

$$1) a) (4x+5)^2 = 16x^2 + 40x + 25 \quad b) (2x^2 - 5y^3)^2 = 4x^4 - 20x^2y^3 + 25y^6$$

$$c) (2+7x^2y)(2-7x^2y) = 4 - 49x^4y^2 \quad d) \left(1 + \frac{x}{2}\right) \cdot \left(1 - \frac{x}{2}\right) = 1 - \frac{x^2}{4}$$

$$e) \left(\frac{3x}{2} - \frac{1}{x}\right)^2 = \frac{9x^2}{4} - \frac{1}{x^2}$$

$$2) a) (3x-1)^2 - (2x+5)(2x-5) = 9x^2 - 6x + 1 - (4x^2 - 25) = 9x^2 - 6x + 1 - 4x^2 + 25 = 5x^2 - 6x + 26$$

$$b) (3x-1)^2 - (5x^2+3x)^2 - (-x+2x^2) \cdot (2x^2+x) = 9x^2 - 6x + 1 - (25x^4 + 30x^3 + 9x^2) - (2x^2-x) \cdot (2x^2+x) = 9x^2 - 6x + 1 - 25x^4 - 30x^3 - 9x^2 - (4x^4 - x^2) = 9x^2 - 6x + 1 - 25x^4 - 30x^3 - 9x^2 - 4x^4 + x^2 = -29x^4 - 30x^3 + x^2 - 6x + 1$$

$$3) a) 4x^2 + 8x + 4 = (2x+2)^2$$

$$b) x^2 - 6x + 9 = (x-3)^2$$

$$c) 9x^2 - 36 = (3x-6) \cdot (3x+6)$$

$$d) a^2 - 2a + 1 = (a-1)^2$$

$$e) x^2 + 2xy + y^2 = (x+y)^2$$

$$f) a^2 - 16 = (a-4)(a+4)$$

$$4) a) 4x^2 - 6x + 2x^3 = 2x \cdot (2x - 3 + x^2)$$

$$b) -3xy - 2xy^2 - 10x^2yz = xy \cdot (-3 - 2y - 10xz)$$

$$c) 2ab^2 - 4a^3b + 8a^4b^3 = 2ab(b - 2a^2 + 4a^3b^2)$$

$$d) 6x^3y^2 - 3x^2yz + 9xy^3z^2 = 3xy(2x^2y - xz + 3y^2z^2)$$

$$e) -2x \cdot (x-3)^2 + 4x^2(x-3) = 2x(x-3) \cdot (-(x-3) + 2x) = 2x(x-3)(-x+3+2x) = 2x(x-3)(x+3)$$

$$5) a) x^3 - 7x^2 + 4x + 12 = (x+1)(x-2)(x-6)$$

	1	-7	4	12
-1		-1	+8	-12
	1	-8	12	0

$$x^2 - 8x + 12 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm 4}{2} \rightarrow \begin{matrix} x=6 \\ x=2 \end{matrix}$$

$$b) x^6 + 6x^5 + 9x^4 - x^2 - 6x - 9 = (x-1)(x+3)^2 \cdot (x^2+x+1)$$

$$\begin{array}{r|rrrrrr} 1 & 1 & 6 & 9 & -1 & -6 & -9 \\ & 1 & 7 & 16 & 15 & 9 & 0 \\ \hline -3 & 1 & 4 & 4 & 3 & 0 \\ \hline -3 & 1 & 1 & 1 & 0 \end{array}$$

$$x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} \neq$$

$$c) 6x^3 + 7x^2 - 9x + 2 = (x+2) \cdot (2x-1) (3x-1)$$

$$\begin{array}{r|rrrr} 6 & 7 & -9 & 2 \\ -2 & 6 & -5 & 1 & 0 \end{array}$$

$$6x^2 - 5x + 1 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 6 \cdot 1}}{2 \cdot 6} = \frac{5 \pm \sqrt{1}}{12} = \frac{5 \pm 1}{12} \begin{matrix} \nearrow x = \frac{1}{2} \\ \searrow x = \frac{1}{3} \end{matrix}$$

$$6) a) 3x - 2(x+3) = x - 3(x+1) \Rightarrow 3x - 2x - 6 = x - 3x - 3 \Rightarrow 3x - 2x - x + 3x = 6 - 3$$

$$\Rightarrow 3x = 3 \Rightarrow x = \frac{3}{3} \Rightarrow x = 1$$

$$b) 4(2x-7) - 3 \cdot (3x+1) = 2 - (7-x) \Rightarrow 8x - 28 - 9x - 3 = 2 - 7 + x \Rightarrow$$

$$\Rightarrow 8x - 9x - x = 2 - 7 + 28 + 3 \Rightarrow -2x = 26 \Rightarrow x = \frac{26}{-2} \Rightarrow x = -13$$

$$c) \frac{3x+2}{5} - \frac{4x-1}{10} + \frac{5x-2}{8} = \frac{x+1}{4} \Rightarrow \frac{24x+16}{40} - \frac{16x-4}{40} + \frac{25x-10}{40} = \frac{10x+10}{40}$$

$$\Rightarrow 24x + 16 - 16x + 4 + 25x - 10 = 10x + 10 \Rightarrow 24x - 16x + 25x - 10x = 10 - 16 - 4 + 10$$

$$\Rightarrow 23x = 0 \Rightarrow x = \frac{0}{23} \Rightarrow x = 0$$

$$d) \frac{2(3-x)}{3} - \frac{4 \cdot (x-2)}{5} = 8 \Rightarrow \frac{6-2x}{3} - \frac{4x-8}{5} = 8 \Rightarrow \frac{30-10x}{15} - \frac{12x-24}{15} = \frac{120}{15}$$

$$\Rightarrow 30 - 10x - 12x + 24 = 120 \Rightarrow -10x - 12x = 120 - 30 - 24 \Rightarrow -22x = 66 \Rightarrow x = \frac{66}{-22} \Rightarrow$$

$$\Rightarrow x = -3$$

$$e) 2 \cdot (x-7) - \frac{x+3}{4} = x-5 \Rightarrow 2x-14 - \frac{x+3}{4} = x-5 \Rightarrow \frac{8x-56}{4} - \frac{x+3}{4} = \frac{4x-20}{4}$$

$$\Rightarrow 8x-56 - x-3 = 4x-20 \Rightarrow 8x-x-4x = -20+56+3 \Rightarrow 3x=39 \Rightarrow x=\frac{39}{3}$$

$$\Rightarrow x=13$$

$$f) \frac{1}{3} - \frac{4 \cdot (x-5)}{3} - \frac{3x}{4} = \frac{1}{2} - \frac{2(x+3)}{3} \Rightarrow \frac{1}{3} - \frac{4x-20}{3} - \frac{3x}{4} = \frac{1}{2} - \frac{2x+6}{3}$$

$$\Rightarrow \frac{4}{12} - \frac{16x-80}{12} - \frac{9x}{12} = \frac{6}{12} - \frac{8x+24}{12} \Rightarrow 4-16x+80-9x=6-8x-24$$

$$\Rightarrow -16x-9x+8x=6-24-4-80 \Rightarrow -17x=-102 \Rightarrow x=\frac{-102}{-17} \Rightarrow x=6$$

$$7) a) 3x^2-12x=0 \Rightarrow x \cdot (3x-12)=0 \rightarrow x=0$$

$$3x-12=0$$

$$3x=12$$

$$x=\frac{12}{3} \Rightarrow x=4$$

$$b) 2x^2-8=0 \Rightarrow 2x^2=8 \Rightarrow x^2=\frac{8}{2} \Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{4} \Rightarrow x=\pm 2$$

$$c) 4x^2+28x+49=0$$

$$x=\frac{-28 \pm \sqrt{28^2-4 \cdot 4 \cdot 49}}{2 \cdot 4} = \frac{-28 \pm \sqrt{0}}{8} = \frac{-28}{8} = -\frac{7}{2}$$

$$d) -2x^2+3x+2=0$$

$$x=\frac{-3 \pm \sqrt{3^2-4 \cdot 2 \cdot (-2)}}{2 \cdot (-2)} = \frac{-3 \pm \sqrt{25}}{-4} = \frac{-3 \pm 5}{-4} \rightarrow x=2$$

$$x=-\frac{1}{2}$$

$$e) (x+4)^2+(x-3)^2=(x+5)^2 \Rightarrow x^2+8x+16+x^2-6x+9=x^2+10x+25$$

$$x^2+8x+16+x^2-6x+9-x^2-10x-25=0 \Rightarrow x^2-8x=0 \Rightarrow x \cdot (x-8)=0$$

$$\rightarrow x=0$$

$$\rightarrow x-8=0$$

$$x=8$$

$$f) (x-1) \cdot (x+2) - (2x-3) \cdot (x+4) = x-14 \Rightarrow x^2 - x + 2x - 2 - (2x^2 - 3x + 8x - 12) = x-14$$

$$x^2 + x - 2 - 2x^2 - 5x + 12 = x - 14 = 0 \Rightarrow -x^2 - 5x + 24 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot (-1) \cdot 24}}{2 \cdot (-1)} = \frac{5 \pm \sqrt{121}}{-2} = \frac{5 \pm 11}{-2} \rightarrow \begin{cases} x = -8 \\ x = 3 \end{cases}$$

$$g) \frac{x}{3}(x-1) - \frac{x}{4} \cdot (x+1) + \frac{3x+4}{12} = 0 \Rightarrow \frac{x^2}{3} - \frac{x}{3} - \frac{x^2}{4} - \frac{x}{4} + \frac{3x+4}{12} = 0$$

$$\Rightarrow \frac{4x^2}{12} - \frac{4x}{12} - \frac{3x^2}{12} - \frac{3x}{12} + \frac{3x+4}{12} = 0 \Rightarrow 4x^2 - 4x - 3x^2 - 3x + 3x + 4 = 0$$

$$x^2 - 4x + 4 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{4 \pm \sqrt{0}}{2} = \frac{4 \pm 0}{2} = 2$$

$$8) a) x^4 - 10x^3 + 5x^2 + 40x - 36 = 0$$

	1	-10	5	40	-36
1		1	-9	-4	36
	1	-9	-4	36	0
2		2	-14	-36	
	1	-7	-18	0	

$$(x-1)(x-2)(x-9)(x+2) = 0$$

Solution

$$x = 1$$

$$x = 2$$

$$x = 9$$

$$x = -2$$

$$x^2 - 7x - 18 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot (-18)}}{2 \cdot 1} = \frac{7 \pm \sqrt{121}}{2} = \frac{7 \pm 11}{2} \rightarrow \begin{cases} x = 9 \\ x = -2 \end{cases}$$

$$b) 6x^3 + x^2 - 10x + 3 = 0$$

	6	1	-10	3
1		6	7	-3
	6	7	-3	0

$$(x-1)(2x+3) \cdot (3x-1) = 0$$

Solution

$$x = 1, x = -\frac{3}{2}, x = \frac{1}{3}$$

$$6x^2 + 7x - 3 = 0$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 6 \cdot (-3)}}{2 \cdot 6} = \frac{-7 \pm \sqrt{121}}{12} = \frac{-7 \pm 11}{12} \rightarrow \begin{cases} x = -\frac{3}{2} \\ x = \frac{1}{3} \end{cases}$$

$$c) (x^2 - 1) \cdot (2x + 3) \cdot (x^2 - x) = 0$$

$$\textcircled{\text{I}} \quad x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm \sqrt{1} \Rightarrow x = \pm 1$$

$$\textcircled{\text{II}} \quad 2x + 3 = 0 \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$$

$$\textcircled{\text{III}} \quad x^2 - x = 0 \Rightarrow x \cdot (x - 1) = 0 \rightarrow \begin{array}{l} x = 0 \\ x - 1 = 0 \\ x = 1 \end{array}$$

Soluciones

$$x = \pm 1$$

$$x = -\frac{3}{2}$$

$$x = 0$$

$$d) (x^2 - 6x + 9) (3x - 1) (x^2 - 6x) = 0$$

$$\textcircled{\text{I}} \quad x^2 - 6x + 9 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} = \frac{+6 \pm 0}{2} = 3$$

$$\textcircled{\text{II}} \quad 3x - 1 = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$

$$\textcircled{\text{III}} \quad x^2 - 6x = 0 \Rightarrow x(x - 6) = 0 \rightarrow \begin{array}{l} x = 0 \\ x - 6 = 0 \\ x = 6 \end{array}$$

Soluciones

$$x = 3$$

$$x = \frac{1}{3}$$

$$x = 0$$

$$x = 6$$

	Ahora	3 años
9.) Ignacio	x	$x + 3$
Miguel	$x + 4$	$x + 4 + 3$

$$x + 3 + x + 4 + 3 = 20$$

$$2x + 10 = 20$$

$$2x = 20 - 10$$

$$2x = 10 \Rightarrow x = \frac{10}{2} = 5$$

Ignacio tiene 5 años y Miguel tiene 9.

10) x : número de amigos

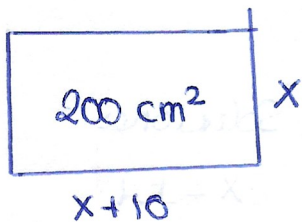
$$16 \cdot x = 12 \cdot (x + 3)$$

$$16x = 12x + 36 \Rightarrow 16x - 12x = 36 \Rightarrow 4x = 36$$

$$\Rightarrow x = \frac{36}{4} \Rightarrow x = 9$$

Hay 9 amigos.

11)



$$x \cdot (x+10) = 200$$

$$x^2 + 10x = 200 \Rightarrow x^2 + 10x - 200 = 0$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 1 \cdot (-200)}}{2 \cdot 1} = \frac{-10 \pm \sqrt{900}}{2} = \frac{-10 \pm 30}{2}$$

$x = -20$ No puede ser
 $x = 10$

Un lab mide 10 cm y el otro 20 cm

12) Edad de Pedro x

Hoy 13 años $x-13$ En 11 años $x+11$

$$x+11 = \frac{(x-13)^2}{2}$$

$$\Rightarrow x^2 - 28x + 147 = 0$$

$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4 \cdot 1 \cdot 147}}{2 \cdot 1} = \frac{28 \pm \sqrt{196}}{2} = \frac{28 \pm 14}{2}$$

$x = 21$
 $x = 7$ No puede ser porque es menor de 13

Pedro tiene 21 años.

$$13) a) \begin{cases} -2x + y = -1 & \text{I} \\ 4x - 2y = 2 & \text{II} \end{cases}$$

$$\text{I} \quad -2x + y = -1$$

$$y = -1 + 2x$$

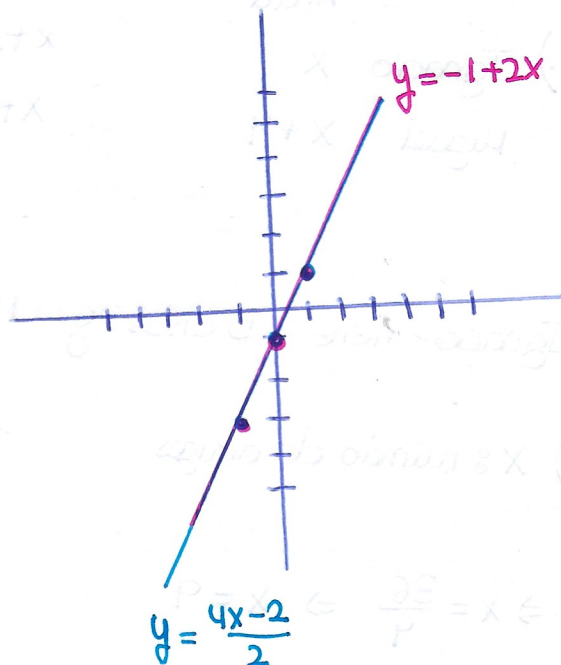
x	y
0	-1
1	1
-1	-3

$$\text{II} \quad 4x - 2y = 2$$

$$4x - 2 = 2y$$

$$\frac{4x-2}{2} = y$$

x	y
1	1
0	-1
-1	-3



$$b) \begin{cases} 2x + 3y = 12 & \textcircled{I} \\ 3x - 2y = 5 & \textcircled{II} \end{cases}$$

$$\textcircled{I} \quad 2x + 3y = 12$$

$$3y = 12 - 2x$$

$$y = \frac{12 - 2x}{3}$$

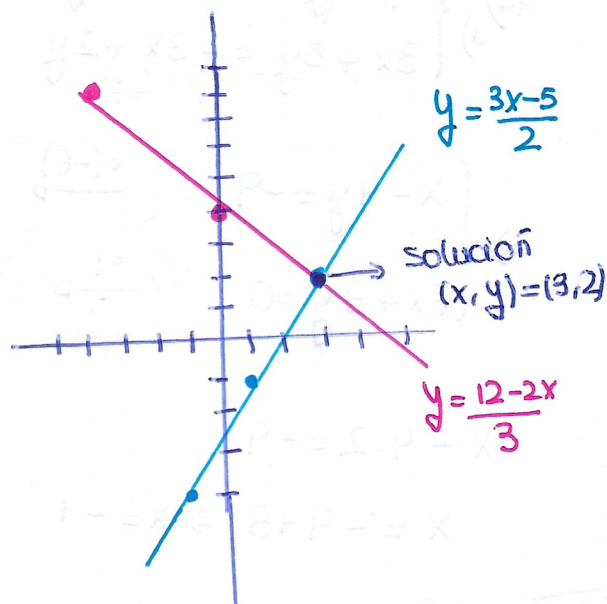
x	y
0	4
3	2
-3	0

$$\textcircled{II} \quad 3x - 2y = 5$$

$$3x - 5 = 2y$$

$$y = \frac{3x - 5}{2}$$

x	y
1	-1
3	2
-1	-4



$$14) \begin{cases} 2x + y = 3 \\ 3x - 4y = 10 \end{cases} \rightarrow y = 3 - 2x$$

$$3x - 4(3 - 2x) = 10 \Rightarrow 3x - 12 + 8x = 10 \Rightarrow$$

$$\Rightarrow 11x = 10 + 12 \Rightarrow 11x = 22 \Rightarrow x = \frac{22}{11} = 2$$

$$y = 3 - 2 \cdot 2 = 3 - 4 = -1$$

$$(x, y) = (2, -1)$$

$$15) \begin{cases} 3x - y = 7 \\ 2x + y = 13 \end{cases} \rightarrow 3x - 7 = y$$

$$y = 13 - 2x$$

$$3x - 7 = 13 - 2x$$

$$3x + 2x = 13 + 7$$

$$5x = 20$$

$$x = \frac{20}{5} = 4$$

$$y = 13 - 2 \cdot 4 = 5$$

$$(x, y) = (4, 5)$$

$$16) \begin{cases} 2x + 3y = 5 \\ 6x + 5y = 3 \end{cases} \xrightarrow{\begin{smallmatrix} \cdot (-3) \\ \cdot 1 \end{smallmatrix}} \begin{cases} -6x - 9y = -15 \\ 6x + 5y = 3 \end{cases}$$

$$\underline{-4y = -12} \Rightarrow y = \frac{-12}{-4} = 3$$

$$2x + 3 \cdot 3 = 5 \Rightarrow 2x = 5 - 9 \Rightarrow 2x = -4 \Rightarrow x = -4/2 = -2$$

$$(x, y) = (-2, 3)$$

$$17) a) \begin{cases} 4x - y = 3(x - 3 + y) \\ 3x + 5y = -3x + 2y \end{cases} \Rightarrow \begin{cases} 4x - y = 3x - 9 + 3y \\ 3x + 5y = -3x + 2y \end{cases} \Rightarrow \begin{cases} 4x - 3x - y - 3y = -9 \\ 3x + 5y + 3x - 2y = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x - 4y = -9 \\ 6x + 3y = 0 \end{cases} \xrightarrow{\cdot(-6)} \begin{matrix} -6x + 24y = 54 \\ 6x + 3y = 0 \end{matrix} \xrightarrow{\cdot 1} \frac{27y = 54}{27y = 54} \Rightarrow y = \frac{54}{27} = 2$$

$$x - 4 \cdot 2 = -9$$

$$x = -9 + 8 \Rightarrow x = -1$$

$$(x, y) = (-1, 2)$$

$$b) \begin{cases} \frac{x}{2} - \frac{x+y}{6} = \frac{11}{6} \\ \frac{2x-3y}{5} - \frac{1}{10} = \frac{33}{10} \end{cases} \Rightarrow \begin{cases} \frac{3x}{6} - \frac{x+y}{6} = \frac{11}{6} \\ \frac{4x-6y}{10} - \frac{1}{10} = \frac{33}{10} \end{cases} \Rightarrow \begin{cases} 3x - x - y = 11 \\ 4x - 6y - 1 = 33 \end{cases}$$

$$\Rightarrow \begin{cases} 2x - y = 11 \\ 4x - 6y = 33 + 1 \end{cases} \Rightarrow \begin{cases} 2x - y = 11 \\ 4x - 6y = 34 \end{cases} \xrightarrow{\cdot(-6)} \begin{matrix} -12x + 6y = -66 \\ 4x - 6y = 34 \end{matrix} \xrightarrow{\cdot 1} \frac{-8x}{-8x} = \frac{-32}{-8} \Rightarrow x = \frac{-32}{-8} = 4$$

$$2 \cdot 4 - y = 11 \Rightarrow 8 - 11 = y \Rightarrow y = -3$$

$$(x, y) = (4, -3)$$

18) x: libras de 15 €
y: libras de 12 €

$$\begin{cases} x + y = 125 \\ 15x + 12y = 1680 \end{cases} \rightarrow x = 125 - y$$

$$15 \cdot (125 - y) + 12y = 1680 \Rightarrow 1875 - 15y + 12y = 1680 \Rightarrow -3y = 1680 - 1875$$

$$\Rightarrow -3y = -195 \Rightarrow y = \frac{-195}{-3} = 65 \quad x = 125 - 65 = 60$$

Vende 60 libras de 15 € y 65 libras de 12 €

19) x : gallinas
 y : cerdos

$$\begin{cases} x + y = 50 \\ 2x + 4y = 134 \end{cases} \rightarrow x = 50 - y$$

$$2 \cdot (50 - y) + 4y = 134 \Rightarrow 100 - 2y + 4y = 134 \Rightarrow 2y = 134 - 100 \Rightarrow$$

$$2y = 34 \Rightarrow y = \frac{34}{2} = 17 \quad x = 50 - 17 = 33$$

Hay 33 gallinas y 17 cerdos

20) x : edad de Sara
 y : edad del padre

$$\begin{cases} 2x = \frac{y}{4} \\ x + 2 = \frac{y+2}{6} \end{cases} \Rightarrow \begin{cases} \frac{8x}{4} = \frac{y}{4} \\ \frac{6x+12}{6} = \frac{y+2}{6} \end{cases} \Rightarrow$$

$$\begin{cases} 8x = y \\ 6x + 12 = y + 2 \end{cases} \Rightarrow \begin{cases} 8x = y \\ 6x + 12 - 2 = y - 2 \end{cases} \Rightarrow \begin{cases} 8x = y \\ 6x + 10 = y \end{cases}$$

$$8x = 6x + 10 \Rightarrow 8x - 6x = 10 \Rightarrow 2x = 10 \Rightarrow x = \frac{10}{2} = 5$$

$$y = 8 \cdot 5 = 40$$

Sara tiene 5 años y su padre tiene 40

21) x : precio de la mercha

$$3 \cdot 5 + 7 \cdot 4 = x \cdot (7 + 5)$$

$$15 + 28 = 12x$$

$$43 = 12x \Rightarrow x = \frac{43}{12} = 3,58 \text{ €}$$

El precio de la mercha es de 3,58 €

22) a) $\begin{cases} x^2 - xy + y^2 = 7 \\ x + y = 5 \Rightarrow x = 5 - y \end{cases}$

$$(5-y)^2 - (5-y) \cdot y + y^2 = 7$$

$$25 - 10y + y^2 - 5y + y^2 + y^2 = 7$$

$$\Rightarrow 3y^2 - 15y + 25 - 7 = 0 \Rightarrow 3y^2 - 15y + 18 = 0 \Rightarrow y^2 - 5y + 6 = 0$$

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2} \rightarrow y = \frac{5+1}{2} = 3$$

$$\rightarrow y = \frac{5-1}{2} = 2$$

$$(x, y) = (2, 3)$$

$$(x, y) = (3, 2)$$

Si $y = 3$ $x = 5 - 3 = 2$ Si $y = 2$ $x = 5 - 2 = 3$

$$b) \begin{cases} x \cdot y = 8 \\ x + y = 6 \Rightarrow x = 6 - y \end{cases} \quad \begin{aligned} (6 - y) \cdot y &= 8 \\ 6y - y^2 &= 8 \end{aligned}$$

$$y^2 - 6y + 8 = 0$$

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2} \rightarrow \begin{aligned} y &= \frac{6+2}{2} = 4 \\ y &= \frac{6-2}{2} = 2 \end{aligned}$$

$$\text{si } y = 4 \quad x = 6 - 4 = 2$$

$$\text{si } y = 2 \quad x = 6 - 2 = 4$$

$$(x, y) = (2, 4)$$

$$(x, y) = (4, 2)$$

23. No entra

24. No entra