

SISTEMAS ECUACIONES

⑤ a)
$$\begin{cases} x + 3y - 2z = 6 \\ 2x + 3y - 2z = 8 \\ 4x + 2y - 6z = 6 \end{cases} \xrightarrow[2E_1 - E_2]{\implies} \begin{cases} x + 3y - 2z = 6 \\ 3y - 2z = 4 \\ 10y - 2z = 18 \end{cases} \xrightarrow[4E_1 - E_3]{\implies} \begin{cases} x + 3y - 2z = 6 \\ 3y - 2z = 4 \\ 10y - 2z = 18 \end{cases} \quad \textcircled{1}$$

$$\begin{aligned} x + 3y - 2z &= 6 \\ 3y - 2z &= 4 \\ -14z &= -14 \end{aligned} \implies \boxed{z = 1}$$

$\implies 10E_2 - 3E_3$

$$3 \cdot y - 2 \cdot 1 = 4$$

$$3y = 4 + 2$$

$$\boxed{y = 2}$$

$$x = 6 - 6 + 2$$

$$\boxed{x = 2}$$

Solución : $x = 2$
 $y = 2$
 $z = 1$

S. Compatible determinado

$$b) \left\{ \begin{array}{l} x + 2y - 3z = 3 \\ 3x - 2y + z = 7 \\ 5x + 2y - 5z = 1 \end{array} \right. \xrightarrow{\substack{3E_1 - E_2 \\ 5E_1 - E_3}} \left\{ \begin{array}{l} x + 2y - 3z = 3 \\ 8y - 10z = 2 \\ 8y - 10z = 14 \end{array} \right.$$

$$\begin{array}{l} x + 2y - 3z = 3 \\ 8y - 10z = 2 \end{array}$$

$$\xrightarrow{E_2 - E_3} \quad 0z = -12 \quad \Rightarrow \text{No tiene solución}$$

S. I

Revisar.

$$c) \left\{ \begin{array}{l} 2x + y - 2z = 8 \\ 2x - 4y + 3z = -2 \\ 4x - y + 6z = -4 \end{array} \right. \xrightarrow{\substack{E_1 - E_2 \\ 2E_1 - E_3}} \left\{ \begin{array}{l} 2x + y - 2z = 8 \\ 5y - 5z = 10 \\ 3y - 10z = 20 \end{array} \right.$$

$$\begin{array}{l} 2x + y - 2z = 8 \\ 5y - 5z = 10 \end{array}$$

$$\xrightarrow{E_2 - 5E_3} \quad 35z = -70 \quad \Rightarrow \boxed{z = -2}$$

$$5y + 10 = 10 \quad \Rightarrow \boxed{y = 0}$$

$$2x + 4 = 8 \quad \Rightarrow \boxed{x = 2}$$

Soluciones
 $x = 2$ S.C.D
 $y = 0$
 $z = -2$

$$\begin{array}{l}
 d) \quad x + 3y - 2z = -6 \\
 \quad \quad 2x - 3y + 5z = 6 \\
 \quad \quad 5x - 3y + 8z = 6
 \end{array}
 \begin{array}{l}
 \Rightarrow \\
 \xrightarrow{2E_1 - E_2} \\
 \xrightarrow{5E_1 - E_3}
 \end{array}
 \begin{array}{l}
 x + 3y - 2z = -6 \\
 9y - 9z = -18 \\
 18y - 18z = -36
 \end{array}
 \begin{array}{l}
 \textcircled{2} \\
 \Rightarrow
 \end{array}$$

$$\begin{array}{l}
 x + 3y - 2z = -6 \\
 9y - 9z = -18
 \end{array}$$

$\xrightarrow{2E_2 - E_3}$

$$0z = 0 \Rightarrow \boxed{z = \lambda} \text{ infinitas soluciones}$$

$$\begin{array}{l}
 9 \cdot y - 9\lambda = -18 \\
 y = \frac{-18 + 9\lambda}{9}
 \end{array}$$

$$\boxed{y = \lambda - 2}$$

$$x + 3\lambda - 6 - 2\lambda = -6$$

$$\boxed{x = -\lambda}$$

Soluciones:

$$x = -\lambda$$

$$y = \lambda - 2$$

$$z = \lambda$$

S.C.I

$$\begin{array}{l}
 x + 2y - 2z = 4 \\
 e) \quad 2x + 5y - 2z = 10 \\
 4x + 9y - 6z = 18
 \end{array}
 \left. \begin{array}{l}
 \xrightarrow{2E_1 - E_2} \\
 \xrightarrow{4E_1 - E_3}
 \end{array} \right\}
 \begin{array}{l}
 x + 2y - 2z = 4 \\
 -y - 2z = -2 \\
 -y - 2z = -2
 \end{array}$$

$$\begin{array}{l}
 x + 2y - 2z = 4 \\
 -y - 2z = -2 \\
 0z = 0
 \end{array}$$

$E_2 - E_3 \Rightarrow$

$z = \lambda$ infinitas soluciones

$$-y - 2\lambda = -2$$

$$-y = -2 + 2\lambda$$

$$y = 2 - 2\lambda$$

$$x + 4 - 4\lambda - 2\lambda = 4$$

$$\begin{array}{l}
 x = 6\lambda \\
 y = 2 - 2\lambda \\
 z = \lambda
 \end{array}$$

$$x = +6\lambda$$

S.C.I

$$\begin{array}{l}
 f) \quad x + 2y - z = -5 \\
 5x - y + 2z = 11 \\
 6x + y + z = 5
 \end{array}
 \left. \begin{array}{l}
 \xrightarrow{5E_1 - E_2} \\
 \xrightarrow{6E_1 - E_3}
 \end{array} \right\}
 \begin{array}{l}
 x + 2y - z = -5 \\
 11y - 7z = -36 \\
 11y - 7z = -35 \Rightarrow
 \end{array}$$

$$x + 2y - z = -5$$

$$11y - 7z = -36$$

$$0z = -1$$

∅ solución S.I

$E_2 - E_3 \Rightarrow$

$$g) \begin{cases} 2x + y - z = 0 \\ 3x + 2y - 2z = 15 \\ x + y - z = 7 \end{cases}$$

$$\begin{cases} x + y - z = 7 \\ 2x + y - z = 0 \\ 3x + 2y - 2z = 15 \end{cases}$$

$$\begin{aligned} & \xrightarrow{2E_1 - E_2} \\ & \xrightarrow{3E_1 - E_3} \end{aligned} \begin{cases} x + y - z = 7 \\ y - z = 14 \\ y - z = 6 \end{cases}$$

$$\begin{aligned} & \xrightarrow{E_2 - E_3} \end{aligned} \begin{cases} x + y - z = 7 \\ y - z = 14 \\ 0z = 8 \end{cases}$$

No tiene solución
S.I

$$h) \begin{cases} x + 3y - 2z = 4 \\ 2x + 2y + z = 3 \\ 3x + 2y + z = 5 \end{cases} \xrightarrow{\begin{matrix} 2E_1 - E_2 \\ 3E_1 - E_3 \end{matrix}}$$

$$\begin{cases} x + 3y - 2z = 4 \\ 4y - 5z = 5 \\ 7y - 7z = 7 \end{cases}$$

$$\begin{cases} x + 3y - 2z = 4 \\ 4y - 5z = 5 \\ -7z = +7 \end{cases}$$

$$\xrightarrow{7E_2 - 4E_3}$$

$$\boxed{z = -1}$$

$$4y + 5 = 5$$

$$\boxed{y = 0}$$

$$x + 2 = 4$$

$$\boxed{x = 2}$$

Soluciones: $x = 2$
 $y = 0$
 $z = -1$ S.C.D.

6

a) $2x + y = 8$
 $2x + 3y^2 = 22$

$$\begin{array}{r} -2x - y = -8 \\ +3y^2 = 22 \\ \hline 3y^2 - y - 14 = 0 \end{array}$$

$$y = \frac{+1 \pm \sqrt{1^2 - 4 \cdot 3 \cdot (-14)}}{2 \cdot 3} = \frac{1 \pm \sqrt{169}}{6} = \begin{cases} y_1 = \frac{1+13}{6} = \frac{7}{3} \\ y_2 = -2 \end{cases}$$

$$\boxed{y_1 = \frac{7}{3}}$$

$$2x + \frac{7}{3} = 8$$

$$2x = \frac{24}{3} - \frac{7}{3}$$

$$\boxed{x_1 = \frac{17}{6}}$$

$$\boxed{y_2 = -2}$$

$$2x - 2 = 8$$

$$2x = 10$$

$$\boxed{x_2 = 5}$$

Sol: $x_1 = \frac{17}{6}$

$$y_1 = \frac{7}{3}$$

$$x_2 = 5$$

$$y_2 = -2$$

$$2x = -8 - 5y \Rightarrow x = \frac{-8 - 5y}{2}$$

b) $2x + 5y = -8$
 $xy - 3x = -5$

$$\left(\frac{-8 - 5y}{2} - \frac{5y}{2}\right)y - 3\left(\frac{-8 - 5y}{2} - \frac{5y}{2}\right) = -5$$

$$\frac{-5y^2}{2} - \frac{8}{2}y + \frac{15y}{2} + \frac{24}{2} = \frac{-10}{2}$$

$$-5y^2 + 7y + 34 = 0$$

⇒

$$y = \frac{-7 \pm \sqrt{49 - 4 \cdot (-5) \cdot 34}}{2 \cdot (-5)} = \frac{-7 \pm 27}{-10} = \begin{cases} y_1 = \frac{20}{-10} = -2 & \textcircled{4} \\ y_2 = \frac{-34}{-10} = \frac{17}{5} \end{cases}$$

$$y_1 = -2$$

$$x = \frac{-8 - 5 \cdot (-2)}{2}$$

$$x_1 = 1$$

$$y_2 = \frac{17}{5}$$

$$x = \frac{-8 - 5 \cdot \left(\frac{17}{5}\right)}{2}$$

$$x_2 = \frac{-25}{2}$$

Soluções: $x_1 = 1$ $y_1 = -2$
 $x_2 = \frac{-25}{2}$ $y_2 = \frac{17}{5}$

c) $x + y = 0 \Rightarrow x = -y$
 $xy = 1$

$$-y^2 = 1$$

$$y^2 = -1$$

$$y = \pm \sqrt{-1}$$

$$y = \pm i$$

Soluções

$$y_1 = i$$

$$x_1 = -i$$

$$y_2 = -i$$

$$x_2 = i$$

$$d) 2x - y = 3 \Rightarrow y = 2x - 3$$

$$x^2 - y^2 = 3$$

$$x^2 - (2x - 3) = 3$$

$$x^2 - (4x^2 - 12x + 9) = 3$$

$$-3x^2 + 12x - 12 = 0$$

$$x^2 - 4x + 4 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 4}}{2} = 2$$

$$x = 2 \quad y = 1$$

$$e) \left. \begin{array}{l} x + y = 11 \\ \frac{1}{x} + \frac{6}{y+1} = 1 \end{array} \right\} x = 11 - y$$

$$\frac{1}{11-y} + \frac{6}{y+1} = 1$$

$$\frac{y+1}{(11-y)(y+1)} + \frac{66-6y}{(11-y)(y+1)} = \frac{(11-y)(y+1)}{(11-y)(y+1)}$$

$$-5y + 67 = 11y + 11 - y^2 - y$$

$$y^2 - 5y - 10y + 67 - 11 = 0$$

$$y^2 - 15y + 56 = 0$$

$$y = \frac{15 \pm \sqrt{225 - 4 \cdot 1 \cdot (+56)}}{2} = \frac{15 \pm 1}{2} = \begin{cases} y_1 = 8 \\ y_2 = 7 \end{cases}$$

$y_1 = 8$ $y_2 = 7$ solutions

$x_1 = 3$ $x_2 = 4$

4) $x y = -3 \implies x = \frac{-3}{y}$
 $x^2 + 2y^2 = 19$

$$\frac{9}{y^2} + 2y^2 = 19$$

$$\frac{9}{y^2} + \frac{2y^4}{y^2} = \frac{19y^2}{y^2}$$

$$2y^4 - 19y^2 + 9 = 0$$

$t = y^2$
 $t^2 = y^4$

$$2t^2 - 19t + 9 = 0$$

$$t = \frac{19 \pm \sqrt{19^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 2} = \frac{19 \pm 17}{4} = \begin{cases} t_1 = 9 \\ t_2 = \frac{1}{2} \end{cases}$$

$t_1 = 9$

$y^2 = 9$

$y_1 = 3 \implies x_1 = -1$

$y_2 = -3 \implies x_2 = 1$

$y_3 = \frac{\sqrt{2}}{2}$

$\implies x_3 = \frac{-6\sqrt{2}}{2} \implies x_3 = -3\sqrt{2}$

$y_4 = -\frac{\sqrt{2}}{2}$

$\implies x_4 = \frac{6\sqrt{2}}{2} \implies x_4 = 3\sqrt{2}$

$t_2 = \frac{1}{2}$

$y^2 = \frac{1}{2}$

$y = \pm \sqrt{\frac{1}{2}} \implies y = \pm \frac{\sqrt{2}}{2}$

Soluciones

$$x_1 = -1 \quad y_1 = 3$$

$$x_2 = 1 \quad y_2 = -3$$

$$x_3 = -3\sqrt{2} \quad y_3 = \frac{\sqrt{2}}{2}$$

$$x_4 = 3\sqrt{2} \quad y_4 = -\frac{\sqrt{2}}{2}$$

7.

$$a) \quad \begin{aligned} 2^x + 5^y &= 9 & 2^x + 5^y &= 9 \\ 2^{x+2} + 5^{y+1} &= 41 \Rightarrow 2^2 \cdot 2^x + 5 \cdot 5^y &= 41 \end{aligned}$$

$$2^x = t \quad 5^y = z$$

$$t + z = 9 \Rightarrow t = 9 - z$$

$$4t + 5z = 41$$

$$4(9 - z) + 5z = 41$$

$$36 - 4z + 5z = 41$$

$$\boxed{z = 5} \Rightarrow 5^y = 5 \Rightarrow \boxed{y = 1}$$

$$t = 9 - 5$$

$$\boxed{t = 4}$$

$$\Rightarrow 2^x = 2^2 \Rightarrow \boxed{x = 2}$$

Soluciones $\boxed{\begin{matrix} x = 2 \\ y = 1 \end{matrix}}$

$$b) \begin{cases} 2^x - 2^y = 1016 \\ 2^{x-y} = 128 \end{cases} \Rightarrow \frac{2^x}{2^y} = 128$$

$$2^x = t \quad 2^y = z$$

$$t - z = 1016 \quad \Rightarrow \quad 128 \cdot z - z = 1016 \Rightarrow$$

$$\frac{t}{z} = 128 \Rightarrow t = 128 \cdot z$$

$$\Rightarrow \boxed{z = 8} \Rightarrow 2^y = 2^3 \quad \boxed{y = 3}$$

$$t = 128 \cdot 8 \Rightarrow t = 1024 \Rightarrow 2^x = 2^{10} \quad \boxed{x = 10}$$

Soluciones $x = 10$
 $y = 3$

$$c) \begin{cases} 3^x + 7^y = 16 \\ 3^{x-1} - 7^{y+2} = -340 \end{cases} \Rightarrow \begin{cases} t + z = 16 \Rightarrow t = 16 - z \\ \frac{3^x}{3} - 7^2 \cdot 7^y = -340 \end{cases}$$

$$\boxed{3^x = t \quad 7^y = z}$$

$$\frac{t}{3} - 49 \cdot z = -340$$

$$\frac{16 - z}{3} - \frac{147z}{3} = -\frac{1020}{3}$$

$$-148z = -1036$$

$$\boxed{z = 7} \Rightarrow 7^y = 7 \Rightarrow \boxed{y = 1}$$

$$t = 16 - 7 \Rightarrow t = 9 \Rightarrow 3^x = 3^2 \Rightarrow \boxed{x = 2}$$

Soluciones $x = 2$
 $y = 1$

$$d) \left. \begin{aligned} \frac{1}{2^x} + \frac{1}{2^y} &= \frac{x+y}{8} \\ x+y &= 5 \end{aligned} \right\} \begin{aligned} \frac{1}{2^x} + \frac{1}{2^y} &= \frac{5}{8} \\ x &= 5-y \end{aligned}$$

$$\frac{1}{2^{5-y}} + \frac{1}{2^y} = \frac{5}{8}$$

$$\frac{1}{\frac{2^5}{2^y}} + \frac{1}{2^y} = \frac{5}{8}$$

$$\frac{2^y}{2^5} + \frac{1}{2^y} = \frac{5}{8}$$

$$\frac{1}{2^y} + \frac{2^y}{32} = \frac{5}{8}$$

$$z = 2^y$$

$$\frac{1}{\mu} + \frac{\mu}{32} = \frac{5}{8}$$

$$\frac{32}{32\mu} + \frac{\mu^2}{32\mu} = \frac{4 \cdot 5 \mu}{32\mu}$$

$$32 + \mu^2 - 20\mu = 0$$

$$\mu^2 - 20\mu + 32 = 0$$

$$\mu = \frac{20 \pm \sqrt{400 - 4 \cdot 1 \cdot 32}}{2 \cdot 1} = \frac{20 \pm \sqrt{272}}{2} = \frac{20 \pm \sqrt{2^4 \cdot 17}}{2} =$$

$$= 5 \pm 2\sqrt{17} \quad \left\{ \begin{array}{l} \mu_1 = 5 + 2\sqrt{17} \\ \mu_2 = 5 - 2\sqrt{17} \end{array} \right.$$

$$2^y = 5 + 2\sqrt{17}$$

$$\log_2 2^y = \log_2 (5 + 2\sqrt{17})$$

$$y_1 = \log_2 (5 + 2\sqrt{17})$$

$$x_1 = 5 - \log_2 (5 + 2\sqrt{17})$$

$$2^y = 5 - 2\sqrt{17}$$

$$\log_2 2^y = \log_2 (5 - 2\sqrt{17})$$

$$y_2 = \log_2 (5 - 2\sqrt{17})$$

$$x_2 = 5 - \log_2 (5 - 2\sqrt{17})$$

$$\begin{array}{l} \text{a) } \log x + 3 \log y = 5 \\ \log x - \log y = 3 \end{array} \quad \left\{ \begin{array}{l} \Rightarrow \\ \times (-1) \end{array} \right. \quad \begin{array}{l} \log x + 3 \log y = 5 \\ \log x + \log y = -3 \end{array}$$

$$4 \log y = 2$$

$$\log y = \frac{1}{2}$$

$$y = 10^{1/2}$$

$$y = \sqrt{10}$$

$$\log x - \frac{1}{2} = 3$$

$$\log x = \frac{7}{2}$$

$$x = 10^{7/2}$$

$$x = 10^3 \sqrt{10}$$

$$b) \log_3 x = 7 - \log_3 y$$

$$x - 2y = 27 \Rightarrow x = 27 + 2y$$

$$\log_3 (27 + 2y) = 7 - \log_3 y$$

$$\log_3 (27 + 2y) + \log_3 y = \log_3 3^7$$

$$\log_3 [(27 + 2y) \cdot y] = \log_3 3^7$$

$$2y^2 + 27y - 2187 = 0$$

$$y = \frac{-27 \pm \sqrt{18225}}{2 \cdot 2} = \frac{-27 \pm 135}{4} = \begin{cases} y_1 = 27 \\ y_2 = \frac{81}{2} \end{cases}$$

$$y_1 = 27$$

$$x_1 = 81$$

$$y_2 = \frac{-81}{2}$$

$$x_2 = -54$$

$$c) 3x + 2y = 64$$

$$\log x \rightarrow \log y = 1$$

$$x = \frac{64 - 2y}{3}$$

$$\log \frac{64 - 2y}{3} = \log y = \log 10$$

$$= \log 10$$

$$\log \left[\frac{(64 - 2y)}{3} \right] = \log y$$

$$\frac{64}{3} - \frac{2y}{3} = 10y \Rightarrow 64 - 2y = 30y$$

$$64 = 32y$$

$$\boxed{y = 2}$$

$$\boxed{x = \frac{60}{3}}$$

soluciones

$$\begin{aligned} \text{d) } \log x^3 - 2 \log y &= 6 \\ 2 \log x + \log y^2 &= 2 \end{aligned} \quad \left. \begin{aligned} 3 \log x - 2 \log y &= 6 \\ 2 \log x + 2 \log y &= 2 \end{aligned} \right\} \textcircled{8}$$

$$5 \log x = 8$$

$$\log x = \frac{8}{5}$$

$$x = 10^{8/5}$$

$$2 \cdot \frac{8}{5} + 2 \log y = 2$$

$$2 \log y = \frac{10}{5} - \frac{16}{5}$$

$$\log y = \frac{3}{5}$$

$$y = \sqrt[5]{10^3}$$

$$x = 10^{\frac{8}{5}} \\ x = 10 \sqrt[5]{10^3}$$