

BOLETÍN 1.5 .- CÁLCULO PRÁCTICO DE DERIVADAS

Halla la función derivada de las siguientes funciones:

$$1. f(x) = 3x^2 - 6x + 5$$

$$7. f(x) = x e^x$$

$$2. f(x) = \sqrt{x} + \sqrt[3]{x}$$

$$8. f(x) = x \cdot 2^x$$

$$3. f(x) = \sqrt{2x} + \sqrt[3]{5x}$$

$$9. f(x) = (x^2 + 1) \cdot \log_2 x$$

$$4. f(x) = \frac{1}{x\sqrt{x}}$$

$$10. f(x) = \frac{x^2 + 1}{x^2 - 1}$$

$$5. f(x) = \sin x \cos x$$

$$11. f(x) = \frac{x^3 + 3x^2 - 5x + 3}{x}$$

$$6. f(x) = \operatorname{tg} x$$

$$12. f(x) = \frac{\log x}{x}$$

Halla la función derivada de las siguientes funciones compuestas:

$$13. f(x) = \sin \sqrt{x}$$

$$21. f(x) = \sin (x^2 - 5x + 7)$$

$$14. f(x) = \cos x^2$$

$$22. f(x) = \sqrt[3]{(5x+3)^2} = (5x+3)^{2/3}$$

$$15. f(x) = \operatorname{tg}(2x + \pi)$$

$$23. f(x) = \sin(3x+1) \cdot \cos(3x+1)$$

$$16. f(x) = (\sin x)^2 = \sin^2 x$$

$$24. f(x) = \frac{\log x^2}{x}$$

$$17. f(x) = e^{3x-5}$$

$$25. f(x) = \cos(3x-\pi)$$

$$18. f(x) = \operatorname{arc tg}(x^2 - 2x)$$

$$26. f(x) = \sqrt{1+2x}$$

$$19. f(x) = \ln \sqrt{x}$$

$$27. f(x) = x e^{2x+1}$$

$$20. f(x) = \sin^2 \left(3x + \frac{\pi}{2} \right)$$

$$28. f(x) = \frac{\sin(x^2+1)}{\sqrt{1-x^2}}$$

SOLUCIONES

1. $f'(x) = 6x - 6$

2. $f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}}$

3. $f'(x) = \frac{1}{\sqrt{2x}} + \frac{5}{3\sqrt[3]{5x}}$

4. $f(x) = x^{-3/2} \rightarrow f'(x) = -\frac{3}{2}x^{-5/2} = \frac{-3}{2\sqrt{x^5}} = \frac{-3}{2x^2\sqrt{x}}$

5. $f'(x) = \cos^2 x - \sin^2 x$

6. $f'(x) = 1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$

7. $f'(x) = e^x + x e^x = e^x(1+x)$

8. $f'(x) = 2^x + x \cdot 2^x \cdot \ln 2 = 2^x(1+x \ln 2)$

9. $f'(x) = 2x \log_2 x + (x^2 + 1) \cdot \frac{1}{x} \cdot \frac{1}{\ln 2} = 2x \log_2 x + \frac{(x^2 + 1)}{x \ln 2}$

10. $f'(x) = \frac{2x(x^2 - 1) - (x^2 + 1)2x}{(x^2 - 1)^2} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$

11. $f'(x) = \frac{(3x^2 + 6x - 5)x - (x^3 + 3x^2 - 5x + 3)}{x^2} = \frac{2x^3 + 3x^2 - 3}{x^2} = 2x + 3 - \frac{3}{x^2}$

12. $f'(x) = \frac{[1/(\ln 10)] - \log x}{x^2} = \frac{1 - \ln 10 \log x}{x^2 \ln 10}$

19. $f'(x) = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$

13. $f'(x) = (\cos \sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

O, de otro modo: $\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x \rightarrow f'(x) = \frac{1}{2} \cdot \frac{1}{x}$

14. $f'(x) = (-\sin x^2) \cdot 2x = -2x \sin x^2$

20. $f'(x) = 2 \sin \left(3x + \frac{\pi}{2}\right) \cdot D \left[\sin \left(3x + \frac{\pi}{2}\right)\right]$

15. $f'(x) = [1 + \operatorname{tg}^2(2x + \pi)] \cdot 2$

$D \left[\sin \left(3x + \frac{\pi}{2}\right)\right] = \cos \left(3x + \frac{\pi}{2}\right) \cdot D \left(3x + \frac{\pi}{2}\right) = \cos \left(3x + \frac{\pi}{2}\right) \cdot 3$

16. $f'(x) = 2 \sin x (\cos x) = 2 \sin x \cos x$

Por tanto: $f'(x) = 2 \sin \left(3x + \frac{\pi}{2}\right) \cdot \cos \left(3x + \frac{\pi}{2}\right) \cdot 3$

17. $f'(x) = e^{3x-5} \cdot 3 = 3e^{3x-5}$

21. $f'(x) = (2x-5) \cos(x^2 - 5x + 7)$

18. $f'(x) = \frac{2x-2}{1+(x^2-2x)^2} = \frac{2x-2}{x^4-4x^3+4x^2+1}$

22. $f'(x) = \frac{2}{3} (5x+3)^{-1/3} \cdot 5 = \frac{10}{3\sqrt[3]{5x+3}}$

24. $f(x) = \frac{2 \log x}{x} \rightarrow f'(x) = \frac{2(1 - \ln 10 \log x)}{x^2 \ln 10}$

23. $f'(x) = 3 [\cos^2(3x+1) - \sin^2(3x+1)]$

25. $f'(x) = -3 \sin(3x-\pi)$

26. $f'(x) = \frac{1}{\sqrt{1+2x}}$

27. $f'(x) = e^{2x+1} + x e^{2x+1} \cdot 2 = e^{2x+1}(1+2x)$

28. $f'(x) = \frac{2x\sqrt{1-x^2}\cos(x^2+1) + [x\sin(x^2+1)]/\sqrt{1-x^2}}{1-x^2} = \frac{2x(1-x^2)\cos(x^2+1) + x\sin(x^2+1)}{\sqrt{(1-x^2)^3}}$