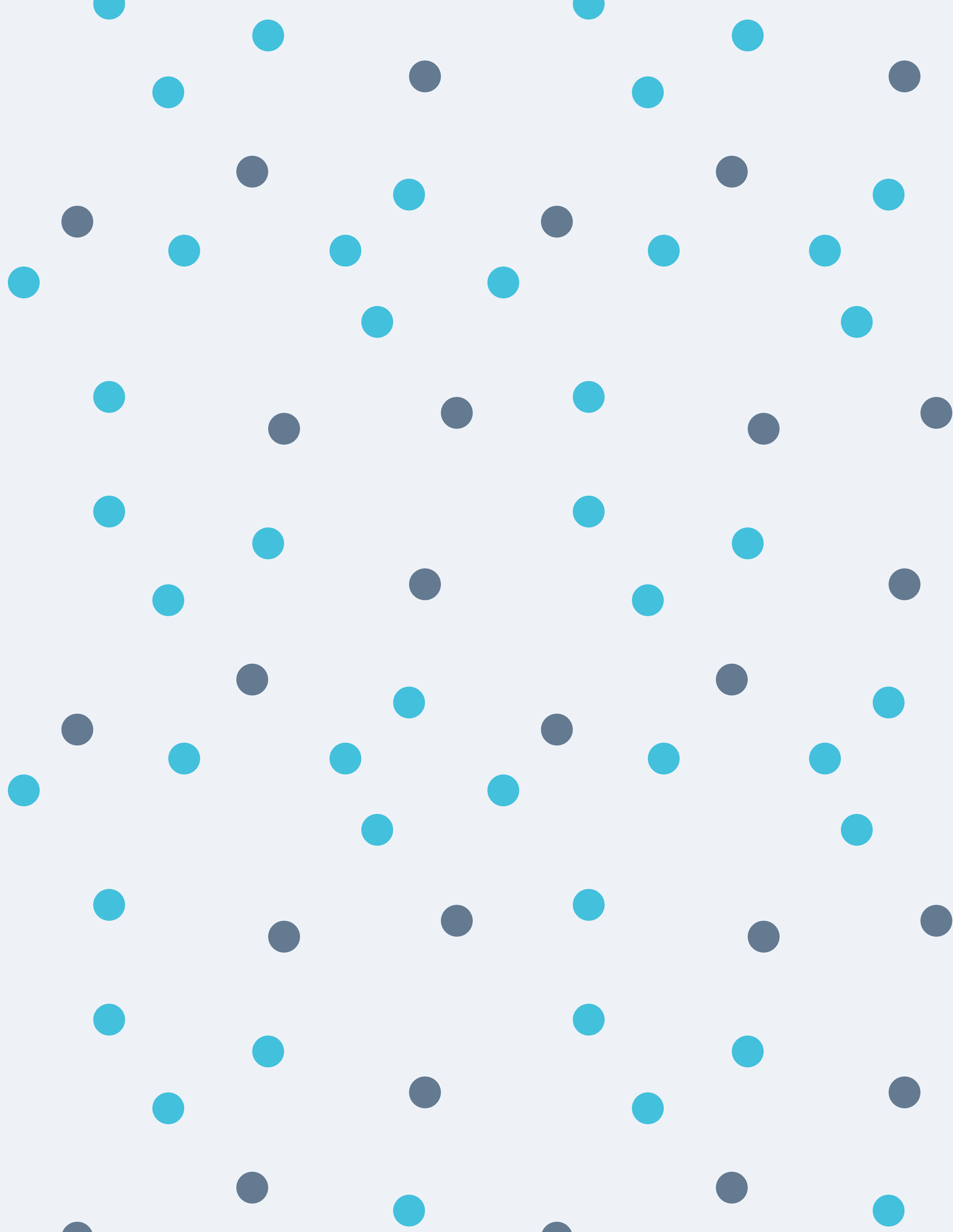


$\boxed{1.-}$ $A(3,1)$ $r: \begin{cases} x = -1-2t \\ y = 5+3t \end{cases}$

$$\left.\begin{array}{l} 3 = -1-2t \rightarrow t = -1 \\ 1 = 5+3t \rightarrow t = -\frac{4}{3} \end{array}\right\} \neq \Rightarrow A \notin r$$

$\boxed{2.-}$ a) $\frac{x+5}{3} = \frac{y-2}{2}$ Continua

$A = (-5, 2)$

$\vec{u} = (3,2) \rightarrow m = \frac{2}{3}$

$\vec{n} = (-2,3)$

Otro punto $\boxed{x=1} \rightarrow \frac{6}{3} = \frac{y-2}{2}$

$4 = y-2 \rightarrow \boxed{y=6}$

$B = (1,6)$

b) $y = 3x+1$ Explícita

$m = 3 \rightarrow \vec{u} = (1,3)$ $\vec{n} = (-3,1)$

$A = (0,1)$

otro punto $x=1$ $y=4 \rightarrow B(1,4)$

c) $(x,y) = (0,3) + t(2,-5) \quad t \in \mathbb{R}$

Ecuación general

$A = (0,3)$

$\vec{u} = (2,-5) \to m = \frac{-5}{2}$)

$\downarrow$

$\vec{n} = (5,2)$

Otro punto $t = 1 \to (x,y) = (2,-2)$

$B = (2,-2)$

d) $5x - 2y + 6 = 0$

Ec. general $\quad \vec{n} = (5,-2) \to \vec{u}\,(2,5)$

$\downarrow$

$m = \frac{5}{2}$

Pto: $x = 0 \to -2y + 6 = 0 \to y = 3$

$A(0,3)$

$x = 2 \to 10 - 2y + 6 = 0 \to y = 8$

$B(2,8)$

e) $\begin{cases} x = 4 + 2t \\ y = 3 - 2t \end{cases}$ $t \in \mathbb{R}$

Paramètrica

$P(4,3)$ $\vec{u}\,(2,-2) \rightarrow m = -1$

$\perp$

$\vec{n} = (2,2)$

otro punto $t = 1 \rightarrow x = 6$ $B(6,1)$

$y = 1$

f) $y - 3 = -2(x+5)$

Punto-Pendiente

$A = (-5,3)$

$m = -2 \rightarrow \vec{u} = (1,-2) \rightarrow \vec{n} = (2,1)$

otro punto

$x = 0 \rightarrow y - 3 = -10$ $y = 7$ $\rightarrow B(0,7)$

g) $3(x-1)+2(y+1)=0$

Normal

$\vec{n}(3,2) \rightarrow \vec{u}(-2,3) \rightarrow m=-\frac{3}{2}$

Pb $x=1 \rightarrow 2y+2 \Rightarrow y=-1 \quad A=(1,-1)$

Pb $y=-1 \quad 3x-3=0 \quad x=1 \quad B=(1,0)$

$\boxed{3.-}$ a) $P(2,-5) \quad \vec{u}(-1,2) \rightarrow m=-2$

- Vect: $(x,y)=(2,-5)+t(-1,2)$
- Param. $\begin{cases} x = 2-t \\ y=-5+2t \end{cases} \quad t \in \mathbb{R}$
- Continua: $\dfrac{x-2}{-1}=\dfrac{y+5}{2}$
- General: $2x-4=-y-5 \; ; \; 2x+y+1=0$
- Explícita: $y=-2x-1$
- Punto-Pendiente $y+5=-2(x-2)$

b) $A(-2,-4)$ $B(3,-2)$

$\vec{u}=(5,2)$ $m=\frac{2}{5}$

- Vect → $(x,y)=(-2,-4)+t(5,2)$
- Param → $\begin{cases} x=-2+5t \\ y=-4+2t \end{cases}$
- Cont $\frac{x+2}{5}=\frac{y+4}{2}$
- General $2x+4=5y+20$; $2x-5y-16=0$
- Explicita: $y=\frac{2}{5}x-\frac{16}{5}$
- Pto-Pendiente: $(y+4)=\frac{2}{5}(x+2)$

c) $A(-5,4)$; $m=-2$ → $\vec{u}=(1,-2)$

- Vect $(x,y)=(-5,4)+t(1,-2)$
- Param $\begin{cases} x=-5+t \\ y=4-2t \end{cases}$
- Cont $\frac{x+5}{1}=\frac{y-4}{-2}$
- General $-2x-10=y-4$; $2x+y+6=0$
- Exp. $y=-2x-6$
- Pto-Pendiente; $y-4=-2(x+5)$

d) $\boxed{P(0,0)}$ Pto medio de $\underbrace{A(1,-3);\ B(5,2)}$

$\boxed{M=\left(3,-\frac{1}{2}\right)}$

$\vec{u}=\left(3,-\frac{1}{2}\right)$

- Vectorial: $(x,y)=t\left(3,-\frac{1}{2}\right)$
- Paramétrica: $\begin{cases} x=3t \\ y=-\frac{1}{2}t \end{cases}$
- continua $\frac{x}{3}=\frac{y}{-1/2}$
- General $-\frac{1}{2}x=3y \rightarrow x+6y=0$
- explícita $y=-\frac{1}{6}x$
- Pto - Pendiente $y=-\frac{1}{6}x$

$\boxed{4.-}$ $A(-2,3)$; $B(2,-3)$; $C(-2,6)$

$r \rightarrow$ Pasa por $(-2,6) \rightarrow \vec{u}_r=(0,-3)$

$r: \begin{matrix} x=-2 \\ y=6-3t \end{matrix}$

Para pertenecer a r la primera coordenada debe ser $-2 \Rightarrow B\notin r$

$A, C \in r$

$\boxed{5.-}$ $r: (k+5)x - (3+k)y = 1-k$

$k \,/\, P = (2,3) \in r$

$(k+5)\cdot 2 - (3+k)\cdot 3 = 1-k$

$2k + 10 - 9 - 3k = 1-k$; $0k = 0$

$\Rightarrow \forall k \quad P \in r$

$\boxed{6.-}$ $r: 2x - 3y + 5 = 0$

a) $\vec{n}_r(2,-3)$; $\vec{u}_r = (3,2)$; $\vec{n} = \frac{2}{3}$

$P \to x=0 \to -3y+5=0 \to y = \frac{5}{3} \to P\left(0, \frac{5}{3}\right)$

b) $s \parallel r \qquad A=(2,0) \in s$

$\vec{u}_s = \vec{u}_r$

$$s: \begin{cases} x = 2+3t \\ y = 2t \end{cases}$$

c) $t \perp r \qquad O(0,0) \in t$

$\vec{u}_t = \vec{n}_r = (2,-3)$

$$t \begin{cases} x = 2t \\ y = -3t \end{cases}$$

$\boxed{7.-}$ $r: 4x + 2y - 5 = 0$

a) $\vec{n}_r(4,2) \rightarrow \vec{u}_r(-2,4) \rightarrow m = \frac{4}{-2} = -2$

$P \rightarrow x = 0 \quad 2y = 5 \quad y = \frac{5}{2} \rightarrow P\left(0, \frac{5}{2}\right)$

b) $s \parallel r \quad P(0,0) \in s$

$\Downarrow$

$\vec{u}_s = \vec{u}_r \qquad s: \begin{cases} x = -2t \\ y = 4t \end{cases}$

c) $t \perp r \quad A(-2,3) \in t$

$\Downarrow$

$\vec{u}_t = \vec{n}_r \qquad t: \begin{cases} x = -2 + 4t \\ y = 3 + 2t \end{cases}$

$\boxed{8.-}$ $A(-3,2) \qquad s \perp r: 3x - 6y = 1$

$\Downarrow$

$\vec{u}_s = \vec{n}_r = (3,-6)$

$s: \begin{cases} x = -3 + 3t \\ y = 2 - 6t \end{cases}$

9.- Segmento de extremos $A(3,1)$ $B(7,4)$

$\vec{u}_r = (4,3)$

$\downarrow$

$\vec{n}_r = (-3,4)$

Pto medio $P\left(5, \frac{5}{2}\right)$

$$s: \begin{cases} x = 5 - 3t \\ y = \frac{5}{2} + 4t \end{cases}$$

10.- $r: 5x + 4y - 3 = 0 \rightarrow \vec{n}_r = (5,4)$

$t \perp r$ pasando por $x = 1$ en $s: 6(x-1) - (y-1) = 0$

$\downarrow$

$\vec{u}_t = \vec{n}_r$

$6 \cdot 0 - (y-1) = 0$

$y = 1$

$\boxed{P(1,1)}$

$$t: \begin{cases} x = 1 + 5t \\ y = 1 + 4t \end{cases}$$

11.- $A(-1,3)$; $B(4,0)$; $C(-1,2)$

a) A ; $\vec{BC} = (-5, 2)$ $\quad \frac{x+1}{-5} = \frac{y-3}{2}$; $2x + 2 = -5y + 15$

$2x + 5y = 13$; $\boxed{y = -\frac{2}{5}x + \frac{13}{5}}$

b) $\vec{AC} = (0, -1)$ $\quad \vec{n}_{AC} = (1, 0)$ $\quad B(4,0)$

$\rightarrow m = 0$

$y = 0(x-4) \rightarrow \boxed{y = 0}$

$\boxed{12.-}$ Paralela a

a) $r: 2x+5y-5=0$ $\quad P(-2,6)$ $\quad \vec{n}_r=(2,5) \rightarrow \vec{u}_r=(-5,2)$

$s \parallel r \Rightarrow \vec{u}_s=\vec{u}_r$

$$s: \begin{cases} x=-2-5t \\ y=6+2t \end{cases}$$

b) Eje abscisas $\quad B(-1,4)$

$\longrightarrow \quad \vec{u}_r=(1,0)$

$$t: \begin{cases} x=-1+t \\ y=4 \end{cases}$$

c) Eje ordenadas $\quad B(-1,4)$

$\uparrow \quad \vec{u}_r=(0,1)$

$$p: \begin{cases} x=-1 \\ y=4+t \end{cases}$$

d) $r: 2x-y+12=0$ $\quad P(0,0)$

$\vec{n}_r=(2,-1) \rightarrow \vec{u}_r=(1,2)$

$$q: \begin{cases} x=t \\ y=2t \end{cases}$$

$\boxed{13.-}$ Perpendicular a:

a) $r: x-2y-3=0 \quad P(2,-1)$

$\vec{u_s} = \vec{n_r} = (1,-2)$

$s: \begin{cases} x = 2+t \\ y = -1-2t \end{cases}$

b) Eje abscisas $\quad B(-4,8)$

$* \vec{u_r} = (1,0) \rightarrow \vec{n_r} = (0,1)$

$\vec{u_t} = \vec{n_r} = (0,1)$

$t: \begin{cases} x = -4 \\ y = 8+t \end{cases}$

c) Eje ordenadas $\quad C(-1,3)$

$* \vec{u_r} = (0,1) \rightarrow \vec{n_r} = (1,0)$

$\vec{u_q} = \vec{n_r} = (1,0)$

$q: \begin{cases} x = -1+t \\ y = 3 \end{cases}$

Se puede trabajar con vectores directores o normales

$\boxed{11\ 4/,-}$

a) $r: 4x + 2y - 5 = 0 \qquad \vec{n}_r = (4,2) \rightarrow \vec{u}_r\ (-2,4)$

$s: \dfrac{x-3}{-1} = \dfrac{y-2}{2} \qquad \vec{u}_s = (-1,2)$

$$\left.\begin{array}{l} m_r = \dfrac{4}{-2} = -2 \\ \\ m_s = \dfrac{2}{-1} = -2 \end{array}\right\} \Rightarrow \text{Paralelas o Coincidentes}$$

$P_s = (3,2) \in r\ ?$

$4 \cdot 3 + 2 \cdot 2 - 5 = 12 + 4 - 5 \neq 0$

$P_s \notin r \Rightarrow$ Paralelas

b) $r: 2x + y - 5 = 0 \rightarrow \vec{n}_r = (2,1)$

$s: 4x + 3y = 11 \rightarrow \vec{n}_s = (4,3)$

Vectores normales no proporcionales $\Rightarrow$

$\Rightarrow$ secantes

c) $r: -\frac{1}{2}x + \frac{3}{4}y + \frac{5}{4} = 0 \qquad \vec{n}_r = \left(-\frac{1}{2}, \frac{3}{4}\right)$

$s: \ 2x - 3y - 5 = 0 \qquad \vec{n}_s = (2,-3)$

$\dfrac{-1/2}{2} = \dfrac{3/4}{-3} \Rightarrow$ Proporcionales $\Rightarrow$ Paralelas o Coincidentes

- Tomamos un $P \in r$ $\rightarrow$ si $P \in S \Rightarrow$ coincidentes

  $\searrow$ si $P \notin S \Rightarrow$ paralelas

r: $-\frac{1}{2}x + \frac{3}{4}y + \frac{5}{4} = 0$

$x = 0$; $+\frac{3}{4}y = -\frac{5}{4}$ $\rightarrow$ $\boxed{y = \frac{-5}{3}}$

$P = (0, \frac{-5}{3})$

s: $2x - 3y - 5 = 0$; $2 \cdot 0 - 3\left(\frac{-5}{3}\right) - 5 = 0$

$\Downarrow$

$P \in S$

$\Rightarrow$ coincidentes

$\boxed{115.-}$ 1º Intersección de $r: x - 3y + 1 = 0$

$s: 2x + y - 12 = 0$

Resuelvo el sistema: $x = 3y - 1$

$2(3y - 1) + y - 12 = 0$

$6y - 2 + y - 12 = 0$; $7y = 14$ $\boxed{y = 2}$

$x = 3 \cdot 2 - 1$ $\boxed{x = 5}$ $Q(5, 2)$

$P = (3, -2)$ $Q(5, 2)$ $\vec{u}_t = (2, 4)$

$$t: \begin{cases} x = 3 + 2t \\ y = -2 + 4t \end{cases}$$

$\boxed{1/6.-}$ Calcular k

a) $r: kx + 2y - 3 = 0$
$s: x + ky + 1 = 0$ paralelas

$\vec{n}_r = (k, 2)$ $\vec{n}_s = (1, k)$

$r \parallel s \Longleftrightarrow \vec{n}_r; \vec{n}_s$ proporcionales

$$\frac{k}{1} = \frac{2}{k} \rightarrow k^2 = 2 \quad \boxed{k = \pm\sqrt{2}}$$

b) $r: x - ky + 1 = 0$
$s: kx + 4y - 3 = 0$ paralelas

$\vec{n}_r = (1, -k)$
$\vec{n}_s = (k, 4)$

$$\frac{1}{k} = \frac{-k}{4} ; \quad -k^2 = 4$$

Imposible

c) $r: kx - 2y - 4k = 0$

$s: x - 3y - 4 = 0$

coincidents

1: Paral·les $\vec{n}_r = (k, -2)$

$\vec{n}_s = (1, -3)$

$\frac{k}{1} = \frac{-2}{-3} \rightarrow -3k = -2; \quad \boxed{k = \frac{2}{3}}$

Per ademés coincidents

$P \in r \quad x = 0 \quad -2y - 4k = 0; \quad \boxed{y = -2k}$

$P(0, -2k)$

$P \in s; \quad 0 - 3(-2k) - 4 = 0$

$+6k = 4; \quad k = +\frac{4}{6} = +\frac{2}{3}$

Efectivament para $k = \frac{2}{3}$ r y s coincidents

d) $r: (k-1)x - 2y + 2k = 0$

$s: (3k-4)x + y + k^2 = 0$

Perpendiculares

Para ser $\perp$ $\quad \vec{n_r} = \vec{u_s}$ — Proporcionales

$\vec{n_r} = (k-1, -2)$

$\vec{n_s} = (3k-4, 1) \rightarrow \vec{u_s} = (-1, 3k-4)$

$\dfrac{k-1}{-1} = \dfrac{-2}{3k-4}$; $(3k-4)(k-1) = +2$

$3k^2 - 3k - 4k + 4 - 2 = 0$

$\boxed{3k^2 - 7k + 2 = 0}$

$$k = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6} \begin{cases} 2 \\ \frac{2}{6} = \frac{1}{3} \end{cases}$$

Si $k = 2$ o $k = \dfrac{1}{3}$ $\quad r \perp s$

$\boxed{17.-}$ $r: x - by = -4b - 1$

$s \rightarrow$ pasa por $P(-1,4)$; $Q(2,3)$

$\boxed{\vec{u_s} = (3,-1)}$

$$s: \begin{cases} x = -1 + 3t \\ y = 4 - t \end{cases}$$

Calcular b para que $\boxed{r = s}$ coincidente

$\vec{u_r} = u_s$ (Proporcionales)

$\vec{n_r} = (1, -b) \Rightarrow \boxed{\vec{u_r} = (b, 1)}$

$\frac{3}{b} = \frac{-1}{1} \rightarrow \boxed{b = -3}$

Coincidente si $P \in s \Rightarrow P \in r$

¿$P(-1,4) \in r$?

$-1 - 4b = -4b - 1$ exacto

$\Rightarrow P \in r$

Si $\boxed{b = -3}$ r, s coincidente

$\boxed{18.-}$ $r: 5x + my + 1 = 0$

$s: -x - y + 3 = 0$

$m \,/\, r \parallel s$

$\vec{n}_r = (5, m)$

$\vec{n}_s = (-1, -1)$

$\vec{n}_r$ prop $\vec{n}_s$

$\frac{5}{-1} = \frac{m}{-1} \rightarrow \boxed{m = 5}$

Si $m = 5 \Rightarrow$ paralelas

Si $m \neq 5 \Rightarrow$ secantes

¿Coincidentes? $P \in r$ $\left(y = 0 \rightarrow x = -\frac{1}{5}\right)$

$P\left(-\frac{1}{5}, 0\right)$

¿$P \in s$? $\frac{1}{5} + 3 \neq 0 \rightarrow P \notin s$

Nunca serán coincidentes

$\boxed{19.}$ $r: 3x - my + 2 = 0$

$s: -6x - y + n = 0$

Posición Relativa

$\vec{n}_r = (3, -m)$

$\vec{n}_s = (-6, -1)$

$\frac{3}{-6} = \frac{-m}{-1}$; $6m = -3$ $\boxed{m = -\frac{1}{2}}$

Si $m \neq -\frac{1}{2}$ secantes

Si $m = -\frac{1}{2}$ paralelas o coincidentes

$P \in r$ $(y = 0 \rightarrow x = -\frac{2}{3})$ $P(-\frac{2}{3}, 0)$

$P \in s$ ¿$-6(-\frac{2}{3}) - 0 + n = 0$?

$4 + n = 0$ $n = -4$

- Si $m = -\frac{1}{2}$; $n = -4 \rightarrow$ Coincidentes
- Si $m = -\frac{1}{2}$; $n \neq -4 \rightarrow$ Paralelas

$\boxed{20,-}$

$r: \frac{1}{2}x - \frac{3}{5}y + 6 = 0$

$s: \frac{5}{6}x - \frac{3}{5}y + 8 = 0$

$r \perp s \iff \vec{n}_r = \vec{u}_s$

$\perp$ prop.

$$\boxed{\vec{n}_r = \left(\frac{1}{2}, -\frac{3}{5}\right)}$$

$\vec{n}_s = \left(\frac{5}{6}, -\frac{3}{5}\right) \rightarrow \boxed{\vec{u}_s = \left(\frac{3}{5}, \frac{5}{6}\right)}$

$$\text{è } \frac{1/2}{3/5} = \frac{-3/5}{5/6} \text{ ?} \rightarrow \text{NO} \Rightarrow \text{non son } \perp$$

b- $r: \frac{2}{3}x + \frac{1}{4}y = 0$ $\qquad \vec{n}_r = \left(\frac{2}{3}, \frac{1}{4}\right)$

$$s: \begin{cases} x = 2 - 3t \\ y = 3 - \frac{9}{8}t \end{cases} \qquad \vec{u}_s = \left(-3, -\frac{9}{8}\right)$$

Serán proporcionales si:

$$¿\ \frac{2/3}{-3} = \frac{1/4}{-9/8}\ ?$$

$$-\frac{2}{9} = -\frac{8}{36}$$

$-72 = 72 \Rightarrow$ son $\perp$

$\boxed{21-}$ $P(-4,3)$

$A(1,-3)$ $B(-1,2)$ → Pto Medio $T = \left(0, -\frac{1}{2}\right)$

→ Distancia entre 2 puntos = módulo del vector entre esos puntos

$$\vec{PT} = \left(4, -\frac{7}{2}\right)$$

$$d(P,T) = |\vec{PT}| = \sqrt{16 + \frac{49}{4}} = \sqrt{\frac{113}{4}} = \frac{\sqrt{113}}{2}$$

$\boxed{22.-}$ r que pasa por $A(-2,3)$
$B(2,2)$

$\vec{u}_r = (4,-1)$

$$r:\begin{cases} x = -2+4t \\ y = 3-t \end{cases}$$

$$\frac{x+2}{4} = \frac{y-3}{-1}$$

$$-x-2 = 4y-12$$

$$\boxed{x+4y-10=0}$$

$d(P,r)$ $\quad P(-4,3)$

$$d(P,r) = \frac{|-4+4\cdot 3-10|}{\sqrt{1^2+4^2}} = \frac{|-2|}{3} = \frac{2}{3}$$

$\boxed{23.-}$ $A(2,-3)$; $B(-2,5)$

$$d(A,B) = |\vec{AB}| = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}\ u.$$

$\vec{AB} = (-4,8)$

$\boxed{24.-}$ $d(P,r)$

a) $P(-3,4)$ $r: 2x+3y-5=0$

$$d(P,r)=\frac{|-6+12-5|}{\sqrt{4+9}}=\frac{1}{\sqrt{13}}\,u$$

b) $P(0,2)$ $r: y=-2x+5$

$$2x+y-5=0$$

$$d(P,r)=\frac{|0+2-5|}{\sqrt{4+1}}=\frac{3}{\sqrt{5}}\,u$$

c) $P\left(\frac{1}{2},-3\right)$ $r: 2x-2y=-3$

$$2x-2y+3=0$$

$$d(P,r)=\frac{|1+6+3|}{\sqrt{4+4}}=\frac{10}{\sqrt{8}}=\frac{5}{\sqrt{2}}\,u.$$

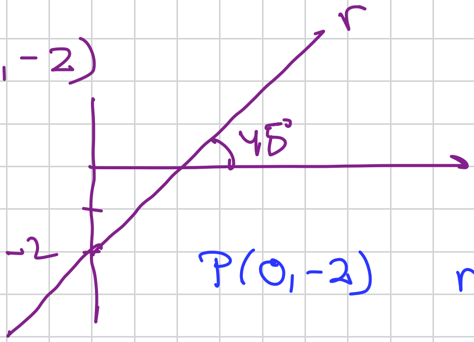
d) $P(1,-2)$ $r: \begin{cases} x=1+2t \\ y=-2-2t \end{cases}$ $t\in\mathbb{R}$

$$\frac{x-1}{2}=\frac{y+2}{-2}\;;\; -2x+2=2y+4;\; \boxed{2x+2y+2=0}$$

$$d(P,r)=\frac{|2-4+2|}{\sqrt{4+4}}=0$$

Per

e) $P(3,-2)$



$P(0,-2)$ $\quad m=1 \to \vec{u}=(1,1)$

$\vec{n}=(-1,1)$

$r: -x+y+t=0$

Per ; $-0-2+t=0$ $\boxed{t=2}$

$\boxed{r: -x+y+2=0}$

$$d(P,r)=\frac{|-3-2+2|}{\sqrt{1+1}}=\frac{3}{\sqrt{2}}\ u.$$

$\boxed{25.-}$

$r: x+2y-4=0$

$r \cap OX$; $y=0$

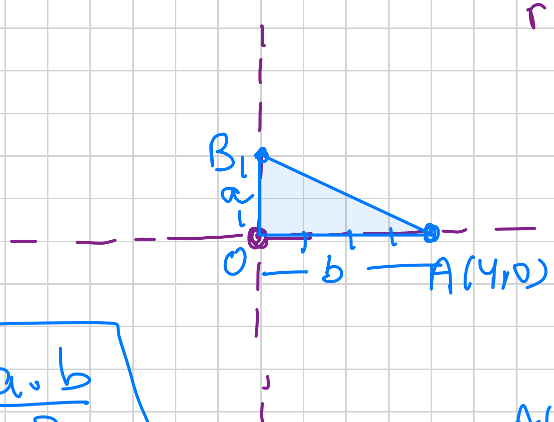
$x=4$

$r \cap OY$; $x=0$

$2y=4$

$y=2$

$A(4,0)$; $B(0,2)$



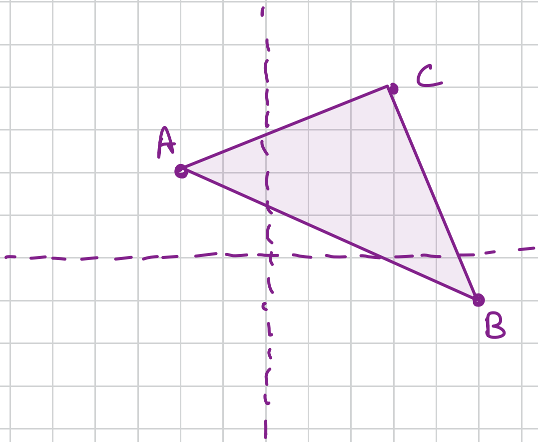
$$\boxed{A_T=\frac{a \cdot b}{2}}$$

$a = |\vec{OB}| = 2$

$b = |\vec{OA}| = 4$

$A_T = \dfrac{2 \cdot 4}{2} = 4\ u^2$

$\boxed{26.-}$ $A(-2,2)$; $B(5,-1)$ $C(3,4)$



Perímetro: $|\vec{AB}| + |\vec{BC}| + |\vec{CA}|$

$\vec{AB} = (7,-3)$ $|\vec{AB}| = \sqrt{58}$

$\vec{BC} = (-2,5)$ $|\vec{BC}| = \sqrt{29}$

$\vec{CA} = (-5,-2)$ $|\vec{CA}| = \sqrt{29}$

Perímetro $= 2\sqrt{29} + \sqrt{58}\ u$

$\vec{CA} \cdot \vec{BC} = 0 \Rightarrow$ son $\perp \Rightarrow$ Triángulo Rectángulo

Área $= \dfrac{|\vec{CA}| \cdot |\vec{BC}|}{2} = \dfrac{\sqrt{29} \cdot \sqrt{29}}{2} = \dfrac{29}{2} = 14'5\ u^2$

$\boxed{27.-}$ Comprueba que son paralelas y la distancia entre ellas

a) $r: 2x - y - 7 = 0$ $\qquad \vec{n_r} = (2, -1)$

$s: 2x - y - 8 = 0$ $\qquad \vec{n_s} = (2, -1)$ $\quad \| \Rightarrow r \| s$

Per $P(0, -7)$

$$d(r, s) = d(P, s) = \frac{|0 + 7 - 8|}{\sqrt{4+1}} = \frac{1}{\sqrt{5}} \text{ u.}$$

b) $r: 2x - 3y - 2 = 0$ $\qquad \vec{n_r}\ (2, -3)$

$s: -\frac{2}{3}x + y - 2 = 0$ $\qquad \vec{n_s} = \left(-\frac{2}{3}, 1\right)$

$$¿\frac{2}{-2/3} = \frac{-3}{1}? \quad 2 = 2 \Rightarrow r \| s$$

$Q \in s \rightarrow Q(0, 2)$

$$d(r, s) = d(Q, r) = \frac{|0 - 6 - 2|}{\sqrt{4+9}} = \frac{4}{\sqrt{13}} \text{ u.}$$

c) $r: \begin{cases} x = 1+2t \\ y = 3-2t \end{cases}$ $\quad$ $s: \begin{cases} x = 2-2t \\ y = 1+t \end{cases}$

$\vec{u_r} = (2,-2)$ $\quad$ $\vec{u_s} = (-2,1)$

$$\frac{2}{-2} \neq \frac{-2}{1} \Rightarrow \text{Secantes}$$

No son proporcionales

$\boxed{128.-}$ Angulo $\alpha = \measuredangle(u_r, u_s)$ -

a) $r: 3x+y-1=0$ $\quad$ $\vec{n_r} = (3,1)$ $\quad$ $|\vec{n_r}| = \sqrt{10}$

$s: x-y-3=0$ $\quad$ $\vec{n_s} = (1,-1)$ $\quad$ $\vec{n_s} = \sqrt{2}$

$$\cos\alpha = \frac{|\vec{n_r}\cdot\vec{n_s}|}{|\vec{n_r}|\,|\vec{n_s}|} = \frac{|3-1|}{\underbrace{\sqrt{10}\sqrt{2}}_{=\sqrt{20}}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \alpha = 63^\circ 26' 5{,}82''$$

b) $r: \; x - 2y - 2 = 0$ $\qquad \vec{n}_r = (1, -2)$ $\qquad |\vec{n}_r| = \sqrt{5}$

$s: \; -x - y - 2 = 0$ $\qquad \vec{n}_s = (-1, -1)$ $\qquad |\vec{n}_s| = \sqrt{2}$

$$\cos\alpha = \frac{|-1+2|}{\sqrt{5}\cdot\sqrt{2}} = \frac{1}{\sqrt{10}} \rightarrow \alpha = 71^\circ\, 33'\, 54{,}18''$$

c) $r: \; y = -x + 2y \;;\quad x - y = 0$

$s: \; y = -\frac{1}{2} - 3x \qquad 3x + y + \frac{1}{2} = 0$

$\vec{n}_r = (1, -1)$ $\qquad |\vec{n}_r| = \sqrt{2}$

$\vec{n}_s = (3, 1)$ $\qquad |\vec{n}_s| = \sqrt{10}$

$$\cos\alpha = \frac{|3-1|}{\sqrt{2}\cdot\sqrt{10}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\alpha = 63^\circ\, 26'\, 5{,}86''$$

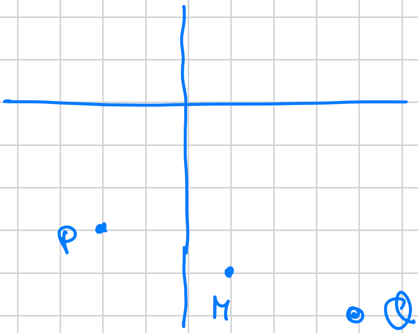
d) $r: \begin{cases} x = 1 + 2t \\ y = 3 - 2t \end{cases}$ $\qquad s: \begin{cases} x = 2 - 2t \\ y = 1 + t \end{cases}$

$\vec{u}_r = (2, -2)$ $\qquad \vec{u}_s = (-2, 1)$

$|\vec{u}_r| = \sqrt{8}$ $\qquad |\vec{u}_s| = \sqrt{5}$

$$\cos\alpha = \frac{|-4-2|}{\sqrt{8}\cdot\sqrt{5}} = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}} \rightarrow \alpha = 18^\circ\, 26'\, 5{,}82''$$

29.- Simétrico de $P(-2,-3)$ respecto de $M(1,-4)$



M punto medio de PQ

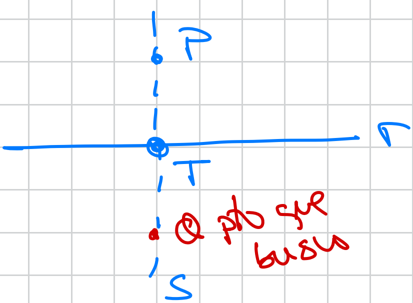
$P(-2,-3)$ $Q(a,b)$ $M=(1,-4)$

$\frac{-2+a}{2}=1 \rightarrow a=4$

$\frac{-3+b}{2}=-4 \rightarrow b=-5$

$\Rightarrow Q=(4,-5)$

30.- Simétrico de $P(5,0)$ respecto a $r: 2x+y=5$



Necesito una $s \perp r$ pasando por P

$\vec{u}_s = \vec{n}_r = (2,1)$

$\frac{x-5}{2} = \frac{y}{1} \rightarrow \boxed{x-2y-5=0}$

$T = r \cap s$ $\left\{\begin{array}{l} 2x+y=5 \\ x-2y=5 \end{array}\right.$ $\left\{\begin{array}{l} 2\cancel{x}+y=5 \\ -2\cancel{x}+4y=-10 \end{array}\right.$

$5y=-5$ $y=1$

$x-2=5$ $\boxed{x=7}$

$\boxed{T=(7,1)}$

T es el punto medio de $\overline{PQ}$

$T=(7,1)$ $P(5,0)$ $Q(a,b)$

$\frac{5+a}{2}=7 \rightarrow a=9$

$\frac{0+b}{2}=1 \rightarrow b=2$

$\boxed{Q=(9,2)}$

$\boxed{31.-}$ Simétrico de $P(1,1)$ respecto a $r: x-3y=9$

$\downarrow Q$ $\vec{n}_r(1,-3)$

$s \perp r$ pasando por P

$s: \begin{cases} \frac{x-1}{1}=\frac{y-1}{-3} \end{cases}$; $-3x+3=y-1$

$\vec{u}_s=\vec{n}_r$

$s: \boxed{3x+y-4=0}$

$T=r\cap s$ $\begin{cases} x-3y=9 \\ 3x+y=4 \end{cases}$ $\begin{matrix} x-\cancel{3y}=9 \\ +9x+\cancel{3y}=12 \end{matrix}$

$10x=21$ $x=\frac{21}{10}$

$\frac{21}{10}-3y=9$

$\frac{21}{10}-9=3y \rightarrow \frac{-69}{10}=3y \rightarrow y=-\frac{23}{10}$

$T=\left(\frac{21}{10},-\frac{23}{10}\right)$

T será el punto medio de $\overline{PQ}$ siendo Q el simétrico de P respecto de r

$$\left.\begin{array}{l} P=(1,1) \\ T=\left(\frac{21}{10},-\frac{23}{10}\right) \\ Q=(a,b) \end{array}\right\} \quad \begin{array}{l} \frac{1+a}{2}=\frac{21}{10};\ \to a=\frac{16}{5} \\ \frac{1+b}{2}=\frac{-23}{10};\ \to b=\frac{-18}{5} \end{array}$$

$$Q=\left(\frac{16}{5},-\frac{18}{5}\right)$$

$\boxed{32.-}$ Simétrico de $P(3,2)$ respecto de $r: 2x+y-3=0$

$s \perp r$ pasando por P $\quad \vec{u}_s=\vec{n}_r=(2,1)$

$$\frac{x-3}{2}=\frac{y-2}{1} \to x-3=2y-4 \to \boxed{s: x-2y+1=0}$$

$$T=r\cap s \left\{\begin{array}{l} 2x+y=3 \\ x-2y=-1 \end{array}\right. \quad \begin{array}{r} 4x+2y=6 \\ x-2y=-1 \\ \hline 5x=5 \to \boxed{x=1} \end{array}$$

$$1-2y=-1$$
$$2=2y: \boxed{y=1}$$

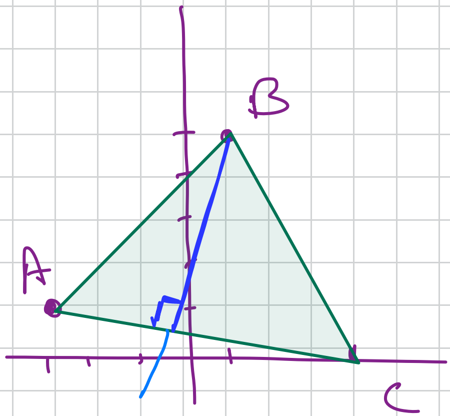
$$T=(1,1)$$

T es el punto medio de $\overline{PQ}$ (Q simétrico buscado)
$\quad (a,b)$

$$\frac{3+a}{2}=1 \to 3+a=2;\ \boxed{a=-1}$$
$$\frac{2+b}{2}=1 \to 2+b=2;\ \boxed{b=0}$$

$$\boxed{Q=(-1,0)}$$

33.- $A = (-3,1)$ $B(1,5)$ $C(4,0)$



a) Altura de B

s: será una recta $\perp \vec{AC}$ pasando por B

$\vec{AC} = (7,-1)$ $\quad n_{\vec{AC}} = (1,7)$

$$s: \begin{cases} x = 1+t \\ y = 5+7t \end{cases} \qquad \frac{x-1}{1} = \frac{y-5}{7}$$

$$7x - 7 = y - 5$$

$$\boxed{s: 7x - y - 2 = 0}$$

b) Mediatriz de $\overline{AB}$

Es una recta $\perp \overline{AB}$ pasando por M (punto medio)

$M=(-1,3)$

$\vec{AB}=(4,4)$ $\quad n_{\vec{AB}}=(-4,4)$

$t:\left\{ \frac{x+1}{-4}=\frac{y-3}{4} \right.$; $4x+4=-4y+12$

$\boxed{t:\ 4x+4y-8=0}$

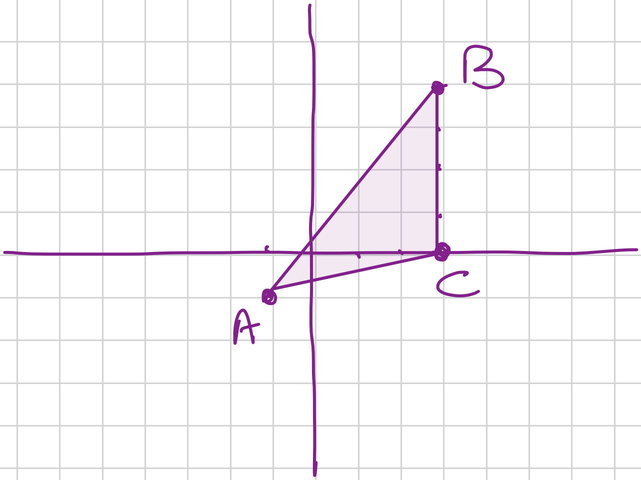
c) Mediana de AB

Recta que pasa por el M anterior y C

$M(-1,3)$; $C=(4,0)$ $\quad \vec{MC}=(5,-3)$

$\frac{x-4}{5}=\frac{y}{-3}$; $-3x+12=5y$; $\boxed{3x+5y-12=0}$

$\boxed{34.-}$ $A(-1,1)$; $B(3,4)$; $C(3,0)$



a) Mediatriz BC: r

Recta ⊥ BC pasando por su punto medio

$\vec{BC} = (0,-4)$ $\quad n_{\vec{BC}} = (4,0)$

Pto medio $M = (3,2)$

$$r: \left\{ \frac{x-3}{4} = \frac{y-2}{0} \right. ; \quad 4y = 8 \quad \boxed{y = 2}$$

Mediatriz

b) Mediatriz AC

Recta $s \perp \vec{AC}$ pasando por el punto medio

$\vec{AC} = (4, -1)$ $\quad$ $\vec{n}_{\vec{AC}} = (1, 4)$

Pto medio $T = (1, \frac{1}{2})$

$$s: \left\{ \frac{x-1}{1} = \frac{y - 1/2}{4} \right.$$

$$4x - 4 = y - \frac{1}{2}$$

$$s: \ 4x - y - \frac{7}{2} = 0$$

$$s: \ \boxed{8x - 2y - 7 = 0}$$

Mediatriz

a) Circuncentro

↓ Punto donde se cortan las mediatrices

$$\left.\begin{aligned} r&: y=2 \\ s&: 8x-2y-7=0 \end{aligned}\right\} \quad 8x=11 \quad x=\frac{11}{8}$$

$$\boxed{C=\left(\frac{11}{8},2\right)}$$

$-3x+12=-9$

35.- $A(-5,4)$; $B(4,1)$; $C(-1,-2)$

a) Ecuaciones de los lados

Lado AB: $\vec{AB}=(9,-3)$ $B(4,1)$

$r: \dfrac{x-4}{9}=\dfrac{y-1}{-3}$ $\quad r: 3x+9y-21=0$

$$\boxed{r: x+3y-7=0}$$

Lado BC : $\vec{BC}(-5,-3)$ $C(-1,-2)$

$$s: \frac{x+1}{-5} = \frac{y+2}{-3}$$

$$\boxed{s: 3x - 5y - 7 = 0}$$

Lado AC : $\vec{AC}(4,-6)$ $C(-1,-2)$

$$t: \frac{x+1}{4} = \frac{y+2}{-6}$$

$$6x + 4y + 14 = 0$$

$$\boxed{t: 3x + 2y + 7 = 0}$$

b) Mediana de B → r

Recta que pasa por B y el punto medio de AC → T

$T = (-3, 1)$ $B(4,1)$ $\vec{TB}(7,0)$

$$r: \frac{x+3}{7} = \frac{y-1}{0} \rightarrow 7y = 7 \quad \boxed{y = 1}$$

c) Mediana del vértice A → t

Recta que pasa por A y el punto medio de BC → R

$R = \left(\frac{3}{2}, \frac{-1}{2}\right)$ $A(-5,4)$

$\vec{RA} = \left(\frac{-13}{2}, \frac{9}{2}\right) = (-13, 9)$

$t: \left\{ \frac{x+5}{-13} = \frac{y-4}{9} \right.$

$9x + 45 = -13y + 52$

$\boxed{t: 9x + 13y - 7 = 0}$

d) Baricentro

↓

Punto donde se cortan las medianas R

$$R = t \cap r$$

$$\left.\begin{array}{l} r: \; y = 1 \\ t: \; 9x + 13y - 7 = 0 \end{array}\right\}$$

$$9x + 13 - 7 = 0$$

$$9x = -6$$

$$x = -\frac{2}{3}$$

$$\boxed{R = \left(-\frac{2}{3}, 1\right)}$$