CÁLCULO DE DERIVADAS PRÁCTICA 6.1

$\boxed{10}$

a) $f(x) = (x^2+3)^3 \to f'(x) = 3(x^2+3)^2 \cdot 2x$

$f'(x) = 6x \cdot (x^2+3)^2$

b) $f(x) = (x^2+1)^{15} \to f'(x) = 15(x^2+1)^{14} \cdot 2x$

$f'(x) = 30x \cdot (x^2+1)^{14}$

c) $f(x) = \sqrt{\operatorname{sen} x} \to f'(x) = \dfrac{1}{2\sqrt{\operatorname{sen} x}} \cdot \cos x$

$f'(x) = \dfrac{\cos x}{2\sqrt{\operatorname{sen} x}}$

d) $f(x) = e^{x^3-x+1} \to f'(x) = e^{x^3-x+1} \cdot (3x^2-1)$

e) $f(x) = \operatorname{sen} 2x \to f'(x) = \cos(2x) \cdot 2 = 2\cos(2x)$

f) $f(x) = 3^{-x^2+1} \to f'(x) = 3^{-x^2+1} \cdot \ln 3 \cdot (-2x) =$

$= -2\ln 3 \cdot x \cdot 3^{-x^2+1}$

g) $f(x) = \sqrt[3]{e^x} = e^{x/3} \to f'(x) = e^{x/3} \cdot \dfrac{1}{3} = \dfrac{\sqrt[3]{e^x}}{3}$

h) $f(x) = \cos(x^3+4x-1) \to f'(x) = -\operatorname{sen}(x^3+4x-1) \cdot (3x^2+4)$

$f'(x) = -(3x^2+4) \cdot \operatorname{sen}(x^3+4x-1)$

i) $f(x) = \sqrt{3x^2-1} \rightarrow f'(x) = \dfrac{6x}{2\sqrt{3x^2-1}} = \dfrac{3x}{\sqrt{3x^2-1}}$

j) $f(x) = \ln(x^2-1) \rightarrow f'(x) = \dfrac{2x}{x^2-1}$

k) $f(x) = ((\cos x))^{3/2} \rightarrow f'(x) = \frac{3}{2}(\cos x)^{1/2} \cdot (-\operatorname{sen} x) =$

$= -\dfrac{3 \operatorname{sen} x \cdot \sqrt{\cos x}}{2}$

l) $f(x) = 3^x \cdot \operatorname{sen} x^2 \rightarrow f'(x) = 3^x \cdot \ln 3 \cdot \operatorname{sen} x^2 + 3^x \cdot \cos x^2 \cdot 2x$

m) $f(x) = x \cdot e^{2x+1} \rightarrow f'(x) = e^{2x+1} + x \cdot e^{2x+1} \cdot 2x =$

$= e^{2x+1} + 2x^2 e^{2x+1} = (1+2x^2)e^{2x+1}$

n) $f(x) = \sqrt[3]{(5x+3)^2} = (5x+3)^{2/3} \rightarrow f'(x) = \frac{2}{3}(5x+3)^{-1/3} \cdot 5 \Rightarrow$

$\Rightarrow f'(x) = \dfrac{10}{3\sqrt[3]{5x+3}}$

o) $f(x) = x \cdot \operatorname{sen} \frac{x}{2} \rightarrow f'(x) = \operatorname{sen} \frac{x}{2} + x \cdot \cos\left(\frac{x}{2}\right) \cdot \frac{1}{2}$

p) $f(x) = \left(\dfrac{x^2}{x-2}\right)^2 \rightarrow f'(x) = 2 \cdot \left(\dfrac{x^2}{x-2}\right) \cdot \dfrac{2x \cdot (x-2) - x^2 \cdot 1}{(x-2)^2} =$

$= \dfrac{2x^2}{x-2} \cdot \dfrac{2x^2-4x-x^2}{(x-2)^2} = \dfrac{2x^2}{x-2} \cdot \dfrac{x^2-4x}{(x-2)^2} = \dfrac{2x^3(x-4)}{(x-2)^3}$

q) $f(x) = e^{x^2} - \cos 2x \rightarrow f'(x) = e^{x^2} \cdot 2x - (-\operatorname{sen} 2x) \cdot 2 =$

$= 2xe^{x^2} + 2\operatorname{sen}(2x)$

s) $f(x) = x - \sqrt{2-x} \longrightarrow f'(x) = 1 - \frac{1}{2\sqrt{2-x}} \cdot (-1) =$

$$= 1 + \frac{1}{2\sqrt{2-x}}$$

t) $f(x) = 1 + \frac{2}{(x-2)^2} \longrightarrow f'(x) = \frac{-2 \cdot 2(x-2) \cdot 1}{(x-2)^2} = \frac{-4(\cancel{x-2})}{(x-2)^{\cancel{2}}}$

$\Longrightarrow f'(x) = \frac{-4}{x-2}$

u) $f(x) = (\operatorname{sen} x)^2 - 3x^2 \longrightarrow f'(x) = 2 \operatorname{sen} x \cdot \cos x - 6x$

v) $f(x) = \ln(2x-1) \longrightarrow f'(x) = \frac{1}{2x-1} \cdot 2 = \frac{2}{2x-1}$

w) $f(x) = 2e^{-x}(x+1) \longrightarrow f'(x) = 2e^{-x} \cdot (-1)(x+1) + 2e^{-x} \cdot 1 =$

$= -2e^{-x}(x+1) + 2e^{-x} = 2e^{-x} \cdot [-1(x+1)+1] =$

$= 2e^{-x} \cdot [-x-1+1] = -2xe^{-x}$

x) $f(x) = (2x-1)^3 \longrightarrow f'(x) = 3(2x-1)^2 \cdot 2 = 6(2x-1)^2$

y) $f(x) = e^{\sqrt{x}} \longrightarrow f'(x) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

z) $f(x) = \log_2(x^2+x+1) \longrightarrow f'(x) = \frac{2x+1}{x^2+x+1} \cdot \frac{1}{\ln 2}$

aa) $f(x) = (x-\sqrt{x})^4 \longrightarrow f'(x) = 4(x-\sqrt{x})^3 \cdot \left(1 - \frac{1}{2\sqrt{x}}\right)$

ab) $f(x)=\sqrt[3]{x^2-4x+1} \to f'(x)=\dfrac{2x-4}{3\sqrt[3]{(x^2-4x+1)^2}}$

ac) $f(x)=\cos^4 x \to f'(x)=4\cos^3 x\cdot(-\operatorname{sen} x)=$
$=-4\cos^3 x\cdot \operatorname{sen} x$

ad) $f(x)=\cos^2(5x-3) \to f'(x)=2\cos(5x-3)\cdot[-\operatorname{sen}(5x-3)]\cdot 5$
$\Rightarrow f'(x)=-10\cos(5x-3)\cdot\operatorname{sen}(5x-3)$

ae) $f(x)=\log_3(x^2-\sqrt{x}) \to f'(x)=\dfrac{1}{x^2-\sqrt{x}}\cdot\left(2x-\dfrac{1}{2\sqrt{x}}\right)\cdot\dfrac{1}{\ln 3}$

af) $f(x)=\ln\left(\dfrac{x-1}{x}\right) \to f'(x)=\dfrac{1}{\frac{x-1}{x}}\cdot\dfrac{1\cdot x-(x-1)\cdot 1}{x^2}=$

$=\dfrac{\cancel{x}}{x-1}\cdot\dfrac{\cancel{x}-\cancel{x}+1}{x^{\cancel{2}}}=\dfrac{1}{(x-1)\cdot x}$

ag) $f(x)=(\ln x)^2\cdot\cos x \to f'(x)=2\ln x\cdot\dfrac{1}{x}\cos x+(\ln x)^2\cdot(-\operatorname{sen} x)$
$\Rightarrow f'(x)=\dfrac{2\cdot\cos x\cdot\ln x}{x}-\operatorname{sen} x\cdot(\ln x)^2$

ah) $f(x)=\dfrac{1}{(x-2)^5}=(x-2)^{-5} \to f'(x)=-5(x-2)^{-6}\cdot 1 \Rightarrow$
$\Rightarrow f'(x)=-\dfrac{5}{(x-2)^6}$

ai) $f(x)=\dfrac{2x-1}{\cos(x^2+1)}$

$f'(x)=\dfrac{2\cdot\cos(x^2+1)-(2x-1)\cdot[-\operatorname{sen}(x^2+1)]\cdot 2x}{[\cos(x^2+1)]^2}=$

$$= \frac{2\cdot\cos(x^2+1) + 2x\cdot(2x-1)\cdot \operatorname{sen}(x^2+1)}{[\cos(x^2+1)]^2}$$

aj) $f(x) = \ln\left(\frac{x+1}{x-1}\right) \rightarrow f'(x) = \frac{1}{\frac{x+1}{\cancel{x-1}}} \cdot \frac{1(x-1)-(x+1)\cdot 1}{(x-1)^{\cancel{2}}} =$

$$= \frac{\cancel{x}-1-\cancel{x}-1}{(x+1)(x-1)} = \frac{-2}{x^2-1}$$

ak) $f(x) = \sqrt{\operatorname{sen}x - x} \rightarrow f'(x) = \frac{1}{2\sqrt{\operatorname{sen}x - x}} \cdot (\cos x - 1)$

$$\Rightarrow f'(x) = \frac{\cos x - 1}{2\sqrt{\operatorname{sen}x - x}}$$

al) $f(x) = 5^{x+\cos x} \rightarrow f'(x) = 5^{x+\cos x} \cdot (1-\operatorname{sen}x)\cdot \ln 5$

am) $f(x) = \frac{\ln(x^3-4x)}{x} \rightarrow f'(x) = \frac{\frac{3x^2-4}{x^3-4x}\cdot x - \ln(x^3-4x)\cdot 1}{x^2} =$

$$= \frac{\frac{3x^2-4}{x^2-4} - \ln(x^3-4x)}{x^2}$$

an) $f(x) = \frac{1+\ln x^2}{x^3} \rightarrow f'(x) = \frac{\frac{1}{x^2}\cdot 2x\cdot x^3 - \ln(x^2)\cdot 3x^2}{(x^3)^2}$

$$= \frac{2x^2 - 3x^2\ln(x^2)}{x^6} = \frac{2-3\ln(x^2)}{x^4}$$

ao) $f(x) = \operatorname{sen}^2 x \cdot \cos^2 x$

$f'(x) = 2\operatorname{sen}x\cdot\cos x\cdot\cos^2 x + \operatorname{sen}^2 x\cdot 2\cos x\cdot(-\operatorname{sen}x) =$

$$= 2\operatorname{sen}x\cdot\cos^3 x - 2\operatorname{sen}^2 x\cdot\cos x$$

ap) $f(x) = \sqrt[4]{x^2 - e^{-x}} \rightarrow f'(x) = \frac{1}{4\sqrt[4]{(x^2-e^{-x})^3}} \cdot (2x + e^{-x})$

$$f'(x) = \frac{2x + e^{-x}}{4\sqrt[4]{(x^2-e^{-x})^3}}$$

aq) $f(x) = \operatorname{sen}^2\sqrt{x} \rightarrow f'(x) = 2\operatorname{sen}\sqrt{x} \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$$f'(x) = \frac{\operatorname{sen}\sqrt{x} \cdot \cos\sqrt{x}}{\sqrt{x}}$$

ar) $f(x) = (1 + 5\sqrt{x})^3 \rightarrow f'(x) = 3(1+5\sqrt{x})^2 \cdot \frac{5}{2\sqrt{x}} \Rightarrow$

$$\Rightarrow f'(x) = \frac{15(1+5\sqrt{x})^2}{2\sqrt{x}}$$