EJERCICIOS DE TRIGONOMETRÍA I

CAMBIOS DE UNIDADES

1) a) $45^\circ = \pi/4$ rad ; b) $120^\circ = \frac{120^\circ}{180^\circ}\cdot\pi \text{ rad} = \frac{2\pi}{3}$ rad

c) $690^\circ = \frac{690^\circ}{180^\circ}\pi \text{ rad} = \frac{13}{6}\pi$ rad ; d) $1470^\circ = \frac{1470}{180}\pi \text{ rad} = \frac{49}{6}\pi$ rad

2) a) $3 \text{ rad} = \frac{3\cdot 180^\circ}{\pi} = 171^\circ 53' 14.4''$; b) $2'5 \text{ rad} = \frac{2'5\cdot 180^\circ}{\pi} = 143^\circ 14' 22''$

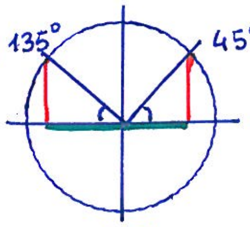
c) $\frac{7\pi}{2} \text{ rad} = \frac{7}{2}\cdot 180^\circ = 630^\circ$; d) $\frac{\pi}{5} \text{ rad} = \frac{180^\circ}{5} = 36^\circ$

3) $\frac{3\pi}{4}$ rad + 0'5 rectos + $50^\circ 40' 3'' = \left(\frac{3\pi}{4} + 0'5\cdot\pi + 50^\circ 40' 3''\cdot\frac{\pi}{180^\circ}\right)$ rad =

$= (2'3562 + 1'5708 + 0'8843)$ rad $= 4'8113$ rad

OPERAR CON ÁNGULOS CONOCIDOS

4) a)

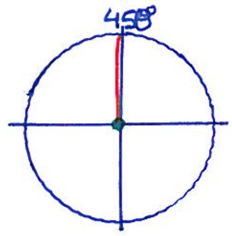


$\cos(135^\circ) = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\sqrt{2}/2$; $\sec 135^\circ = -\sqrt{2}$

$\text{sen}(135^\circ) = \text{sen}(180^\circ - 45^\circ) = \text{sen}\, 45^\circ = \sqrt{2}/2$; $\text{cosec}\, 135^\circ = \sqrt{2}$

$\text{tg}(135^\circ) = \frac{\text{sen}\, 135^\circ}{\cos 135^\circ} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$; $\text{cotg}\, 135^\circ = -1$

b)



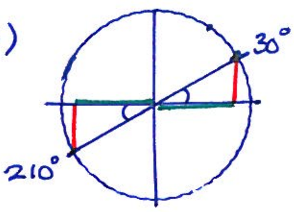
$450^\circ = 360^\circ + 90^\circ$

$\text{Sen}\, 450^\circ = \text{sen}\, 90^\circ = 1$; $\text{cosec}\, 450^\circ = 1$

$\cos 450^\circ = \cos 90^\circ = 0$; $\sec 450^\circ = \frac{1}{0}\ \nexists$

$\text{tg}\, 450^\circ = \frac{\text{sen}\, 90^\circ}{\cos 90^\circ} = \frac{1}{0}\ \nexists$; $\text{cotg}\, 450^\circ = \frac{0}{1} = 0$

c)

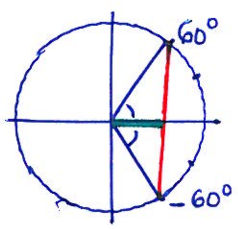


$\text{sen}\, 210^\circ = \text{sen}(180^\circ + 30^\circ) = -\text{sen}\, 30^\circ = -1/2$; $\text{cosec}\, 210^\circ = -2$

$\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\sqrt{3}/2$; $\sec 210^\circ = \frac{-2\sqrt{3}}{3}$

$\text{tg}\, 210^\circ = \frac{\text{sen}\, 210^\circ}{\cos 210^\circ} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$; $\text{cotg}\, 210^\circ = \sqrt{3}$

d)

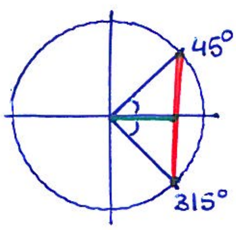


$\text{sen}(-60^\circ) = -\text{sen}\, 60^\circ = -\sqrt{3}/2$; $\text{cosec}(-60^\circ) = \frac{-2\sqrt{3}}{3}$

$\cos(-60^\circ) = +\cos 60^\circ = 1/2$; $\sec(-60^\circ) = 2$

$\text{tg}(-60^\circ) = \frac{\text{sen}\, 60^\circ}{\cos 60^\circ} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$; $\text{cotg}(-60^\circ) = -\frac{\sqrt{3}}{3}$

5) a)



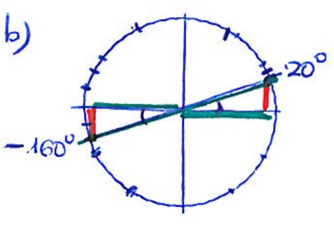
$1035^\circ = 2\cdot 360^\circ + 315^\circ$

$\text{Sen}(1035^\circ) = \text{sen}(315^\circ) = -\text{sen}\, 45^\circ = -\sqrt{2}/2$; $\text{cosec}(1035^\circ) = -\sqrt{2}$

$\cos(1035^\circ) = \cos(315^\circ) = \cos 45^\circ = \sqrt{2}/2$; $\sec 1035^\circ = \sqrt{2}$

$\text{tg}(1035^\circ) = \frac{\text{Sen}\, 1035^\circ}{\cos 1035^\circ} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$; $\text{cotg}\, 1035^\circ = -1$

b)



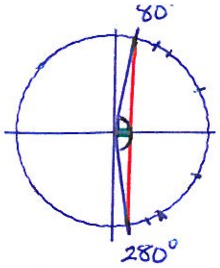
$-3400^\circ = -(9\cdot 360^\circ + 160^\circ)$

$\text{Sen}(-3400^\circ) = \text{sen}(-160^\circ) = -\text{sen}\, 20^\circ = -0'34$

$\cos(-3400^\circ) = \cos(-160^\circ) = -\cos 20^\circ = -0'93$

$\text{tg}(-3400^\circ) = \frac{\text{sen}(-3400^\circ)}{\cos(-3400^\circ)} = \text{tg}\, 20^\circ = 0'36$

c)



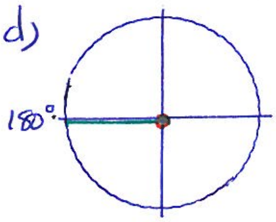
$10000° = 27 \cdot 360° + 280°$

$sen\ 10000° = sen\ 280° = -sen\ 80° = -0'98$

$cos\ 10000° = cos\ 280° = cos\ 80° = 0'17$

$tg\ 10000° = \frac{sen\ 10000}{cos\ 10000} = \frac{-sen\ 80°}{cos\ 80°} = -tg\ 80° = 5'67$

d)



$2700° = 7 \cdot 360° + 180°$

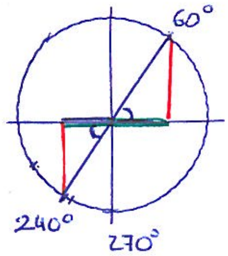
$sen\ 2700° = sen\ 180° = 0$

$cos\ 2700° = cos\ 180° = -1$

$tg\ 2700° = \frac{sen\ 2700°}{cos\ 2700°} = \frac{0}{-1} = 0$

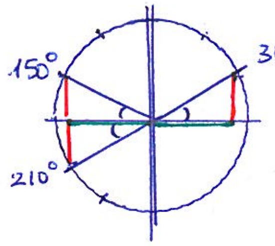
6) a) $2 \cdot tg\ 30° + 5 \cdot tg\ 240° - \underbrace{cos\ 270°}_{0} = 2 \cdot \frac{sen\ 30°}{cos\ 30°} + 5 \cdot \frac{+sen\ 60°}{+cos\ 60°} = 2 \cdot \frac{1/2}{\sqrt{3}/2} + 5 \cdot \frac{\sqrt{3}/2}{1/2} =$

$= \frac{2}{\sqrt{3}} + 5\sqrt{3} = \frac{2\sqrt{3}}{3} + 5\sqrt{3} = \boxed{\frac{17\sqrt{3}}{3}}$

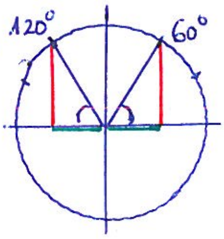


b) $cos\ 60° + sen\ 150° + sen\ 210° + cos\ 240° =$

$\frac{1}{2} + \cancel{sen\ 30°} - \cancel{sen\ 30°} - cos\ 60° = \frac{1}{2} - \frac{1}{2} = \boxed{0}$



7)

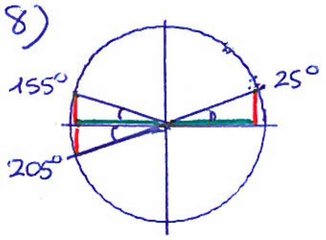


$sen\ 120° = sen\ 60° = \sqrt{3}/2$

$cos\ 120° = -cos\ 60° = -1/2$

$tg\ 120° = \frac{sen\ 120°}{cos\ 120°} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

8)



$sen\ 155° = sen(180° - 25°) = sen\ 25° = 0'42$

$cos\ 155° = cos(180° - 25°) = -cos\ 25° = -0'91$

$tg\ 155° = \frac{sen\ 25°}{-cos\ 25°} = -tg\ 25° = -0'47$

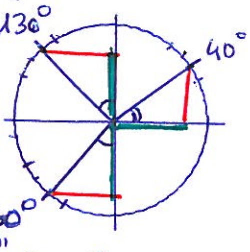
$sen\ 205° = sen(180° + 25°) = -sen\ 25° = -0'42$

$cos\ 205° = cos(180° + 25°) = -cos\ 25° = -0'91$

$tg\ 205° = \frac{+sen\ 25°}{+cos\ 25°} = tg\ 25° = 0'47$

9)

$90° + 40°$ ‖ $130°$



$230°$ ‖ $270° - 40°$

$sen\ 130° = sen(90° + 40°) = cos\ 40° = 0'77$

$cos\ 130° = cos(90° + 40°) = -sen\ 40° = -0'64$

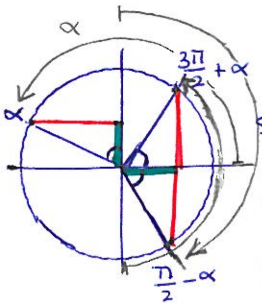
$tg\ 130° = \frac{cos\ 40°}{-sen\ 40°} = -\frac{1}{tg\ 40°} = -\frac{1}{0'84} = -1'19$

$sen\ 230° = sen(270° - 40°) = -cos\ 40° = -0'77$

$cos\ 230° = cos(270° - 40°) = -sen\ 40° = -0'64$

$tg\ 230° = \frac{+cos\ 40°}{+sen\ 40°} = \frac{1}{tg\ 40°} = \frac{1}{0'84} = 1'19$

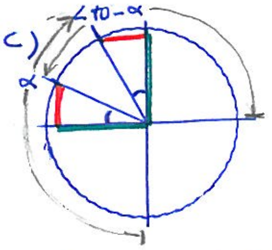
10)



$\left.\begin{matrix} sec\ \alpha = -4 \\ 0 < \alpha < \pi \end{matrix}\right\} \Rightarrow \left. cos\ \alpha = -1/4 < 0 \right\} \Rightarrow \alpha \in 2^{\underline{o}}$ cuadrante. $\left\{\begin{matrix} cos\ \alpha < 0 \\ sen\ \alpha > 0 \\ tg\ \alpha < 0 \end{matrix}\right.$

a) $cosec\left(\frac{3\pi}{2} + \alpha\right) = \frac{1}{sen\left(\frac{3\pi}{2} + \alpha\right)} = \frac{1}{-cos\ \alpha} = \frac{1}{-(-1/4)} = \boxed{4}$

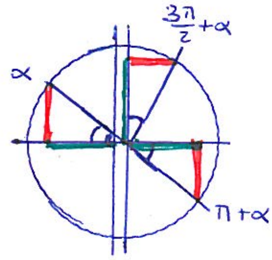
b) $sen\left(\frac{\pi}{2} - \alpha\right) = cos\ \alpha = \boxed{-1/4}$

c) 

$$tg(630^\circ-\alpha)=tg\left(360^\circ+(270^\circ-\alpha)\right)=\frac{sen(270^\circ-\alpha)}{\cos(270^\circ-\alpha)}=\frac{-\cos\alpha}{-sen\,\alpha}=\frac{-1/4}{\sqrt{15}/4}=-\frac{1}{\sqrt{15}}=\boxed{\frac{-\sqrt{15}}{15}}$$

$$\cos^2\alpha+sen^2\alpha=1\Rightarrow sen^2\alpha=1-\left(-1/4\right)^2=\frac{15}{16}\Rightarrow sen\,\alpha=\frac{\sqrt{15}}{4}$$

11) $\left.\begin{array}{l} sen\,\alpha=2/3>0 \\ \pi/2<\alpha<\frac{3\pi}{2}\end{array}\right\}\Rightarrow \frac{\pi}{2}<\alpha<\pi$ ($\alpha\in 2^{\circ}$ cuadrante) $\begin{cases} sen\,\alpha>0 \\ \cos\alpha<0 \\ tg\,\alpha<0\end{cases}$

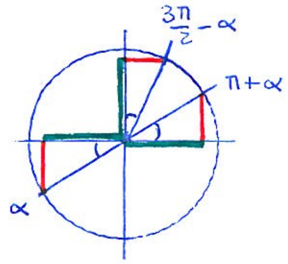


a) $\cos\left(\frac{3\pi}{2}+\alpha\right)=sen\,\alpha=\frac{2}{3}$

b) $tg(\pi+\alpha)=\frac{sen(\pi+\alpha)}{\cos(\pi+\alpha)}=\frac{-sen\,\alpha}{-\cos\alpha}=\frac{2/3}{-\sqrt{5}/3}=\frac{-2}{\sqrt{5}}=\boxed{\frac{-2\sqrt{5}}{5}}$

$sen^2\alpha+\cos^2\alpha=1\Rightarrow\cos^2\alpha=1-(2/3)^2=5/9\Rightarrow\cos\alpha=\frac{-\sqrt{5}}{3}$

12) $\left.\begin{array}{l}\cos\alpha=-2/3 \\ \pi<\alpha<2\pi\end{array}\right\}\Rightarrow\alpha\in 3^{er}$ cuadrante $\begin{cases} sen\,\alpha<0 \\ \cos\alpha<0 \\ tg\,\alpha>0\end{cases}$



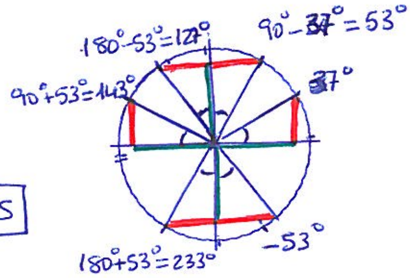
a) $\cos\left(\frac{3\pi}{2}-\alpha\right)=-sen\,\alpha=\boxed{\frac{\sqrt{5}}{3}}$

$sen^2\alpha+\cos^2\alpha=1\Rightarrow sen^2\alpha=1-\left(-2/3\right)^2=\frac{5}{9}\Rightarrow sen\,\alpha=-\frac{\sqrt{5}}{3}$

b) $tg(\pi+\alpha)=\frac{sen(\pi+\alpha)}{\cos(\pi+\alpha)}=\frac{-sen\,\alpha}{-\cos\alpha}=\frac{+\sqrt{5}/3}{+2/3}=\boxed{\frac{\sqrt{5}}{2}}$

13) $\cos 53^\circ=0'6$

$sen^2 53^\circ+\cos^2 53^\circ=1\Rightarrow sen^2 53^\circ=1-(0'6)^2=0'64\Rightarrow sen\,53^\circ=0'8$



a) $\cos 37^\circ=\cos(90^\circ-53^\circ)=sen\,53^\circ=\boxed{0'8}$

b) $sen(143^\circ)=sen(90^\circ+53^\circ)=\cos 53^\circ=\boxed{0'6}$

c) $tg\,127^\circ=\frac{sen\,127^\circ}{\cos 127^\circ}=\frac{sen(180^\circ-53^\circ)}{\cos(180^\circ-53^\circ)}=\frac{sen\,53^\circ}{-\cos 53^\circ}=\frac{0'8}{-0'6}=\boxed{-4/3}$

d) $cotg\,233^\circ=\frac{\cos 233^\circ}{sen\,233^\circ}=\frac{\cos(180^\circ+53^\circ)}{sen(180^\circ+53^\circ)}=\frac{+\cos 53^\circ}{+sen\,53^\circ}=\frac{0'6}{0'8}=\frac{3}{4}=\boxed{0'75}$

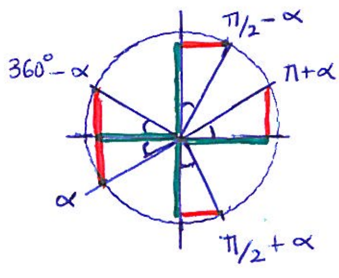
e) $\sec(-53^\circ)=\frac{1}{\cos(-53^\circ)}=\frac{1}{\cos 53^\circ}=\frac{1}{0'6}=\frac{10}{6}=\boxed{5/3}$

14) $tg\,\alpha=\frac{1}{2}$; $\alpha\in 3^{er}$ cuadrante

a) $sen(\pi/2+\alpha)=\cos\alpha=\boxed{\frac{-2\sqrt{5}}{5}}$

$\frac{sen^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}\Rightarrow tg^2\alpha+1=\frac{1}{\cos^2\alpha}\Rightarrow\left(\frac{1}{2}\right)^2+1=\frac{1}{\cos^2\alpha}\Rightarrow\cos\alpha=\frac{-2}{\sqrt{5}}$

b) $\cos(\pi+\alpha)=-\cos\alpha=\boxed{\frac{2\sqrt{5}}{5}}$



c) $tg(\pi/2-\alpha)=\frac{sen(\pi/2-\alpha)}{\cos(\pi/2-\alpha)}=\frac{+\cos\alpha}{+sen\,\alpha}=\frac{+\frac{2\sqrt{5}}{5}}{+\sqrt{5}/5}=\boxed{2}$

$tg\,\alpha=\frac{sen\,\alpha}{\cos\alpha}\Rightarrow\frac{1}{2}=\frac{sen\,\alpha}{-2/\sqrt{5}}\Rightarrow sen\,\alpha=-\frac{1}{\sqrt{5}}=\boxed{\frac{-\sqrt{5}}{5}}$

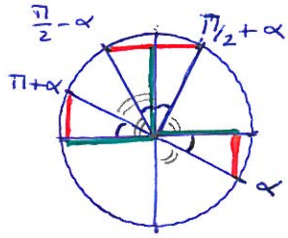
d) $\sec(360^\circ-\alpha)=\frac{1}{\cos(360^\circ-\alpha)}=\frac{1}{\cos\alpha}=\boxed{\frac{-\sqrt{5}}{2}}$

15) $\cot\alpha = -2 \Rightarrow \operatorname{tg}\alpha = -1/2 < 0$
$\pi < \alpha < 2\pi$ $\Big\} \Rightarrow \alpha \in 4^{\circ}$ cuadrante $\begin{cases} \operatorname{sen}\alpha < 0 \\ \cos\alpha > 0 \\ \operatorname{tg}\alpha < 0 \end{cases}$

a) $\cos\left(\pi/2 + \alpha\right) = -\operatorname{sen}\alpha = \boxed{\dfrac{\sqrt{5}}{5}}$

$$\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha} + \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{1}{\operatorname{sen}^2\alpha} \Rightarrow 1 + \cot^2\alpha = \frac{1}{\operatorname{sen}^2\alpha} \Rightarrow 1 + (-2)^2 = \frac{1}{\operatorname{sen}^2\alpha} \Rightarrow \operatorname{sen}\alpha = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$$

b) $\operatorname{sen}(\pi + \alpha) = -\operatorname{sen}\alpha = \boxed{\dfrac{\sqrt{5}}{5}}$

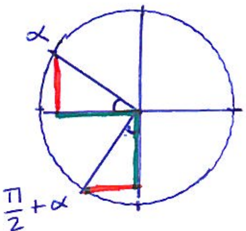


c) $\cot\left(\pi/2 - \alpha\right) = \dfrac{\cos(\pi/2 - \alpha)}{\operatorname{sen}(\pi/2 - \alpha)} = \dfrac{\operatorname{sen}\alpha}{-\cos\alpha} = \dfrac{-\sqrt{5}/5}{2\sqrt{5}/5} = -1/2$

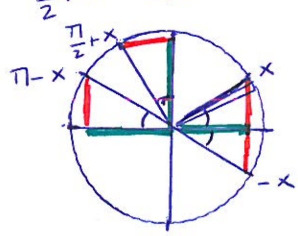
$$\cot\alpha = \frac{\cos\alpha}{\operatorname{sen}\alpha} \Rightarrow (-2) = \frac{\cos\alpha}{-\sqrt{5}/5} \Rightarrow \cos\alpha = \frac{2\sqrt{5}}{5}$$

16) $\cos\alpha = \operatorname{sen}\left(\pi/2 + \alpha\right) = -1/3$

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \operatorname{sen}^2\alpha + 1/9 = 1 \Rightarrow \operatorname{sen}^2\alpha = 8/9 \Rightarrow \operatorname{sen}\alpha = \frac{\sqrt{8}}{3}$$

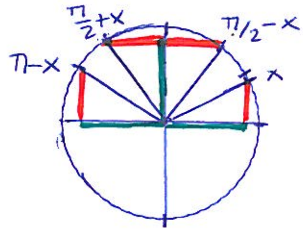


17) $$\frac{\operatorname{sen}\left(\frac{\pi}{2} + x\right) + \cos(\pi - x) + \operatorname{sen}(\pi - x)}{\cos(-x) + \operatorname{sen}(-x)} = \frac{\cancel{\cos x} - \cancel{\cos x} + \operatorname{sen} x}{\cos x - \operatorname{sen} x} = \boxed{\frac{1}{\cot x - 1}}$$

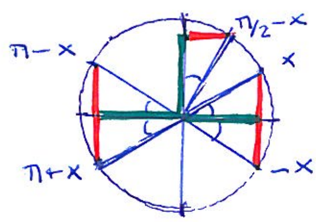


18) $$\frac{\cot(\pi/2 - x) \cdot \operatorname{sen}(\pi/2 + x)}{2\operatorname{tg}(\pi - x)} = \frac{\dfrac{\cos(\pi/2 - x)}{\operatorname{sen}(\pi/2 - x)} \cdot \operatorname{sen}(\pi/2 + x)}{2 \cdot \dfrac{\operatorname{sen}(\pi - x)}{\cos(\pi - x)}} =$$

$$= \frac{\dfrac{\operatorname{sen} x}{\cancel{\cos x}} \cdot \cancel{\cos x}}{2 \cdot \dfrac{\operatorname{sen} x}{-\cos x}} = \frac{-\cancel{\operatorname{sen} x} \cdot \cos x}{2 \cdot \cancel{\operatorname{sen} x}} = \boxed{\frac{-\cos x}{2}}$$



19) $$\frac{\operatorname{tg}(\pi - x) \cdot \cos(-x)}{\cot(\pi - x) \cdot \cos(\pi/2 - x)} = \frac{\dfrac{\operatorname{sen}(\pi - x)}{\cos(\pi - x)} \cdot \cos(-x)}{\dfrac{\cos(\pi + x)}{\operatorname{sen}(\pi + x)} \quad \cos(\pi/2 - x)} =$$



$$= \frac{\dfrac{\operatorname{sen} x}{-\cancel{\cos x}} \cdot \cancel{\cos x}}{\dfrac{+\cos x}{+\cancel{\operatorname{sen} x}} \cdot \cancel{\operatorname{sen} x}} = \frac{-\operatorname{sen} x}{\cos x} = \boxed{-\operatorname{tg} x}$$

FÓRMULAS TRIGONOMÉTRICAS

20) $\left.\begin{array}{l} 90^\circ < \alpha < 360^\circ \\ tg\,\alpha = 3/4 > 0 \end{array}\right\} \Rightarrow \alpha \in 3^{er}$ cuadrante $\begin{cases} sen\,\alpha < 0 \\ cos\,\alpha < 0 \\ tg\,\alpha > 0 \end{cases}$

a) $sen\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$

- $90^\circ < \alpha < 360^\circ \Rightarrow 45^\circ < \frac{\alpha}{2} < 180^\circ \Rightarrow sen\left(\frac{\alpha}{2}\right) > 0$
- $sen^2\alpha + cos^2\alpha = 1 \Rightarrow tg^2\alpha + 1 = \frac{1}{cos^2\alpha} \Rightarrow \left(\frac{3}{4}\right)^2 + 1 = \frac{1}{cos^2\alpha} \Rightarrow \frac{25}{16} = \frac{1}{cos^2\alpha} \Rightarrow$

$\Rightarrow cos^2\alpha = \frac{16}{25} \underset{\alpha \in 3^{er} \text{ cuadr.}}{\Rightarrow} cos\,\alpha = -\frac{4}{5}$

b) $tg\left(\alpha + \frac{3\pi}{4}\right) = \frac{tg\,\alpha + tg(3\pi/4)}{1 - tg\,\alpha \cdot tg(3\pi/4)} = \frac{\frac{3}{4} + (-1)}{1 + 3/4} = \frac{-1/4}{7/4} = \boxed{\frac{-1}{7}}$

21) $\left.\begin{array}{l} cos\,x = -4/5 \\ \pi < x < 2\pi \end{array}\right\} \Rightarrow x \in 3^{er}$ cuadrante $\begin{cases} sen\,\alpha < 0 \\ cos\,\alpha < 0 \\ tg\,\alpha > 0 \end{cases}$

a) $cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-4/5}{2}} = -\sqrt{\frac{1}{10}} = \boxed{-\frac{\sqrt{10}}{10}}$

$\left[\frac{\pi}{2} < \frac{x}{2} < \pi\right]$

b) $sen\left(\frac{x}{2}\right) = +\sqrt{\frac{1-\cos x}{2}} = +\sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3\sqrt{10}}{10}}$ (No se pide en el ejercicio)

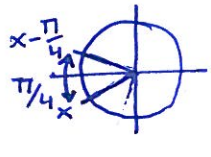
$sen(2x) = 2\,sen\,x \cdot cos\,x = 2 \cdot (-3/5) \cdot (-4/5) = \boxed{\frac{24}{25}}$

$sen^2x + cos^2x = 1 \Rightarrow sen^2x + (-4/5)^2 = 1 \Rightarrow sen^2x = \frac{9}{25} \Rightarrow sen\,x = -3/5$

$[\pi < x < 2\pi \Rightarrow sen\,x < 0]$

22) $\left.\begin{array}{l} sen\,x = -3/5 \\ \pi/2 \le x \le \frac{3\pi}{2} \end{array}\right\} \Rightarrow x \in 3^{er}$ cuadrante $\begin{cases} sen\,x < 0 \\ cos\,x < 0 \\ tg\,x > 0 \end{cases}$

a) $tg\left(x - \frac{\pi}{4}\right) = \frac{tg\,x - tg\left(\frac{\pi}{4}\right)}{1 + tg\,x \cdot tg\left(\frac{\pi}{4}\right)} = \frac{3/4 - 1}{1 + 3/4 \cdot 1} = \frac{-1/4}{7/4} = \boxed{\frac{-1}{7}} \Rightarrow x - \frac{\pi}{4} \in 2^{\circ}$ cuadrante



$sen^2x + cos^2x = 1 \Rightarrow (-3/5)^2 + cos^2x = 1 \Rightarrow cos\,x = -4/5$

$tg\,x = \frac{sen\,x}{cos\,x} = \frac{-3/5}{-4/5} = \frac{3}{4}$

b) $sen\left(\frac{x}{2}\right) = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3\sqrt{10}}{10}}$

$\frac{\pi}{4} \le \frac{x}{2} \le \frac{3\pi}{4} \Rightarrow sen\left(\frac{x}{2}\right) > 0$

23) $\left.\begin{array}{l} cos\,\alpha = -5/13 \\ 180^\circ < \alpha < 360^\circ \end{array}\right\} \Rightarrow \alpha \in 3^{er}$ cuadrante $\begin{cases} sen\,\alpha < 0 \\ cos\,\alpha < 0 \\ tg\,\alpha > 0 \end{cases}$

a) $sen(2\alpha) = 2 \cdot sen\,\alpha \cdot cos\,\alpha = 2 \cdot \left(\frac{-12}{13}\right) \cdot \left(\frac{-5}{13}\right) = \boxed{\frac{120}{169}} > 0 \Rightarrow sen(2\alpha) > 0$ (*)

$sen^2\alpha + cos^2\alpha = 1 \Rightarrow sen^2\alpha + (-5/13)^2 = 1 \Rightarrow sen^2\alpha = 1 - \frac{25}{169} \Rightarrow sen\,\alpha = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$

$180^\circ < \alpha < 360^\circ \Rightarrow 360^\circ < 2\alpha < 720^\circ \longrightarrow$ No se puede decidir el cuadrante

$\left.cos(2\alpha) = cos^2\alpha - sen^2\alpha = (-5/13)^2 - (-12/13)^2 = \frac{25-144}{169} < 0\right\} \Rightarrow \boxed{2\alpha \in 2^{\circ} \text{ cuadrante}}$

(*) $sen(2\alpha) > 0 \Rightarrow 2\alpha \in 1^{\circ}$ ó 2° cuadrante

b) $\operatorname{tg}\left(\frac{\alpha}{2}\right) \underset{\uparrow}{=} -\sqrt{\frac{1-\cos x}{1+\cos x}} = -\sqrt{\frac{1+5/13}{1-5/13}} = -\sqrt{\frac{18}{8}} = -\sqrt{\frac{9}{4}} = \boxed{-\frac{3}{2}}$

$\left[180° < \alpha < 360° \Rightarrow 90° < \frac{\alpha}{2} < 180° \Rightarrow \boxed{\frac{\alpha}{2} \in 2^{\circ} \text{ cuadrante}} \Rightarrow \operatorname{tg}\left(\frac{\alpha}{2}\right) < 0\right]$

24) $\left.\begin{array}{l} \operatorname{sen} x = 3/5 \\ \frac{\pi}{2} < x < \frac{3\pi}{2} \end{array}\right\} \Rightarrow x \in 2^{\circ}$ cuadrante (*) $\left\langle \begin{array}{l} \operatorname{sen} x > 0 \\ \cos x < 0 \\ \operatorname{tg} x < 0 \end{array}\right.$

a) $\operatorname{sen}(2x) = 2 \operatorname{sen} x \cdot \cos x = 2 \cdot \frac{3}{5} \cdot (-4/5) = \boxed{-24/25} \Rightarrow 2x \in 3^{\circ}$ ó 4° cuadrante (*)

· $\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow (3/5)^2 + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow \cos x = -\frac{4}{5}$

· $\cos(2x) = \cos^2 x - \operatorname{sen}^2 x = (-4/5)^2 - (3/5)^2 = 7/25 > 0$

$\left.\begin{array}{l} \operatorname{sen}(2x) < 0 \\ \cos(2x) > 0 \end{array}\right\} \Rightarrow \boxed{2x \in 4^{\circ} \text{ cuadrante}}$

b) $\cos\left(\frac{x}{2}\right) = +\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-4/5}{2}} = \sqrt{\frac{1}{10}} = \boxed{\frac{\sqrt{10}}{10}}$

(*) $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \Rightarrow \cos\left(\frac{x}{2}\right) > 0$

25) $\left.\begin{array}{l} \operatorname{sen} x = -3/5 \\ 90° \leq x \leq 270° \end{array}\right\} \Rightarrow x \in 3^{er}$ cuadrante (*) $\left\langle \begin{array}{l} \operatorname{sen} x < 0 \\ \cos x < 0 \\ \operatorname{tg} x > 0 \end{array}\right.$

a) $\operatorname{sen}(x/2) = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3\sqrt{10}}{10}}$

(*) $180° < x < 270° \Rightarrow 90° < \frac{x}{2} < 135° \Rightarrow \boxed{\frac{x}{2} \in 2^{\circ} \text{ cuadrante}} \Rightarrow \operatorname{sen}\left(\frac{x}{2}\right) > 0$

$\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow (-3/5)^2 + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \frac{9}{25} \Rightarrow \boxed{\cos x = -\frac{4}{5}}$

b) $\cos(2x) = \cos^2 x - \operatorname{sen}^2 x = (-4/5)^2 - (-3/5)^2 = \boxed{\frac{7}{25}} \Rightarrow 2x \in 1^{\circ}$ ó 2° cuadrante

· $\operatorname{sen}(2x) = 2 \cdot \operatorname{sen} x \cdot \cos x = 2 \cdot (-3/5) \cdot (-4/5) = 24/25 > 0$

· $\left.\begin{array}{l} \operatorname{sen}(2x) > 0 \\ \cos(2x) > 0 \end{array}\right\} \Rightarrow 2x \in 1^{er}$ cuadrante.

26) a) $\left.\begin{array}{l} \pi/2 < \alpha < \frac{3\pi}{2} \\ \operatorname{sen}\alpha = 1/3 \end{array}\right\} \Rightarrow \boxed{\alpha \in 2^{\circ} \text{ cuadrante}} \left\langle \begin{array}{l} \operatorname{sen} x > 0 \\ \cos x < 0 \\ \operatorname{tg} x < 0 \end{array}\right.$

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow (1/3)^2 + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow \boxed{\cos\alpha = \frac{-\sqrt{8}}{3}} = \frac{-2\sqrt{2}}{3}$

· $\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{1/3}{-\sqrt{8}/3} = -\frac{1}{\sqrt{8}} = \frac{-\sqrt{8}}{8} = \frac{-2\sqrt{2}}{8} = \frac{-\sqrt{2}}{4}$

· $\sec\alpha = 1/\cos\alpha = \frac{-3\sqrt{8}}{8}$; $\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} = 3$; $\operatorname{cotg}\alpha = \frac{1}{\operatorname{tg}\alpha} = -\sqrt{8} = -2\sqrt{2}$

b) $\frac{\pi}{2} < \alpha < \pi \Rightarrow \pi < 2\alpha < 2\pi$

$\left.\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha = \left(\frac{-\sqrt{8}}{3}\right)^2 - (1/3)^2 = \boxed{\frac{7}{9}} > 0 \right\} \Rightarrow \boxed{2\alpha \in 4^{\circ} \text{ cuadrante}}$

c) $\alpha \in 2^{\circ}$ cuadrante $\Rightarrow \frac{\pi}{2} < \alpha < \pi \Rightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2} \Rightarrow \boxed{\frac{\alpha}{2} \in 1^{\circ} \text{ cuadrante}} \Rightarrow \operatorname{sen}\left(\frac{\alpha}{2}\right) > 0$

$\operatorname{sen}\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1+\sqrt{8}/3}{2}} = \boxed{\sqrt{\frac{3+\sqrt{8}}{6}}}$

d) $\operatorname{tg}\left(\alpha - \frac{\pi}{4}\right) = \frac{\operatorname{tg}\alpha - \operatorname{tg}(\pi/4)}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}(\pi/4)} = \frac{-\sqrt{2}/4 - 1}{1 + \frac{\sqrt{2}}{4} \cdot 1} = \frac{-\sqrt{2}-1}{4+\sqrt{2}} < 0 \Rightarrow \boxed{\alpha - \frac{\pi}{4} \in 2^{\circ} \text{ cuadrante}}$

$\frac{\pi}{2} < \alpha < \pi \Rightarrow \frac{\pi}{4} < \alpha - \frac{\pi}{4} < \frac{3\pi}{4}$

27) $\left.\begin{array}{l} 90° < x < 270° \\ \operatorname{sen} x = -2/5 \end{array}\right\} \Rightarrow x \in 3^{er}$ cuadrante $\left\langle \begin{array}{l} \cos x < 0 \\ \operatorname{sen} x < 0 \\ \operatorname{tg} x > 0 \end{array}\right.$

a) $\operatorname{sen}(2x) = 2 \operatorname{sen} x \cdot \cos x = 2 \cdot \left(-\frac{2}{5}\right) \cdot \left(-\frac{\sqrt{21}}{5}\right) = \boxed{\frac{4\sqrt{21}}{25}}$

$\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow (-2/5)^2 + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \frac{4}{25} \Rightarrow \cos x = -\frac{\sqrt{21}}{5}$

$\cos(2x) = \cos^2 x - \operatorname{sen}^2 x = \left(-\frac{\sqrt{21}}{5}\right)^2 - (-2/5)^2 = \frac{17}{25} > 0$ $\left\langle \left.\begin{array}{l} \cos(2x) > 0 \\ \operatorname{sen}(2x) > 0 \end{array}\right\} \right. \Rightarrow \boxed{2x \in 1^{er} \text{ cuadrante}}$

27) b) $\cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-\sqrt{21}/5}{2}} = \boxed{-\sqrt{\frac{5-\sqrt{21}}{10}}}$

$180^\circ < \alpha < 270^\circ \Rightarrow 90^\circ < \frac{\alpha}{2} < 135^\circ \Rightarrow \boxed{\frac{x}{2} \in 2^{\circ} \text{ cuadrante}} \Rightarrow \cos\left(\frac{x}{2}\right) < 0$

c) $\cot g\,(x+45^\circ) = \frac{1}{tg(x+45^\circ)} = \frac{1 - tg x \cdot tg 45^\circ}{tg x + tg 45^\circ} = \frac{1 - \frac{2\sqrt{21}}{21}\cdot\frac{\sqrt{2}}{2}}{\frac{2\sqrt{21}}{21} + \frac{\sqrt{2}}{2}} = \boxed{\frac{42 - 2\sqrt{42}}{4\sqrt{21} + 21\sqrt{2}}} > 0$

$\Downarrow$

$tg(x+45^\circ) > 0$

$tg\,x = \frac{\operatorname{sen} x}{\cos x} = \frac{-2/5}{-\sqrt{21}/5} = \frac{2\sqrt{21}}{21}$ $\qquad tg(x+45^\circ) > 0$

$180^\circ < x < 270^\circ \Rightarrow 225^\circ < x + 45^\circ < 315^\circ \Rightarrow \boxed{x + 45^\circ \in 3^{er} \text{ cuadrante}}$

SIMPLIFICAR

28) a) $\frac{\cos^2 a - \operatorname{sen}^2 a}{\cos a + \operatorname{sen} a} = \frac{(\cos a - \operatorname{sen} a)\cdot\cancel{(\cos a + \operatorname{sen} a)}}{\cancel{\cos a + \operatorname{sen} a}} = \boxed{\cos a - \operatorname{sen} a}$

b) $\frac{\cos^2 a - \operatorname{sen}^2 a}{\operatorname{sen}^4 a - \cos^4 a} = \frac{\cancel{(-\cos^2 a + \operatorname{sen}^2 a)}\cdot(-1)}{\cancel{(\operatorname{sen}^2 a - \cos^2 a)}\cdot\underbrace{(\operatorname{sen}^2 a + \cos^2 a)}_{1}} = \frac{-1}{1} = \boxed{-1}$

c) $\frac{1}{1-\operatorname{sen} x} + \frac{1}{1+\operatorname{sen} x} - 2 = \frac{(1+\cancel{\operatorname{sen} x}) + (1 - \cancel{\operatorname{sen} x}) - 2(1-\operatorname{sen} x)(1+\operatorname{sen} x)}{(1-\operatorname{sen} x)\cdot(1+\operatorname{sen} x)} =$

$= \frac{2 - 2(1 - \operatorname{sen}^2 x)}{1 - \operatorname{sen}^2 x} = \frac{\cancel{2} - \cancel{2} + 2\operatorname{sen}^2 x}{\cos^2 x} = \boxed{2 \cdot tg^2 x}$

DEMOSTRAR IDENTIDADES

29) a) $\frac{1-\operatorname{sen}^2\alpha}{\cos\alpha} = \cos\alpha \iff \frac{\cos^2\alpha}{\cos\alpha} = \cos\alpha \iff \cos^2\alpha = \cos^2\alpha$ ✓

b) $tg\,x + \frac{1}{tg\,x} = tg\,x \cdot \frac{1}{1-\cos^2 x} \iff \frac{\operatorname{sen} x}{\cos x} + \frac{\cos x}{\operatorname{sen} x} = \frac{\cancel{\operatorname{sen} x}}{\cos x}\cdot\frac{1}{\operatorname{sen}^{\cancel{2}} x} \iff$

$\iff \frac{\overbrace{\operatorname{sen}^2 x + \cos^2 x}^{1}}{\cos x \cdot \operatorname{sen} x} = \frac{1}{\cos x \cdot \operatorname{sen} x} \iff \cos x \cdot \operatorname{sen} x = \cos x \cdot \operatorname{sen} x$ ✓

c) $\underbrace{\cos^2 x + \operatorname{sen}^2 x}_{1} + tg^2 x = \frac{1}{\cos^2 x} \iff 1 + \frac{\operatorname{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \iff \frac{\overbrace{\cos^2 x + \operatorname{sen}^2 x}^{1}}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\iff \cos^2 x = \cos^2 x$ ✓

d) $1 + \frac{1}{tg^2 x} = \frac{1}{\operatorname{sen}^2 x} \iff 1 + \frac{\cos^2 x}{\operatorname{sen}^2 x} = \frac{1}{\operatorname{sen}^2 x} \iff \frac{\overbrace{\operatorname{sen}^2 x + \cos^2 x}^{1}}{\operatorname{sen}^2 x} = \frac{1}{\operatorname{sen}^2 x} \iff$

$\operatorname{sen}^2 x = \operatorname{sen}^2 x$ ✓

e) $\operatorname{sen}^2 a - \cos^2 b = \operatorname{sen}^2 b - \cos^2 a \iff \operatorname{sen}^2 a + \cos^2 a = \operatorname{sen}^2 b + \cos^2 b \iff 1 = 1$ ✓

f) $\operatorname{sen} a \cdot \cos a \cdot \frac{\operatorname{sen} a}{\cos a} \cdot \frac{\cos a}{\operatorname{sen} a} \cdot \frac{1}{\cos a} \cdot \frac{1}{\operatorname{sen} a} = 1 \iff \operatorname{sen}^2 a \cdot \cos^2 a = \operatorname{sen}^2 a \cdot \cos^2 a$ ✓

30) a) $\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\operatorname{sen}\alpha} = \frac{1}{\cos\alpha}\cdot\frac{1}{\operatorname{sen}\alpha} \iff \frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\cos\alpha \cdot \operatorname{sen}\alpha} = \frac{1}{\cos\alpha \cdot \operatorname{sen}\alpha} \iff \cos\alpha \operatorname{sen}\alpha = \cos\alpha \operatorname{sen}\alpha$ ✓

b) $\frac{\cos^2 a}{\operatorname{sen}^2 a} = \cos^2 a + \left(\frac{\cos a}{\operatorname{sen} a}\cdot\cos a\right)^2 \iff \frac{\cos^2 a}{\operatorname{sen}^{\cancel{2}} a} = \frac{\cos^2 a \cdot \operatorname{sen}^2 a + \cos^4 a}{\operatorname{sen}^{\cancel{2}} a} \iff$

$\iff \cos^2 a = \cos^2 a\,(\underbrace{\operatorname{sen}^2 a + \cos^2 a}_{1}) \iff \cos^2 a = \cos^2 a$ ✓

c) $\frac{\cos a}{\operatorname{sen} a}\cdot\frac{1}{\cos a} = \frac{1}{\operatorname{sen} a} \iff \operatorname{sen} a \cdot \cos a = \operatorname{sen} a \cdot \cos a$ ✓

d) $\frac{1}{\cos^2 a} + \frac{1}{\operatorname{sen}^2 a} = \frac{1}{\operatorname{sen}^2 a \cdot \cos^2 a} \iff \frac{\operatorname{sen}^2 a + \cos^2 a}{\cos^2 a \cdot \operatorname{sen}^2 a} = \frac{1}{\operatorname{sen}^2 a \cdot \cos^2 a}$ ✓

e) $\frac{1+\operatorname{sen} x}{\cos x} = \frac{\cos x}{1-\operatorname{sen} x} \iff (1+\operatorname{sen} x)(1-\operatorname{sen} x) = \cos^2 x \iff \overbrace{1-\operatorname{sen}^2 x}^{=\cos^2 x} = \cos^2 x$ ✓

f) $\operatorname{sen}^4 a - \cos^4 a = \operatorname{sen}^2 a - \cos^2 a \iff (\overbrace{\operatorname{sen}^2 a + \cos^2 a}^{=1})(\operatorname{sen}^2 a - \operatorname{sen}^2 a) = \operatorname{sen}^2 a - \cos^2 a$ ✓

g) $\sqrt{1-\operatorname{sen} x} \cdot \sqrt{1+\operatorname{sen} x} = \pm \cos x \iff \sqrt{(1-\operatorname{sen} x)(1+\operatorname{sen} x)} = \pm \cos x \iff \sqrt{1-\operatorname{sen}^2 x} = \pm \cos x$

$\iff \sqrt{\cos^2 x} = \pm \cos x$ ✓

h) $(1+\operatorname{cotg} x) \cdot (1-\operatorname{tg} x) = \frac{\cos^4 x - \operatorname{sen}^4 x}{\operatorname{sen} x \cdot \cos x} \iff \left(1+\frac{\cos x}{\operatorname{sen} x}\right)\left(1-\frac{\operatorname{sen} x}{\cos x}\right) = \frac{(\cos^2 x - \operatorname{sen}^2 x)(\overbrace{\cos^2 x + \operatorname{sen}^2 x}^{=1})}{\operatorname{sen} x \cdot \cos x}$

$\iff \frac{\operatorname{sen} x + \cos x}{\operatorname{sen} x} \cdot \frac{\cos x - \operatorname{sen} x}{\cos x} = \frac{(\cos^2 x - \operatorname{sen}^2 x)}{\operatorname{sen} x \cdot \cos x} \iff \frac{\cos^2 x - \operatorname{sen}^2 x}{\operatorname{sen} x \cos x} = \frac{\cos^2 x - \operatorname{sen}^2 x}{\operatorname{sen} x \cdot \cos x}$ ✓

i) $\operatorname{cotg}(a+b) = \frac{\operatorname{cotg} a \cdot \operatorname{cotg} b - 1}{\operatorname{cotg} a + \operatorname{cotg} b} \iff \frac{1-\operatorname{tg} a \cdot \operatorname{tg} b}{\operatorname{tg} a + \operatorname{tg} b} = \frac{\frac{1}{\operatorname{tg} a} \cdot \frac{1}{\operatorname{tg} b} - 1}{\frac{1}{\operatorname{tg} a} + \frac{1}{\operatorname{tg} b}} \iff$

$\frac{1-\operatorname{tg} a \cdot \operatorname{tg} b}{\operatorname{tg} a + \operatorname{tg} b} = \frac{(1-\operatorname{tg} a \cdot \operatorname{tg} b)/\cancel{\operatorname{tg} a \cdot \operatorname{tg} b}}{(\operatorname{tg} a + \operatorname{tg} b)/\cancel{\operatorname{tg} a \cdot \operatorname{tg} b}}$ ✓

j) $\cos^4 a - \operatorname{sen}^4 a = 2\cos^2 a - 1 \iff (\cos^2 a - \operatorname{sen}^2 a)(\overbrace{\cos^2 a + \operatorname{sen}^2 a}^{=1}) = 2\cos^2 a - 1 \iff$

$\cos^2 a - (1-\cos^2 x) = 2\cos^2 a - 1$ ✓

k) $\operatorname{sen} b \cdot (\cos a \cos b + \operatorname{sen} a \operatorname{sen} b) + \cos b \cdot (\operatorname{sen} a \cos b - \cos a \operatorname{sen} b) = \operatorname{sen} a \iff$

$\operatorname{sen} b \cancel{\cos a} \cos b + \operatorname{sen} a \operatorname{sen}^2 b + \operatorname{sen} a \cos^2 b - \cos b \cdot \cancel{\cos a} \cdot \operatorname{sen} b = \operatorname{sen} a \iff$

$\operatorname{sen} a (\underbrace{\operatorname{sen}^2 b + \cos^2 b}_{=1}) = \operatorname{sen} a$ ✓