

ECUACIONES

BICUADRADAS

a) $x^4 - 5x^2 + 4 = 0 \xrightarrow{x^2=t} t^2 - 5t + 4 = 0 \rightarrow t = \frac{5 \pm \sqrt{25-16}}{2} = \begin{cases} 8/2 = 4 \\ 1/2 = 1 \end{cases}$

$x^2 = 4 \Rightarrow x = \pm 2$
 $x^2 = 1 \Rightarrow x = \pm 1$

b) $x^4 - 3x^2 - 4 = 0 \xrightarrow{x^2=t} t^2 - 3t - 4 = 0 \Rightarrow t = \frac{3 \pm \sqrt{9+16}}{2} = \begin{cases} 4 \\ -1 \end{cases} \Rightarrow \begin{cases} x^2=4 \Rightarrow x=\pm 2 \\ x^2=-1 \Rightarrow x=\pm \sqrt{-1} \end{cases}$

c) $x^4 + 3x^2 + 2 = 0 \xrightarrow{x^2=t} t^2 + 3t + 2 = 0 \Rightarrow t = \frac{-3 \pm \sqrt{9-8}}{2} = \begin{cases} -2 \\ -1 \end{cases} \Rightarrow \begin{cases} x^2=-2 \Rightarrow x=\pm \sqrt{-2} \\ x^2=-1 \Rightarrow x=\pm i \end{cases}$

d) $x^4 - 9x^2 + 8 = 0 \xrightarrow{x^2=t} t^2 - 9t + 8 = 0 \Rightarrow t = \frac{9 \pm \sqrt{81-32}}{2} = \begin{cases} 8 \\ 1 \end{cases} \Rightarrow \begin{cases} x^2=8 \Rightarrow x=\pm \sqrt{8}=\pm 2\sqrt{2} \\ x^2=1 \Rightarrow x=\pm 1 \end{cases}$

CON RADICALES

a) $(\sqrt{5x+6})^2 = (3+2x)^2 \Rightarrow 5x+6 = 9+12x+4x^2 \Rightarrow 4x^2+7x+3=0 \Rightarrow x = \frac{-7 \pm \sqrt{49-48}}{8} = \begin{cases} -1 \\ -3/4 \end{cases}$

Comprobación: $x=-1 \rightarrow \sqrt{5(-1)+6} = 3+2(-1)$

$x=-3/4 \rightarrow \sqrt{5(-3/4)+6} = (3+2(-3/4))$

$\sqrt{-15/4+24} = (3-3/2) = 3/2$

$\Rightarrow$ Soluciones: $\begin{cases} x=-1 \\ x=-3/4 \end{cases}$

b) $x + \sqrt{7-3x} = 1 \Rightarrow (\sqrt{7-3x})^2 = (1-x)^2 \Rightarrow 7-3x = 1-2x+x^2 \Rightarrow x^2+x-6=0 \Rightarrow x = \frac{-1 \pm 5}{2} = \begin{cases} -3 \\ 2 \end{cases}$

Comprobación: $x=-3 \rightarrow -3 + \sqrt{7-3(-3)} = 1$

$x=2 \rightarrow 2 + \sqrt{7-3(2)} \neq 1$

$\Rightarrow$ Solución: $x=-3$

c) $\sqrt{2-5x} + x\sqrt{3} = 0 \Rightarrow (\sqrt{2-5x})^2 = (-x\sqrt{3})^2 \Rightarrow 2-5x = 3x^2 \Rightarrow 3x^2+5x-2=0 \Rightarrow x = \frac{-5 \pm 7}{6} = \begin{cases} -2 \\ 1/3 \end{cases}$

Comprobación: $x=-2 \rightarrow \sqrt{2-5(-2)} + (-2)\sqrt{3} = 0$

$\sqrt{12-2\sqrt{3}} = 2\sqrt{3}-2\sqrt{3}=0$

$x=1/3 \rightarrow \sqrt{2-5(1/3)} + \frac{1}{3}\sqrt{3} = 0$

$\sqrt{2-5/3} + \frac{\sqrt{3}}{3} = \sqrt{1/3} + \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} \neq 0$

$\Rightarrow$ Solución: $x=-2$

d) $\sqrt{2x+3} + \sqrt{x-5} = 0 \Rightarrow (\sqrt{2x+3})^2 = (-\sqrt{x-5})^2 \Rightarrow 2x+3 = x-5 \Rightarrow x=-8$

Comprobación: $\sqrt{2(-8)+3} + \sqrt{(-8)-5} = 0$
 $\sqrt{-13} + \sqrt{-13} \neq 0$ no solución

POLINÓMICAS

a) $x^3 - 7x - 6 = 0 \rightarrow$ Soluciones:

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline -2 & 1 & -1 & -6 & 0 \\ & & 2 & -6 & 0 \\ \hline 3 & 1 & -3 & 3 & 0 \\ & & 4 & -6 & 0 \\ \hline & 1 & 1 & 0 & 0 \end{array}$$

$\begin{cases} x=-1 \\ x=-2 \\ x=3 \end{cases}$

b) $2x^3 - 3x^2 - 9x + 10 = 0 \rightarrow$ Soluciones:

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -9 & 10 \\ & & 2 & -1 & -10 \\ \hline -2 & 2 & -1 & -10 & 0 \\ & & -4 & 10 & 0 \\ \hline & 2 & -5 & 10 & 0 \end{array}$$

$2x-5=0 \Rightarrow x=5/2$

$\begin{cases} x=1 \\ x=-2 \\ x=5/2 \end{cases}$

c) $x^4 - 5x^3 + 5x^2 + 5x - 6 = 0 \rightarrow$ Soluciones:

$$\begin{array}{r|rrrrr} 1 & 1 & -5 & 5 & 5 & -6 \\ & & -4 & -4 & 1 & 6 \\ \hline -1 & 1 & -4 & 1 & 6 & 0 \\ & & -1 & 5 & -6 & 0 \\ \hline 2 & 1 & -5 & 6 & 10 & 0 \\ & & 2 & -6 & 10 & 0 \\ \hline 3 & 1 & -3 & 3 & 10 & 0 \\ & & 4 & -6 & 10 & 0 \\ \hline & 1 & 1 & 0 & 0 & 0 \end{array}$$

$\begin{cases} x=1 \\ x=-1 \\ x=2 \\ x=3 \end{cases}$

d) $3x^3 - 10x^2 + 9x - 2 = 0 \rightarrow$ Soluciones:

$$\begin{array}{r|rrrr} 1 & 3 & -10 & 9 & -2 \\ & & 3 & -7 & 2 \\ \hline 3 & 3 & -7 & 2 & 0 \end{array}$$

$3x^2 - 7x + 2 = 0$

$x = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6} = \begin{cases} 2 \\ 1/3 \end{cases}$

$\begin{cases} x=1 \\ x=2 \\ x=1/3 \end{cases}$

$$e) x^5 - 16x = 0 \Rightarrow x(x^4 - 16) = 0 \quad \begin{cases} x=0 \\ x^4 - 16 = 0 \end{cases} \xrightarrow{x^2 = t} t^2 - 16 = 0 \Rightarrow t = \pm 4 \quad \begin{cases} x^2 = -4 \Rightarrow x = \pm \sqrt{-4} \quad \cancel{\neq} \\ x^2 = 4 \Rightarrow x = \pm 2 \end{cases}$$

$$\text{Soluciones: } \boxed{x=0, x=2, x=-2}$$

$$f) x^3 - 3x^2 + 2x = 0 \Rightarrow x(x^2 - 3x + 2) = 0 \quad \begin{cases} x=0 \\ x^2 - 3x + 2 = 0 \end{cases} \Rightarrow x = \frac{3 \pm \sqrt{9-8}}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

$$\text{Soluciones: } \boxed{x=0, x=1, x=2}$$

$$g) x^3 - x^2 + 4x - 4 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -1 & 4 & -4 \\ & & 1 & 0 & 4 \\ & & & 1 & 0 \end{array} \quad \begin{cases} x^2 + 4 = 0 \\ x^2 = -4 \\ x = \pm \sqrt{-4} \quad \cancel{\neq} \end{cases}$$

$$\text{Solución: } \boxed{x=1}$$

CON DENOMINADORES

$$a) \frac{1}{x} - \frac{1}{x+3} = \frac{3}{10} \Rightarrow \frac{10(x+3) - 10x}{10x(x+3)} = \frac{3x(x+3)}{10x(x+3)} \Rightarrow 10x+30-10x = 3x^2+9x \Rightarrow 3x^2+9x-30=0$$

$$\downarrow$$

$$x^2+3x-10=0$$

$$x = \frac{-3 \pm \sqrt{9+40}}{2} = \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2} = \begin{cases} -5 \\ 2 \end{cases} \quad \text{Soluciones: } \boxed{x=-5, x=2}$$

$$b) \frac{4}{x} + \frac{2(x+1)}{3(x-2)} = 4 \Rightarrow \frac{12(x-2) + 2x(x+1)}{3x(x-2)} = \frac{12x(x-2)}{3x(x-2)} \Rightarrow 12x-24+2x^2+2x = 12x^2-24x \Rightarrow$$

$$\Rightarrow 10x^2-38x+24=0 \Rightarrow 5x^2-19x+12=0 \Rightarrow x = \frac{19 \pm \sqrt{361-240}}{10} = \frac{19 \pm 11}{10} = \begin{cases} 3 \\ \frac{4}{5} \end{cases}$$

$$\text{Soluciones: } \boxed{x=3, x=\frac{4}{5}}$$

$$c) \frac{1}{x} + \frac{1}{x^2} = \frac{3}{4} \Rightarrow \frac{4x+4}{4x^2} = \frac{3x^2}{4x^2} \Rightarrow 3x^2-4x-4=0 \Rightarrow x = \frac{4 \pm \sqrt{16+48}}{6} = \frac{4 \pm 8}{6} = \begin{cases} 2 \\ -\frac{2}{3} \end{cases}$$

$$\text{Soluciones: } \boxed{x=2, x=-\frac{2}{3}}$$

$$d) \frac{x+1}{2} = x - \frac{2x+3}{4} \Rightarrow \frac{2x+2}{4} = \frac{4x-(2x+3)}{4} \Rightarrow 2x+2 = 4x-2x+3 \Rightarrow 0 \neq 1$$

La ecuación no tiene solución

$$e) \frac{(x+1)^2}{16} - \frac{1+x}{2} = \frac{(x-1)^2}{16} - \frac{2+x}{4} \Rightarrow \frac{x^2+2x+1-8(1+x)}{16} = \frac{x^2-2x+1-4(2+x)}{16} \Rightarrow$$

$$\Rightarrow 2x-8-8x = -2x-8-4x \Rightarrow 8x-8x = 8-8 \Rightarrow 0=0$$

Se trata de una identidad, es decir, la igualdad es cierta $\forall x \in \mathbb{R}$.

SISTEMAS DE ECUACIONES NO LINEALES

$$a) \begin{cases} 2x-y-1=0 \\ x^2-7=y+2 \end{cases} \Rightarrow \begin{cases} y=2x-1 \\ y=x^2-9 \end{cases} \Rightarrow x^2-9=2x-1 \Rightarrow x^2-2x-8=0 \Rightarrow x = \frac{2 \pm 6}{2} = \begin{cases} 4 \\ -2 \end{cases}$$

$$\text{Soluciones: } \boxed{\begin{matrix} x=4 \\ y=2x-1=7 \end{matrix}} \quad \boxed{\begin{matrix} x=-2 \\ y=2x-1=-5 \end{matrix}}$$

$$b) \begin{cases} \frac{1}{x} + \frac{1}{y} = 1 - \frac{1}{xy} \\ xy=6 \end{cases} \Rightarrow \frac{y+x}{xy} = \frac{xy-1}{xy} \Rightarrow y + \frac{6}{y} = \frac{6y-1}{y} \Rightarrow \frac{y^2+6}{y} = \frac{6y-1}{y} \Rightarrow y^2+6=6y-1 \Rightarrow y^2-5y+7=0$$

$$\Rightarrow y = \frac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 3 \\ 2 \end{cases} \quad \text{Soluciones: } \boxed{\begin{matrix} y=3 \\ x=6/3=2 \end{matrix}} \quad \boxed{\begin{matrix} y=2 \\ x=6/2=3 \end{matrix}}$$

$$c) \begin{cases} x=2y+1 \\ \sqrt{x+y} - \sqrt{x-y} = 2 \end{cases} \Rightarrow \sqrt{2y+1+y} - \sqrt{2y+1-y} = 2 \Rightarrow (\sqrt{3y+1} - \sqrt{y+1})^2 = 2^2 \Rightarrow$$

$$\Rightarrow 3y+1 + y+1 - 2\sqrt{(3y+1)(y+1)} = 4 \Rightarrow (2\sqrt{3y^2+4y+1})^2 = (2-4y)^2 \Rightarrow$$

$$4(3y^2+4y+1) = (4y-2)^2 \Rightarrow 16y^2+16y+4 = 16y^2-16y+4 \Rightarrow 32y=0 \Rightarrow y=0$$

$$\Rightarrow 4y(y-8)=0 \Rightarrow \begin{cases} y=0 \\ y=8 \end{cases} \quad \text{Soluciones: } \boxed{\begin{matrix} y=0 \\ x=2y+1=1 \end{matrix}} \quad \boxed{\begin{matrix} y=8 \\ x=2y+1=17 \end{matrix}}$$