# ECUACIONES LOGARÍTMICAS

① $\log x^2 = \log 10^3 + \log \frac{x}{10} \longrightarrow \log x^2 = \log \frac{10^3 \cdot x}{10} \longrightarrow x^2 = 100x \longrightarrow$

$\longrightarrow x(x-100)=0 \begin{cases} x=0 \text{ No sirve: } \log 0 \nexists \\ \boxed{x=100} \quad (100^2>0,\ \frac{100}{10}>0) \end{cases}$

② $\log x + \log(x+3) = 2\cdot\log(x+1) \longrightarrow \log\left(x\cdot(x+3)\right) = \log(x+1)^2 \longrightarrow$

$\longrightarrow x^2+3x = x^2+2x+1 \longrightarrow \boxed{x=1} \quad (1>0,\ 1+3>0,\ 1+1>0)$

③ $4\cdot\log\left(\frac{x}{5}\right) + \log\left(\frac{625}{4}\right) = 2\log x \longrightarrow \log\left(\frac{x^4\cdot 5^4}{5^x\cdot 4}\right) = \log x^2 \longrightarrow \frac{x^4}{4} - x^2 = 0$

$\longrightarrow x^2\left(\frac{x^2}{4}-1\right)=0 \begin{cases} x=0 \text{ No sirve: } \log 0 \nexists \\ \boxed{x=2} \quad (2/5>0,\ 2>0) \\ x=-2 \text{ No sirve: } \log(-2/5) \nexists \end{cases}$

④ $\not{2}\log x = \not{2}\cdot\log(x+1) \longrightarrow x = x+1 \longrightarrow 1\neq 0$. La ecuación no tiene solución.

⑤ $(\log x)^2 + \log x - 2 = 0 \xrightarrow{y=\log x} y^2+y-2=0 \longrightarrow y = \begin{cases} 1 \longrightarrow \log x = 1 \longrightarrow \boxed{x=10} \\ -2 \longrightarrow \log x = -2 \end{cases}$

$\downarrow$

$\boxed{x = 10^{-2} = \frac{1}{100}}$

⑦ $\log(16-x^2) = 2\cdot\log(3x-4) \longrightarrow \log(16-x^2) = \log(3x-4)^2$

$\longrightarrow 16-x^2 = 9x^2-24x+16 \longrightarrow 10x^2-24x=0 \longrightarrow x(10x-24)=0 \longrightarrow$

$\longrightarrow x = \begin{cases} 0 \text{ No sirve: } \log(-4) \nexists \\ \boxed{x=\frac{12}{5}} \quad (16-(\frac{12}{5})^2>0,\ 3\cdot\frac{12}{5}-4>0) \end{cases}$

⑧ $\log(35-x^3) = 3\cdot\log(5-x) \longrightarrow \log(35-x^3) = \log(5-x)^3 \longrightarrow$

$\longrightarrow 35-\not{x^3} = 5^3 - 3\cdot 5^2\cdot x + 3\cdot 5x^2 - \not{x^3} \longrightarrow x^2-5x+6=0 \longrightarrow$

$\longrightarrow \begin{cases} \boxed{x=2} \quad (35-2^3>0,\ 5-2>0) \\ \boxed{x=3} \quad (35-3^3>0,\ 5-3>0) \end{cases}$

⑨ $2\cdot(11-x^2) = (5-x)^2 \longrightarrow 3x^2-10x+3=0 \longrightarrow \begin{cases} \boxed{x=3} \quad (11-3^2)=9>0,\ 5-3>0 \\ \boxed{x=1/3} \quad (11-(\frac{1}{3})^2>0 \text{ y } 5-\frac{1}{3}>0) \end{cases}$

⑩ $(\log_5 x)^2 - \frac{7}{2}\cdot\log_5 x + \underbrace{\log_5 125}_{3} = 0 \xrightarrow{y=\log_5 x} y^2 - \frac{7}{2}y + 3 = 0 \longrightarrow 2y^2-7y+6=0$

$\longrightarrow \begin{cases} y=2 \longrightarrow \log_5 x = 2 \longrightarrow \boxed{x=5^2=25} \\ y=3/2 \longrightarrow \log_5 x = \frac{3}{2} \longrightarrow \boxed{x=5^{3/2}=\sqrt{5^3}=5\sqrt{5}} \end{cases}$