12 $\qquad$ 1

c) $\dfrac{1+\cot g\,\alpha}{\operatorname{sen}\alpha+\cos\alpha}=\operatorname{cosec}\alpha$

$$\frac{1+\dfrac{1}{tg\,\alpha}}{\operatorname{sen}\alpha+\cos\alpha}=\operatorname{cosec}\alpha$$

$$\frac{tg\,\alpha+1}{tg\,\alpha\,(\operatorname{sen}\alpha+\cos\alpha)}=\operatorname{cosec}\alpha$$

$$\frac{\cancel{\dfrac{\operatorname{sen}\alpha+\cos\alpha}{\cos\alpha}}}{\dfrac{\operatorname{sen}\alpha}{\cancel{\cos\alpha}}\cdot\cancel{(\operatorname{sen}\alpha+\cos\alpha)}}=\operatorname{cosec}\alpha$$

$$\frac{1}{\operatorname{sen}\alpha}=\operatorname{cosec}\alpha$$

$$\operatorname{cosec}\alpha=\operatorname{cosec}\alpha$$

d) $\sec^2\alpha-1=tg^2\alpha$

$$\frac{1}{\cos^2\alpha}-1=tg^2\alpha$$

$$\frac{1-\cos^2\alpha}{\cos^2\alpha}=tg^2\alpha$$

$$\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}=tg^2\alpha$$

$$tg^2\alpha=tg^2\alpha$$

(14) $\hat{A} = 30°$; $a = 3cm$; $b = 4cm$

Demostrar que existen 2 triangulos

$$\frac{a}{\operatorname{sen}\hat{A}} = \frac{b}{\operatorname{sen}\hat{B}} \Rightarrow \operatorname{sen}\hat{B} = \frac{2}{3}$$

$\hat{B}_1 = 41'81°$ ÁNGULO AGUDO

$\hat{B}_2 = 180 - 41'.81 = 138'19°$ ANGULO OBTUSO

Sabemos que la suma de $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow$

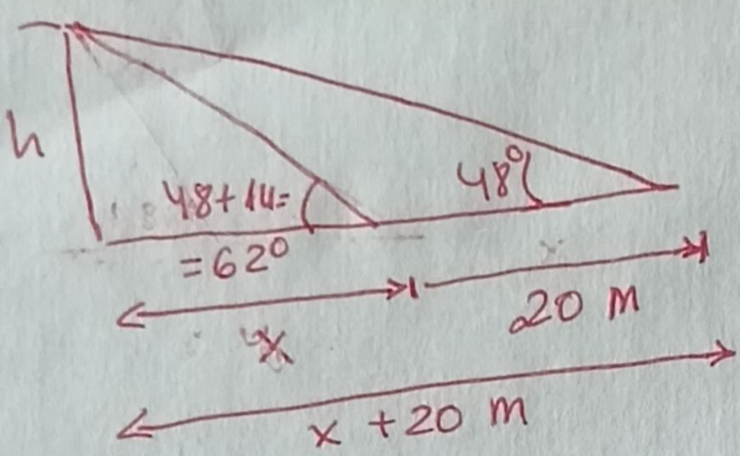
$\hat{A} + \hat{B} < 180°$

$\hat{A} + \hat{B}_1 = 30° + 41'81° = 71'81°$ ✓ $\hat{C}_1 = 108'19°$

$\hat{A} + \hat{B}_2 = 30° + 138'19° = 168'19°$ ✓ $\hat{C}_2 = 11'81°$

si hay dos triangulos

16



$$tg\,48 = \frac{h}{x+20}$$

$$tg\,62 = \frac{h}{x}$$

$$(x+20)\,tg\,48 = h$$

$$h = x \cdot tg\,62$$

$$x \cdot tg\,62 = x\,tg\,48 + 20\,tg\,48$$

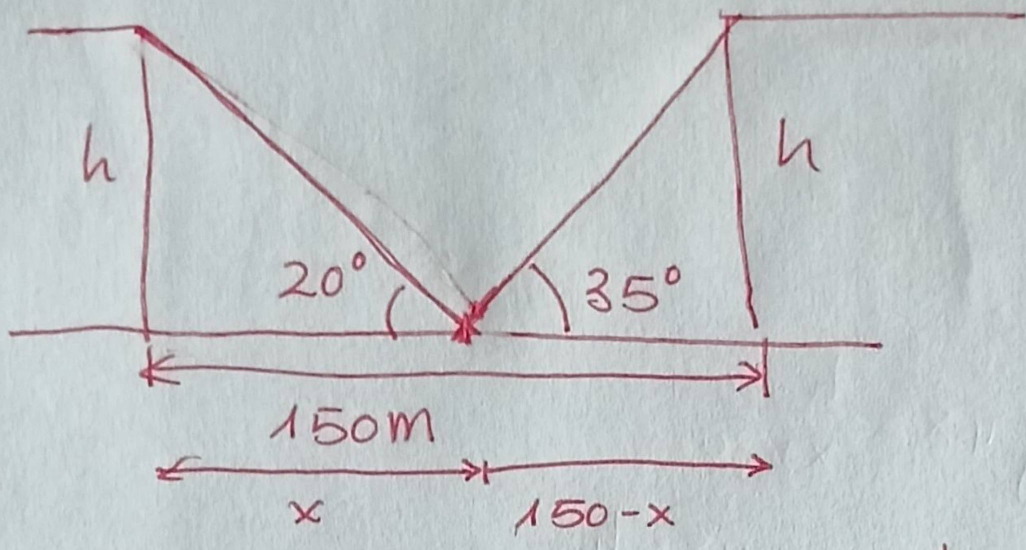
$$x \cdot (tg\,62 - tg\,48) = 20\,tg\,48$$

$$x = \frac{20 \cdot tg\,48}{tg\,62 - tg\,48}$$

$$x = 28'84\ m$$

$$h = 54'24\ m$$

(17)



$$tg\, 20 = \frac{h}{x} \qquad tg\, 35° = \frac{h}{150-x}$$

$$x\, tg\, 20 = h \qquad h = (150-x)\, tg\, 35°$$

$$x \cdot tg\, 20 = (150-x)\, tg\, 35$$

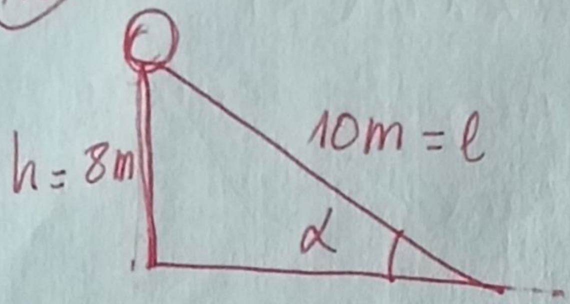
$$x\,(tg\, 20 + tg\, 35) = 150 \cdot tg\, 35$$

$$x = \frac{150 \cdot tg\, 35}{tg\, 20 + tg\, 35}$$

$$x = 98'70\, m$$

$$\boxed{h = 35'92\, m}$$

18



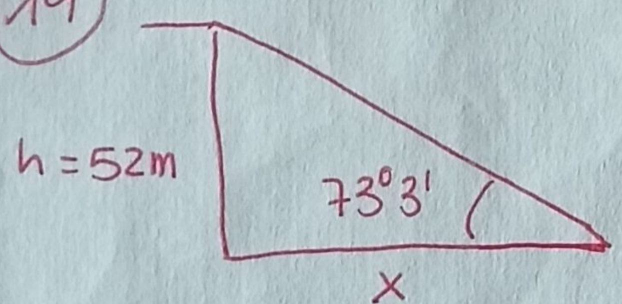
$$\operatorname{Sen} \alpha = \frac{\text{cat op}}{\text{hip}}$$

$$\operatorname{Sen} \alpha = \frac{8}{10}$$

$$\operatorname{Sen} \alpha = 0'8$$

$$\boxed{\alpha = 53'13^\circ}$$

19



$$tg\ \alpha = \frac{\text{cat op}}{\text{cat cont}}$$

$$tg\ 73^\circ 3' = \frac{52}{x}$$

$$x = \frac{52}{tg\ 73^\circ 3'}$$

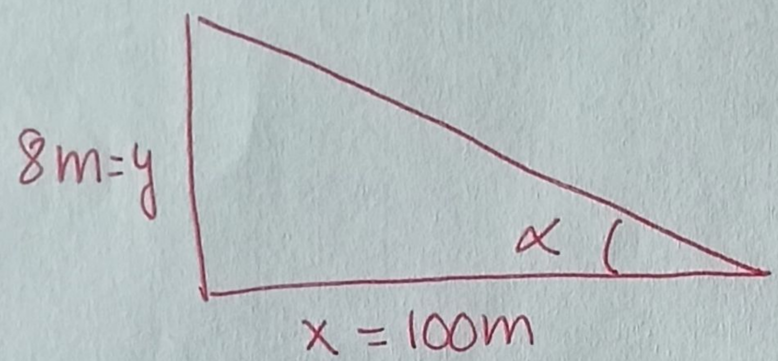
$$x = \frac{52}{tg\ 73'05}$$

$$\boxed{x = 15'85\ m}$$

$$3' \Rightarrow {}^\circ$$

$$\frac{3}{60} = 0'05$$

(20) 8% => 100 m desplazamiento horizontal (x) hay un ascenso de 8 m (y). En un triángulo rectángulo la pendiente equivale a la $tg\,\alpha$:

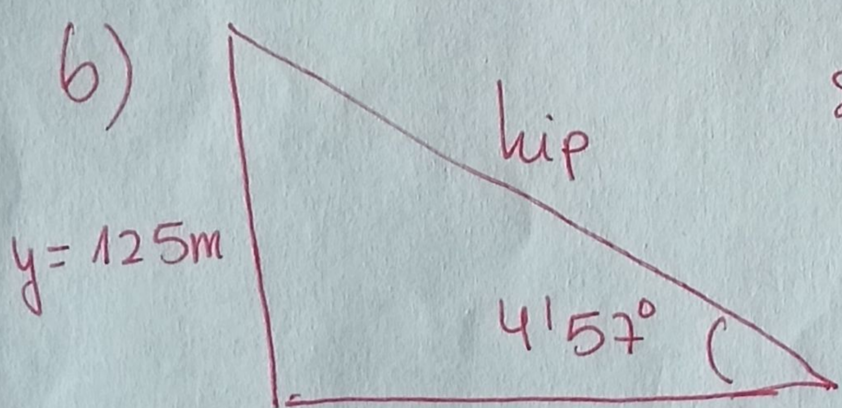


a) $$tg\,\alpha = \frac{y}{x} \Rightarrow tg\,\alpha = 0'08$$

$$\alpha = tg^{-1}\,0'08$$

$$\alpha = 4'57^\circ$$

b)



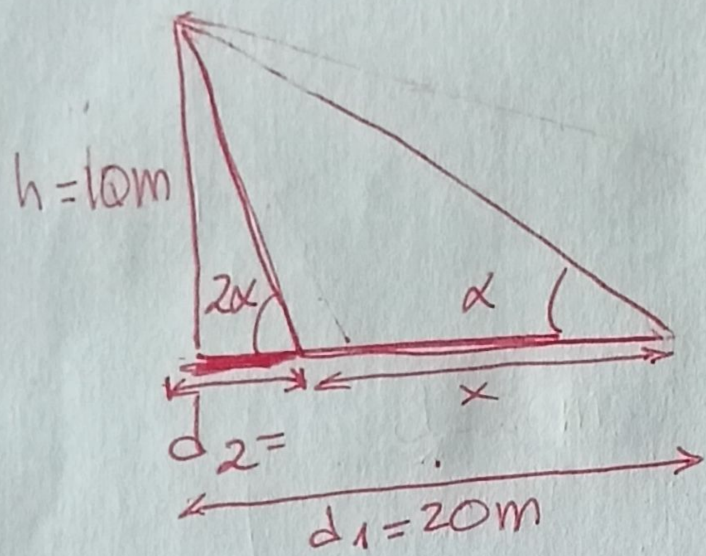
$$\operatorname{sen} 4'57^\circ = \frac{y}{hip}$$

$$hip = \frac{125}{\operatorname{sen} 4'57^\circ}$$

$$\boxed{hip = 1567'49\,m}$$

Debe recorrer 1567'49 m para ascender 125m de altura.

21



$$tg\,\alpha = \frac{10}{20}$$

$$tg\,\alpha = \frac{1}{2}$$

$$tg(2\alpha) = \frac{2\,tg\,\alpha}{1 - tg^2(\alpha)}$$

$$tg(2\alpha) = \frac{2 \cdot 1/2}{1-(1/2)^2} = \frac{4}{3}$$

$$tg\,2\alpha = \frac{10}{d_2} \Rightarrow \frac{4}{3} = \frac{10}{d_2}$$

$$d_2 = \frac{10 \cdot 3}{4}$$

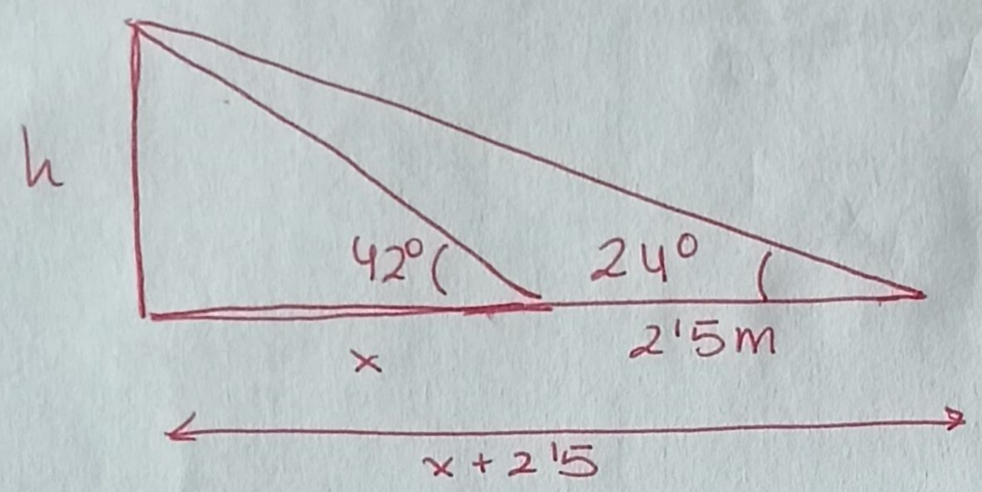
$$\boxed{d_2 = 7'5\,m}$$

Distancia que debemos recorrer hacia la torre (x)
es la diferencia entre

$$x = 20 - 7'5$$

$$\boxed{x = 12'5\,m}$$

(22)



$\text{tg } 42 = \dfrac{h}{x}$ $\qquad$ $\text{tg } 24 = \dfrac{h}{x+2'5}$

$h = x \text{ tg } 42$ $\qquad$ $h = (x+2'5) \text{ tg } 24$

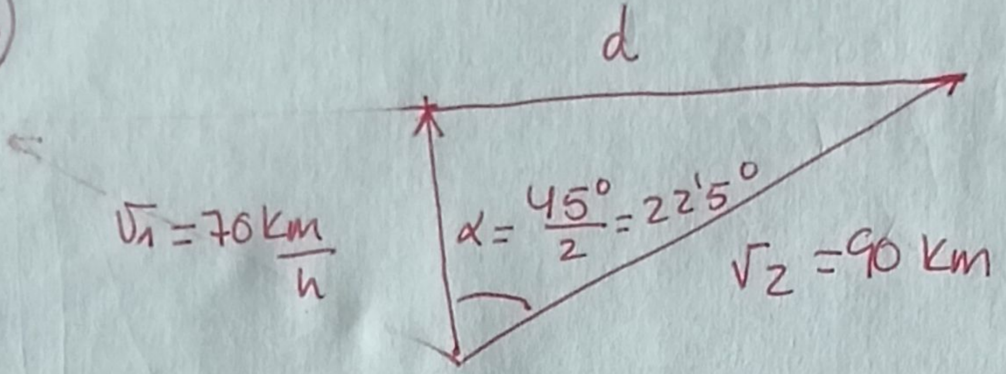
$x \text{ tg } 42 = x \text{ tg } 24 + 2'5 \text{ tg } 24$

$$x = \frac{2'5 \text{ tg } 24}{(\text{tg } 42 - \text{tg } 24)}$$

$x = 2'45 \text{ m}$

$$\boxed{h = 2'20 \text{ m}}$$

23



El rumbo NORNORDESTE se encuentra a la mitad del Norte (0°) y Nordeste (45°) por lo que $\alpha = \frac{45^\circ}{2} \Rightarrow$

$\alpha = 22'5^\circ$

$d_1 = 70 \frac{km}{h} \cdot 0'5h = 35\,km$

$d_2 = 90 \frac{km}{h} \times 0'5 = 45\,km$

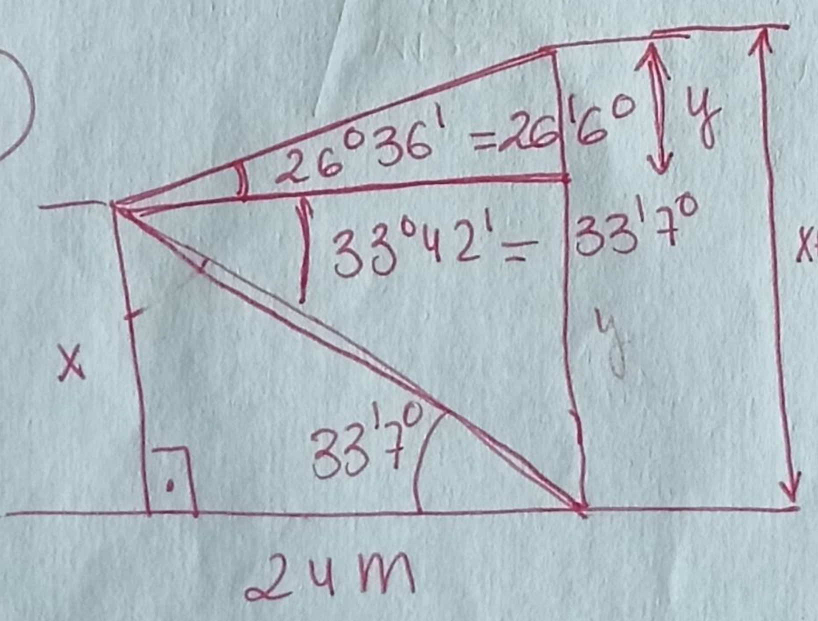
th. coseno

$d^2 = d_1^2 + d_2^2 - 2 d_1 \cdot d_2 \cdot \cos 22'5$

$d = \sqrt{35^2 + 45^2 - 2 \cdot 35 \cdot 45 \cdot \cos 22'5}$

$d = 18'43\,km$ distancia a la que se encuentran los coches despues de media hora

24



$\frac{36'}{60'} = 0'6^\circ$

$\frac{42}{60} = 0'7^\circ$

$tg\, 33'7 = \frac{x}{24} \Rightarrow x = 24 \cdot tg\, 33'7$

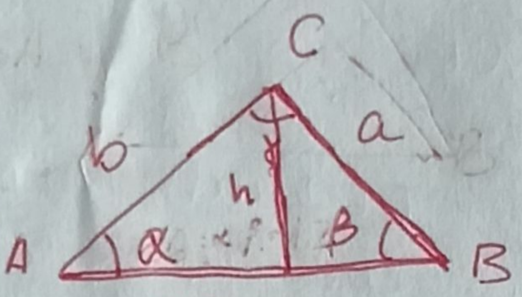
$x = 16\,m$ edificio pequeño

$tg\, 26'6 = \frac{y}{24} \Rightarrow y = 24 \cdot tg\, 26'6$

$y = 12'02\,m$

Edificio grande $28'02\,m$

(25)



$c = 75 \text{ km}$

$\gamma = 180 - 48 = 132^\circ$

$\beta = 12^\circ$

$\alpha = 36^\circ$

$$\frac{a}{\text{sen } 36^\circ} = \frac{b}{\text{sen } 12} = \frac{75}{\text{sen } 132^\circ}$$

$$b = \frac{75 \cdot \text{sen } 12^\circ}{\text{sen } 132^\circ}$$

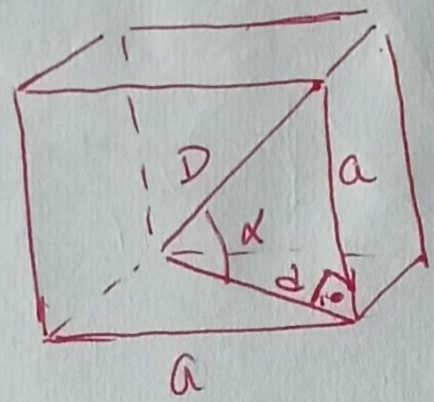
$$\boxed{b = 20'98 \text{ km}}$$

$$a = \frac{75 \cdot \text{sen } 36}{\text{sen } 132}$$

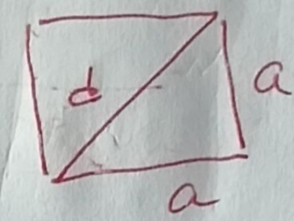
$$\boxed{a = 59'32 \text{ km}}$$

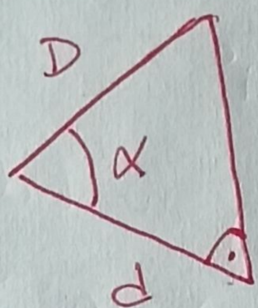
$$\text{sen } 36^\circ = \frac{h}{b} \Rightarrow h = 20'98 \cdot \text{sen } 36$$

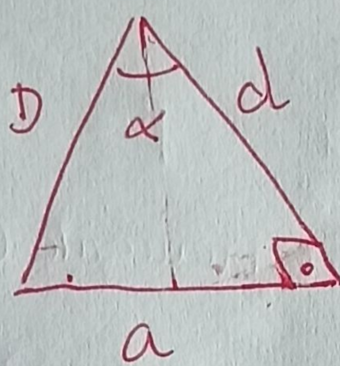
$$\boxed{h = 12'33 \text{ km}}$$



base:







$d^2 = a^2 + a^2$

$d = \sqrt{2a^2}$

$\boxed{d = \sqrt{2}a}$

$D^2 = \sqrt{d^2 + a^2}$

$D = \sqrt{2a^2 + a^2}$

$D = \sqrt{3a^2}$

$\boxed{D = \sqrt{3} \cdot a}$

Usamos th. coseno (tenemos 2 lados)

$a^2 = D^2 + d^2 - 2Dd \cdot \cos\alpha$

$a^2 = (\sqrt{3} \cdot a)^2 + (\sqrt{2} \cdot a)^2 - 2 \cdot \sqrt{3}\, a \cdot \sqrt{2}\, a \cdot \cos\alpha$

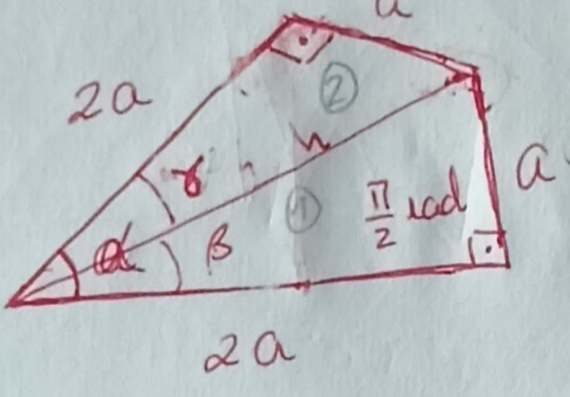
$a^2 = 3a^2 + 2a^2 - 2\sqrt{6}\, a^2 \cdot \cos\alpha$

$\dfrac{-4a^2}{-2\sqrt{6}a^2} = \cos\alpha \Rightarrow \cos\alpha = \dfrac{2}{\sqrt{6}}$

$\cos\alpha = \dfrac{\sqrt{6}}{3} \Rightarrow \alpha = \cos^{-1}\dfrac{\sqrt{6}}{3}$

$\boxed{\alpha = 35'26^\circ}$

(27 c) α ?



① th. Pitagoras

$$h^2 = (2a)^2 + a^2$$

$$h = \sqrt{5}\,a$$

$$tg\,\beta = \frac{a}{2a}$$

$$tg\,\beta = \frac{1}{2} \qquad \beta = 26'57^\circ$$

②

TRIANGULO RECTANGULO ② es como el triangulo ① por sus lados son iguales. Th. COSENO

$$a^2 = (2a)^2 + (a\sqrt{5})^2 - 2(2a)\cdot(a\sqrt{5})\cdot \cos\gamma$$

$$a^2 = 4a^2 + 5a^2 - 4\sqrt{5}a^2 \cdot \cos\gamma$$

$$-8a^2 = -4\sqrt{5}\,a^2 \cos\gamma$$

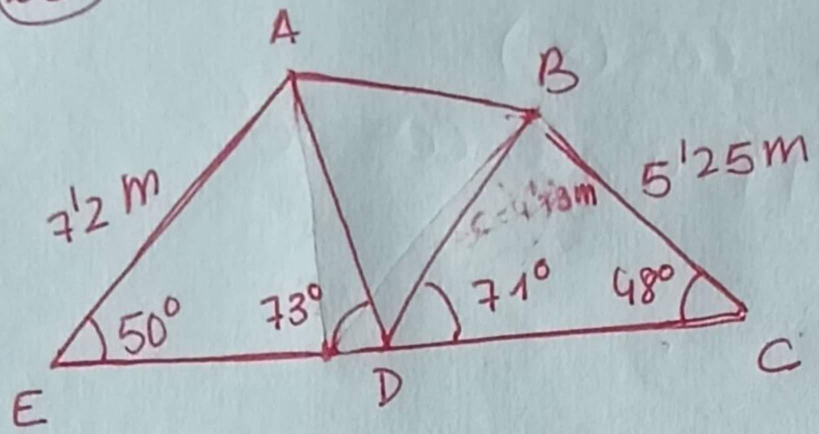
$$\frac{-8\cancel{a^2}}{-4\sqrt{5}\cancel{a^2}} = \cos\gamma \Rightarrow \cos\gamma = \frac{2}{\sqrt{5}} \; ; \; \cos\gamma = \frac{2\sqrt{5}}{5}$$

$$\gamma = \arccos\frac{2\sqrt{5}}{5}$$

$$\gamma = 26'57^\circ$$

$$\alpha = \beta + \gamma = 26'57 + 26'57 = 53'14^\circ$$

28

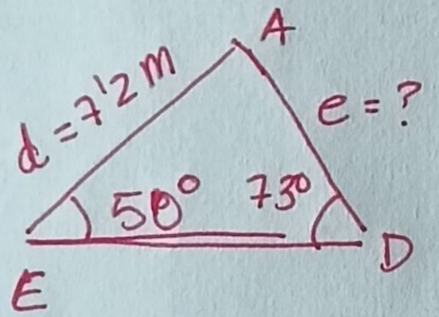


① $\widehat{ADE}$

$d = 7'2m$

$\hat{D} = 73°$

$\hat{E} = 50°$



$$\frac{d}{\operatorname{sen}\hat{D}} = \frac{e}{\operatorname{sen}\hat{E}}$$

$$e = \frac{d \cdot \operatorname{sen}\hat{E}}{\operatorname{sen}\hat{D}}$$

$$e = \frac{7'2 \cdot \operatorname{sen} 50}{\operatorname{sen} 73}$$

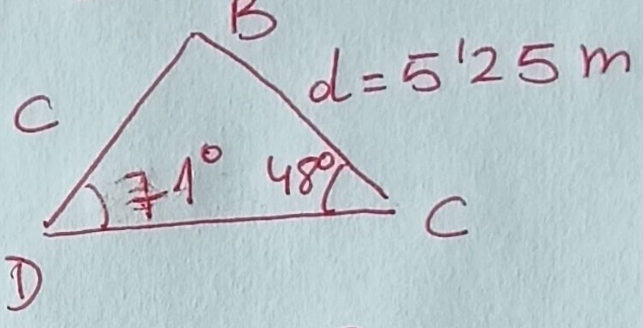
$$\boxed{e = 5'77\,m}$$

② $\widehat{DBC}$

$\hat{D} = 71°$

$\hat{C} = 48°$

$d = 5'25\,m$



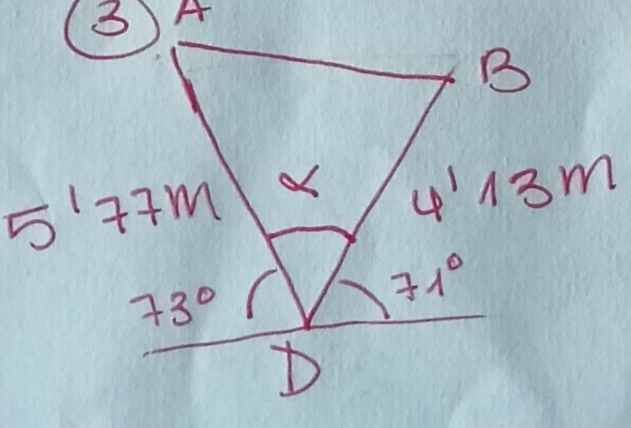
$$\frac{d}{\operatorname{sen}\hat{D}} = \frac{c}{\operatorname{sen}\hat{C}}$$

$$c = \frac{d \operatorname{sen}\hat{C}}{\operatorname{sen}\hat{D}}$$

$$c = \frac{5'25 \cdot \operatorname{sen} 48}{\operatorname{sen} 71}$$

$$\boxed{c = 4'13\,m}$$

③ $\widehat{ABD}$



Calculamos $\overline{AB}$:
sabemos que E, D, C estan en linea recta, formando un angulo de 180° (llano) podemos calcular α:

$\alpha = 180 - 73 - 71 = 36°$

Para calcular $\overline{AB}$ aplicamos th. del coseno

$$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2 + 2 \cdot \overline{AD} \cdot \overline{BD} \cdot \cos(\alpha)$$

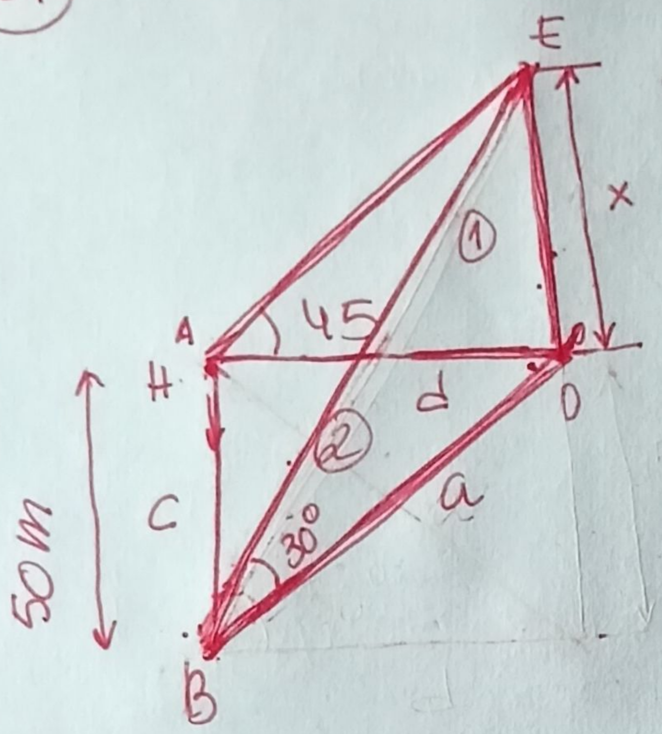
$$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2 + 2 \cdot 5'77 \cdot 4'13 \cdot \cos 36^\circ$$

$$\overline{AB}^2 = (5'77) + (4'13)^2 + 2 \cdot 5'77 \cdot 4'13 \cdot \cos 36^\circ$$

$$\overline{AB}^2 = 11'79$$

$$\boxed{\overline{AB} = 3'43\,m}$$

29



① $tg\,45 = \frac{x}{d}$ $\quad x = d$

② th. de Pitagoras $\quad x = d$
$c = 50m$

$a^2 = c^2 + d^2$

$a^2 = 2500 + x^2$

$a = \sqrt{2500 + x^2}$

$tg\,30 = \frac{x}{a}$

$\frac{\sqrt{3}}{3} = \frac{x}{\sqrt{2500 + x^2}}$

$\left(\sqrt{3} \cdot \sqrt{2500 + x^2}\right)^2 = (3x)^2$

$3(2500 + x^2) = 9 \cdot x^2$

$7500 = 6x^2$

$x^2 = 1250$

$x = \sqrt{1250}$

$x = 35'4$ m tiene la antena.