x \cos x \, dx$$

$$\begin{cases} u = x \;\rightarrow\; du = dx \\ dv = \cos x \, dx \;\rightarrow\; v = \operatorname{sen} x \end{cases}$$

$$3\int x \cos x \, dx = 3\left[x \operatorname{sen} x - \int \operatorname{sen} x \, dx\right] = 3\,[x \operatorname{sen} x + \cos x] + k = 3x \operatorname{sen} x + 3\cos x + k$$

9. $$\int \frac{x^3}{(x+1)^2}\, dx$$

$$I = \int \frac{x^3}{(x+1)^2}\, dx = \int \left(x - 2 + \frac{3x+2}{(x+1)^2}\right) dx = \int (x-2)\, dx + \int \frac{3x+2}{(x+1)^2}\, dx$$

Descomponemos la segunda integral en fracciones simples:

$$\frac{3x+2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \;\rightarrow\; A = 3,\;\; B = -1$$

$$\int \frac{3x+2}{(x+1)^2}\, dx = 3\int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} = 3 \ln|x+1| + \frac{1}{x+1}$$

Sustituimos en I:

$$I = \frac{x^2}{2} - 2x + 3 \ln|x+1| + \frac{1}{x+1} + k$$

10. $$\int x^2 e^{2x}\, dx$$

$$\begin{cases} u = x^2 \;\rightarrow\; du = 2x \, dx \\ dv = e^{2x}\, dx \;\rightarrow\; v = \frac{1}{2} e^{2x} \end{cases}$$

$$\int x^2 e^{2x}\, dx = \frac{x^2}{2} e^{2x} - \underbrace{\int x\, e^{2x}\, dx}_{I_1}$$

$$\begin{cases} u_1 = x \;\rightarrow\; du_1 = dx \\ dv_1 = e^{2x}\, dx \;\rightarrow\; v_1 = \frac{1}{2} e^{2x} \end{cases}$$

$$I_1 = \frac{x}{2} e^{2x} - \int \frac{1}{2} e^{2x}\, dx = \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x}$$

Por tanto:

$$\int x^2 e^{2x}\, dx = \frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} + \frac{1}{4} e^{2x} + k = \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right) e^{2x} + k$$