1 a) $2x - \frac{3x-1}{3} = x + \frac{1}{3}$

$6x - 3x + 1 = 3x + 1$

$6x - 3x - 3x = 1 - 1$

$0x = 0 \Rightarrow$ x tiene infinitas soluciones

b) $\frac{3x-1}{4} - 2x = \frac{2x - \frac{7}{4}}{2} - (3x-1)$

$\frac{3x-1}{4} - 2x = \frac{8x-7}{8} - (3x-1)$

$6x - 2 - 16x = 8x - 7 - 24x + 8$

$6x - \cancel{16x} - \cancel{8x} + \cancel{24x} = -7 + 8 + 2$

$6x = 3$

$\boxed{x = \frac{1}{2}}$

c) $6x^2 - 1 + \frac{2x(-x+3)}{3} = \frac{5x^2-2}{6} - 4x^2 + \frac{59}{6}$

$36x^2 - 6 + 4x(-x+3) = 5x^2 - 2 - 24x^2 + 59$

$36x^2 - 4x^2 - 6 + 12x - 5x^2 + 2 + 24x^2 - 59 = 0$

$51x^2 + 12x - 63 = 0$

$17x^2 + 4x - 21 = 0 \qquad x = \frac{-4 \pm \sqrt{16 - 4 \cdot 17(-21)}}{34} = \frac{-4 \pm 38}{34} =$

$= \begin{cases} x_1 = \frac{-4+38}{34} = 1 \\ x_2 = \frac{-4-38}{34} = \frac{-42}{34} = \frac{-21}{17} \end{cases}$

Soluciones:
$x_1 = 1$
$x_2 = \frac{-21}{17}$

d) $\frac{x^2+1}{2} - \frac{2x-3}{4} + \frac{x^2}{6} = \frac{59}{12}$

$6x^2+6-6x+9+2x^2 = 59$

$8x^2-6x-44=0$

$4x^2-3x-22=0$

$x = \frac{+3 \pm \sqrt{9+359}}{8} = \frac{3 \pm 19}{8} = \begin{cases} \frac{3+19}{8} = \frac{22}{8} = \frac{11}{4} \\ \frac{3-19}{8} = \frac{-16}{2} = -2 \end{cases}$

soluciones $x_1 = \frac{11}{4}$

$x_2 = -2$

e) $6x^4+13x^3-8x^2-17x+6=0$

	6	13	-8	-17	6
-2		-12	-2	20	-6
	6	1	-10	3	0
1		6	7	-3	
	6	7	-3	0	

$x_1 = -2$

$x_2 = 1$

$6x^2+7x-3=0$

$x = \frac{-7 \pm \sqrt{49+72}}{12} = \begin{cases} \frac{-7+11}{12} = \frac{1}{3} = x_3 \\ \frac{-7-11}{12} = -\frac{3}{4} = x_4 \end{cases}$

f) $2x^4 - x^3 - 3x - 18 = 0$

	2	-1	0	-3	-18
2		4	6	12	18
	2	3	6	9	$\boxed{0}$
$-3/2$		-3	0	-9	
	2	0	6	$\boxed{0}$	

$X_1 = 2$

$X_2 = \frac{-3}{2}$

$2x^2 + 6 = 0$

$x^2 = -3$

$x = \pm\sqrt{-3}$ No tiene solución Real

$x = \pm\sqrt{3} \cdot i$ soluciones complejas

$x_3 = +\sqrt{3} \cdot i$

$x_4 = -\sqrt{3} \cdot i$

g) $6x^3 - 7x^2 - 14x + 15 = 0$

	6	-7	-14	+15
1		6	-1	-15
	6	-1	-15	$\boxed{0}$

$x_1 = 1$

$6x^2 - x - 15 = 0$

$$x = \frac{+1 \pm \sqrt{1 - 4 \cdot 6 \cdot (-15)}}{12} = \frac{1 \pm 19}{12} \begin{cases} \frac{20}{12} = \frac{5}{3} = x_2 \\ \frac{-18}{12} = \frac{-3}{2} = x_3 \end{cases}$$

h) $x^4 - 125x^2 + 484 = 0$ Bicuadradas.

$t = x^2$

$t^2 = x^4$

$t^2 - 125t + 484 = 0$

$$t = \frac{125 \pm \sqrt{13689}}{2} = \begin{cases} \frac{125+117}{2} = 121 = t_1 \\ \frac{125-117}{2} = 4 = t_2 \end{cases}$$

$t_1 = 121$

$x^2 = 121$

$x = \pm 11$

$t_2 = 4$

$x^2 = 4$

$x = \pm 2$

$x_1 = 11$

$x_2 = -11$

$x_3 = 2$

$x_4 = -2$

i) $3x^4 - 15x^2 + 12 = 0$ Bicuadradas

$t = x^2$

$t^2 = x^4$

$3t^2 - 15t + 12 = 0$

$t^2 - 5t + 4 = 0$

$$x = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \begin{cases} t_1 = 4 \\ t_2 = 1 \end{cases}$$

$t_1 = 4$

$x^2 = 4$

$x = \pm 2$

$t_2 = 1$

$x^2 = 1$

$x = \pm 1$

Soluciones

$x_1 = 2$

$x_2 = -2$

$x_3 = 1$

$x_4 = -1$

j) $x\,(9x^2-1)(2x+3)=0$

$x=0$

$9x^2-1=0 \Rightarrow 9x^2=1 \Rightarrow x^2=\frac{1}{9}$

$x=\pm\frac{1}{3}$

$2x+3=0$

$x=\frac{-3}{2}$

$x_1=0 \qquad x_2=\frac{1}{3} \qquad x_3=-\frac{1}{3} \qquad x_4=-\frac{3}{2}$

k) $x^4-6x^3+x^2+54x-90=0$

	1	-6	1	54	-90
3		3	-9	-24	90
	1	-3	-8	30	0
-3		-3	18	-30	
	1	-6	10	0	

$x_1=3$

$x_2=-3$

$x^2-6x+10=0$

$$x=\frac{-6\pm\sqrt{36-40}}{2}= \begin{cases} \frac{+6+\sqrt{-4}}{2}=\frac{+6}{2}+\frac{2i}{2}=+3+i \\ \frac{+6-\sqrt{-4}}{2}=\frac{+6}{2}-\frac{2i}{2}=+3-i \end{cases}$$

$x_3=+3+i$

$x_4=+3-i$

l) $x^6+2x^5+22x^4+46x^3-71x^2-100x+100=0$

$$
\begin{array}{r|rrrrrrr}
 & 1 & 2 & 22 & 46 & -71 & -100 & 100 \\
1 & & 1 & 3 & 25 & 71 & 0 & -100 \\
\hline
 & 1 & 3 & 25 & 71 & 0 & -100 & \boxed{0} \\
-2 & & -2 & -2 & -46 & -50 & 100 & \\
\hline
 & 1 & 1 & 23 & 25 & -50 & \boxed{0} & \\
1 & & 1 & 2 & 25 & 50 & & \\
\hline
 & 1 & 2 & 25 & 50 & \boxed{0} & & \\
-2 & & -2 & 0 & -50 & & & \\
\hline
 & 1 & 0 & 25 & \boxed{0} & & &
\end{array}
$$

$x_1 = 1$

$x_2 = -2$

$x_3 = 1$

$x_4 = -2$

$x^2 + 25 = 0$

$x = \pm 5i$

$x_5 = 5i$

$x_6 = -5i$

m) $x^4 - 4x^3 + 2x^2 + 20x + 13 = 0$

$$
\begin{array}{r|rrrrr}
 & 1 & -4 & 2 & 20 & 13 \\
-1 & & -1 & 5 & -7 & -13 \\
\hline
 & 1 & -5 & 7 & 13 & \boxed{0} \\
-1 & & -1 & 6 & -13 & \\
\hline
 & 1 & -6 & 13 & \boxed{0} &
\end{array}
$$

$x_1 = -1$

$x_2 = -1$

$x^2 - 6x + 13 = 0$

$$x = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm \sqrt{-16}}{2} = \begin{cases} \dfrac{6+4i}{2} = 3+2i = x_3 \\ \dfrac{6-4i}{2} = 3-2i = x_4 \end{cases}$$

n) $x^4 - 12x^2 - 64 = 0$

$x^2 = t \qquad t^2 - 12t - 64 = 0$

$x^4 = t^2$

$$t = \frac{+12 \pm \sqrt{144 + 256}}{2} = \frac{+12 \pm \sqrt{400}}{2} = \begin{cases} t_1 = \frac{12+20}{2} = \frac{32}{2} = 16 \\ t_2 = \frac{12-20}{2} = \frac{-8}{2} = -4 \end{cases}$$

$t_1 = 16$

$x^2 = 16$

$x = \pm 4 \quad \begin{cases} x_1 = 4 \\ x_2 = -4 \end{cases}$

$t_2 = -4$

$x^2 = -4$

$x = \pm 2i \quad \begin{cases} x_3 = 2i \\ x_4 = -2i \end{cases}$

2 a) $\frac{2x}{x-2} + \frac{3x}{x+2} = \frac{6x^2}{x^2-4}$

$x - 2$

$x + 2$

$(x^2-4) = (x+2)(x-2)$

m.c.m $[(x-2), (x+2), (x^2-4)] = (x+2)(x-2)$

$$\frac{2x(x+2)}{(x+2)(x-2)} + \frac{3x(x-2)}{(x+2)(x-2)} = \frac{6x^2}{(x+2)(x-2)}$$

$2x^2 + 4x + 3x^2 - 6x = 6x^2$

$-x^2 - 2x = 0$

$-x(x+2) = 0 \quad \begin{cases} \boxed{x_1 = 0} \\ x + 2 = 0 \Rightarrow \boxed{x_2 = -2} \end{cases}$

Comprobamos:

$x = 0 \qquad \frac{2 \cdot 0}{0-2} + \frac{3 \cdot 0}{0+2} = \frac{6 \cdot 0}{0^2 - 4} \Rightarrow \frac{0}{-2} + \frac{0}{2} = \frac{0}{4}$

$0 + 0 = 0$

$0 = 0$

$x=-2$

$$\frac{2\cdot(-2)}{-2-2}+\frac{3\cdot(-2)}{-2+2}=\frac{6(-2)^2}{4-4}$$

$$\frac{-4}{-4}+\frac{-6}{0}=\frac{24}{0}\quad\therefore$$ solución discartada que elimina el denominador

solución : $\boxed{x=0}$

b) $$\frac{x+9}{x}-\frac{5+x}{x+2}=\frac{12x+12}{x^2+2x}$$

$x=x$
$x+2=x+2$
$x^2+2x=x(x+2)$

m.c.m $[x,(x+2),(x^2+2x)]=x(x+2)$

$$\frac{(x+9)(x+2)}{x(x+2)}-\frac{(5+x)\cdot x}{x(x+2)}=\frac{12x+12}{x^2+2x}$$

$$\cancel{x^2}+2x+9x+18-5x-\cancel{x^2}-12x-12=0$$

$$-6x=-6$$

$$\boxed{x=1}$$

comprobamos $x=1$

$$\frac{1+9}{1}-\frac{5+1}{1+2}=\frac{12+12}{1+2}$$

$$10-\frac{6}{3}=\frac{24}{3}$$

$$8=8\ \checkmark$$

solucion $\boxed{x=1}$

c) $\frac{1}{x-a} + \frac{1}{x+a} = \frac{1}{x^2-a^2}$

$$\left.\begin{array}{l} x-a = x-a \\ x+a = x+a \\ x^2-a^2 = (x+a)(x-a) \end{array}\right\} m.c.m\left[(x-a),(x+a),(x^2-a^2)\right] = (x+a)(x-a)$$

$$\frac{x+a}{(x+a)(x-a)} + \frac{x-a}{(x+a)(x-a)} = \frac{1}{(x+a)(x-a)}$$

$$x+\cancel{a}+x-\cancel{a} = 1$$
$$2x = 1$$
$$x = \frac{1}{2}$$

$x = \frac{1}{2}$ no anula el denominador si $a \neq \frac{1}{2}$

Si $a = \frac{1}{2}$ $x \neq \frac{1}{2}$ No sería solución

d) $\frac{x^2+1}{x} + \frac{x}{x^2-1} = x + \frac{7}{6}$

$$\left.\begin{array}{l} x = x \\ x^2-1 = (x+1)(x-1) \\ 6 = 2\cdot 3 \end{array}\right\} m.c.m\left(x,(x^2-1),6\right) = 6\cdot x(x+1)(x-1)$$

$$\frac{(x^2+1)\cdot 6(x+1)(x-1)}{6x(x+1)(x-1)} + \frac{6\cdot x\cdot x}{6x(x+1)(x-1)} = \frac{x(6x(x+1)(x-1))}{6x(x+1)(x-1)} + \frac{7x(x+1)(x-1)}{6x(x+1)(x-1)}$$

$$6(x^2+1)(x^2-1) + 6x^2 = 6x^2(x^2-1) + 7x(x^2-1)$$

$$6(x^4-1) + 6x^2 = 6x^4 - 6x^2 + 7x^3 - 7x$$

$$\cancel{6x^4} - 6 + 6x^2 - \cancel{6x^4} + 6x^2 - 7x^3 + 7x = 0$$

$\Rightarrow$

$$-7x^3+12x^2+7x-6=0$$

$$\begin{array}{r|rrrr} & -7 & +12 & +7 & -6 \\ 2 & & -14 & -4 & 6 \\ \hline & -7 & -2 & 3 & \boxed{0} \end{array} \qquad x_1 = 2$$

$$-7x^2-2x+3=0$$

$$x = \frac{2 \pm \sqrt{4-4\cdot(-7)\cdot 3}}{2\cdot(-7)} = \frac{2 \pm \sqrt{88}}{-14}$$

$$x_2 = \frac{-2-2\sqrt{22}}{14} = -\frac{1}{7}-\frac{\sqrt{22}}{7}$$

$$x_3 = -\frac{1}{7}+\frac{\sqrt{22}}{7}$$

Seran todas soluciones ya que ninguna anula el denominador

$$x_1 = 2 \qquad x_2 = -\frac{1}{7}-\frac{\sqrt{22}}{7} \qquad x_3 = \frac{\sqrt{22}}{7}-\frac{1}{7}$$

e) $x+4+5\sqrt{x-2}=0$

$$\left(5\sqrt{x-2}\right)^2 = (-x-4)^2$$

$$25(x-2) = x^2+16-8x$$

$$25x-50-x^2-16+8x=0$$

$$-x^2+33x-66=0$$

$$x = \frac{-33 \pm \sqrt{1089-4\cdot(-1)\cdot(-66)}}{2\cdot(-1)} = \frac{-33 \pm \sqrt{825}}{-2} = \frac{-33 \pm 5\sqrt{33}}{2}$$

$$x_1 = \frac{-33+5\sqrt{33}}{2} < 0$$

$$x_2 = \frac{-33-5\sqrt{33}}{2} < 0$$

No son soluciones porque no existe raiz de un nº negativo

f) $\left(\sqrt{7x+1}\right)^2 = \left(2\sqrt{x+4}\right)^2$

$7x + 1 = 4(x+4)$

$7x - 4x = 16 - 1$

$3x = 15$

$\boxed{x = 5}$

Comprobar

$\sqrt{7 \cdot 5 + 1} = 2\sqrt{5+4}$

$\sqrt{35+1} = 2 \cdot \sqrt{9}$

$6 = 6$ ✓

Solución $x = 5$

g) $\dfrac{x-1}{\sqrt{x}} = x - \dfrac{5}{2} \Rightarrow \dfrac{x-1}{\sqrt{x}} + \dfrac{5}{2} = x$

$\sqrt{x} = \sqrt{x}$
$2 = 2$ m.c.m $= 2\sqrt{x}$
$1 = 1$

$\dfrac{2(x-1)}{\cancel{2\sqrt{x}}} + \dfrac{5\sqrt{x}}{\cancel{2\sqrt{x}}} = \dfrac{2x\sqrt{x}}{\cancel{2\sqrt{x}}}$

$2x - 2 + 5\sqrt{x} = 2x\sqrt{x}$

$2x - 2 = 2x\sqrt{x} - 5\sqrt{x}$

$2x - 2 = (2x-5)\sqrt{x} \Rightarrow [2x-2]^2 = (2x-5)^2 \cdot \left(\sqrt{x}\right)^2$

$4x^2 - 8x + 4 = (4x^2 - 20x + 25) \cdot x$

$-4x^3 + 24x^2 - 33x + 4 = 0$

$\Rightarrow$

	-4	$+24$	-33	$+4$
4		-16	32	-4
	-4	8	-1	0

$x_1 = 4$

$$-4x^2 + 8x - 1 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 4\cdot(-4)\cdot(-1)}}{2(-4)} = \frac{-8 \pm \sqrt{48}}{-8} = \frac{-8 \pm \sqrt{2^4 \cdot 3}}{-8} =$$

$$= \frac{-8}{-8} \pm \frac{4\sqrt{3}}{-8} = \begin{cases} x_2 = 1 - \frac{\sqrt{3}}{2} \\ x_3 = 1 + \frac{\sqrt{3}}{2} \end{cases}$$

Todas la x son positivas son soluciones de la ecuación

h) $\sqrt{x+4} + \sqrt{x-1} = 5$

$$(\sqrt{x+4})^2 = (5 - \sqrt{x-1})^2$$

$$x + 4 = 25 - 10\sqrt{x-1} + (x-1)$$

$$\cancel{x} + 4 - 25 - \cancel{x} + 1 = -10\sqrt{x-1}$$

$$-20 = -10\sqrt{x-1}$$

$$(2)^2 = (\sqrt{x-1})^2$$

$$4 = x - 1$$

$$\boxed{x = 5}$$

Si es solución

Comprobamos

$$\sqrt{5+4} + \sqrt{5-1} = 5$$

$$3 + 2 = 5$$

$$5 = 5$$

③

a) $\log_x \frac{\sqrt[5]{8}}{2} = -0'4$

$\log_x \frac{2^{3/5}}{2} = -\frac{4}{10}$

$\log_x 2^{-2/5} = -\frac{2}{5}$

$\log_a b = c$

$a^c = b$

$x^{-2/5} = 2^{-2/5}$

$\boxed{x = 2}$

b) $\log_9 \sqrt[5]{27} = 2x - 1$

$\log_9 \sqrt[5]{3 \cdot 9} = 2x - 1$

$\log_{3^2} 3^{3/5} = 2x - 1$

$(3^2)^{(2x-1)} = 3^{3/5}$

$3^{4x-2} = 3^{3/5} \Rightarrow 4x - 2 = \frac{3}{5}$

$4x = \frac{3}{5} + \frac{10}{5}$

$4x = \frac{13}{5} \Rightarrow \boxed{x = \frac{13}{20}}$

c) $\log(2x+3) - \log(x-1) = 2\log 2 + 2\log 3$

$$\log\left[\frac{(2x+3)}{(x-1)}\right] = \log 2^2 + \log 3^2$$

$$\cancel{\log}\left[\frac{2x+3}{x-1}\right] = \cancel{\log} 4 \cdot 9$$

$$\frac{2x+3}{x-1} = 36$$

$$2x+3 = 36x - 36$$

$$-34x = -39$$

$$\boxed{x = \frac{39}{34}}$$

comprobamos y si es solución.

d) $2\log(2x-2) - \log(x-1) = 1$

$$\cancel{\log}\frac{(2x-2)^2}{x-1} = \cancel{\log} 10^1$$

$$4x^2 - 8x + 4 = 10(x-1)$$

$$4x^2 - 8x + 4 - 10x + 10 = 0$$

$$4x^2 - 18x + 14 = 0$$

$$2x^2 - 9x + 7 = 0$$

$$x = \frac{9 \pm \sqrt{63-56}}{2 \cdot 2} = \frac{9 \pm \sqrt{7}}{4}$$

$$x_1 = \frac{9+\sqrt{7}}{4}$$

$$x_2 = \frac{9-\sqrt{7}}{4}$$

e) $\log\left(\frac{2x-2}{2}\right) = 2\log(x-1) - \log x$

$\cancel{\log}(x-1) = \cancel{\log}\frac{(x-1)^2}{x}$

$(x-1)x = x^2-2x+1$

$\cancel{x^2} - x - \cancel{x^2} + 2x - 1 = 0$

$\boxed{x = 1}$ No es solución

Comprobamos

$\log\left(\frac{2-2}{2}\right) = 2\log 0 - \log 1$

No puede ser

f) $\frac{\log(4-x)}{\log(x+2)} = 2$

$\log(4-x) = 2\log(x+2)$

$\cancel{\log}(4-x) = \cancel{\log}(x+2)^2$

$4-x = x^2+4x+4$

$x^2+4x+x \cancel{+4} \cancel{-4} = 0$

$x^2+5x=0$

$x(x+5)=0$ $\begin{cases} x=0 \\ x=-5 \Rightarrow \text{log de un número negativo no tiene solución} \end{cases}$

$$\frac{\log(4)}{\log 2} \stackrel{?}{=} 2$$

$$\frac{\log 2^2}{\log 2} \stackrel{?}{=} 2$$

$$\frac{2\log \cancel{2}^{1}}{\cancel{\log 2}} = 2$$

$$2 \cdot 1 = 2$$

$$2 = 2 \quad \checkmark \qquad x = 0 \text{ es solución}$$

g) $\log_3(x-1) - \log_3(x+2) = 1 - \log_3(x+6)$

$$\cancel{\log_3} \frac{x-1}{x+2} = \cancel{\log_3} \frac{3}{x+6}$$

$$(x-1)(x+6) = 3x+6$$

$$x^2 + 6x - x - 6 = 3x + 6$$

$$x^2 + 2x - 12 = 0$$

$$x = \frac{-2 \pm \sqrt{4+48}}{2} = \frac{-2 \pm \sqrt{52}}{2} = \frac{-2 \pm 2\sqrt{13}}{2}$$

$$x_1 = \frac{-2+2\sqrt{13}}{2} \qquad x_1 = -1 + \sqrt{13}$$

$$x_2 = \frac{-2-2\sqrt{13}}{2} \qquad x_2 = -1 - \sqrt{13}$$

No es solución porque es negativa

h) $\log(x-1) - \frac{1}{2}\log(3x-6) = \log 2$

$$\cancel{\log}\frac{x-1}{\sqrt{3x-6}} = \cancel{\log} 2$$

$$(x-1)^2 = (2\cdot\sqrt{3x-6})^2$$

$$x^2 - 2x + 1 = 4(3x-6)$$

$$x^2 - 2x + 1 - 12x + 24 = 0$$

$$x^2 - 14x + 25 = 0$$

$$x = \frac{14 \pm \sqrt{196-100}}{2} = \begin{cases} x_1 = \frac{14+\sqrt{96}}{2} = \frac{14+2^2\cdot\sqrt{6}}{2} = 7+2\sqrt{6} > 0 \\ x_2 = 7-2\sqrt{6} > 0 \end{cases}$$

$$\begin{array}{r|l} 96 & 2 \\ 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$$

$96 = 2^5\cdot 3$

Comprobar $x_1 = 7+2\sqrt{6}$

$$\log(7+2\sqrt{6}-1) - \frac{1}{2}\log(3(7+2\sqrt{6})-6) = \log 2$$

$$\log(6+2\sqrt{6}) - \frac{1}{2}\log(15+6\sqrt{6}) = \log 2$$

$$\log\frac{6+2\sqrt{6}}{\sqrt{15+6\sqrt{6}}} = \log 2$$

$$0'30102999\ldots = 0'30102999\ldots \checkmark$$

Solución

$$\boxed{\begin{array}{l} x_1 = 7+2\sqrt{6} \\ x_2 = 7-2\sqrt{6} \end{array}}$$

$x_2 = 7-2\sqrt{6}$

$$\log(7-2\sqrt{6}-1) - \frac{1}{2}\log(3(7-2\sqrt{6})-6) = \log 2$$

$$\log(6-2\sqrt{6}) - \log\sqrt{15-6\sqrt{6}} = \log 2$$

$$\log\frac{6-2\sqrt{6}}{\sqrt{15-6\sqrt{6}}} = \log 2$$

$$0'30102999\ldots = 0'30102999\ldots \checkmark$$

i) $\dfrac{\ln(x-1) + \ln(3x)}{2} = \ln\left(2x - \frac{1}{2}\right)$

$\frac{1}{2} \cdot \ln[(x-1)\cdot(3x)] = \ln\left(2x - \frac{1}{2}\right)$

$\cancel{\ln} \sqrt{3x^2 - 3x} = \cancel{\ln}\left(2x - \frac{1}{2}\right)$

$\left(\sqrt{3x^2-3x}\right)^2 = \left(2x - \frac{1}{2}\right)$

$3x^2 - 3x = 4x^2 - 2x + \frac{1}{4}$

$4x^2 - 3x^2 - 2x + 3x + \frac{1}{4} = 0$

$x^2 + x + \frac{1}{4} = 0 \Rightarrow 4x^2 + 4x + 1 = 0$

$x = \dfrac{-4 \pm \sqrt{16 - 4\cdot 4 \cdot 1}}{2\cdot 4} = \dfrac{-4 \pm \sqrt{0}}{8} = \dfrac{-1}{2}$

$x = \frac{-1}{2}$ es un valor negativo no es solución del Ln

j) $x(9x^2 - 1)(2x+3) = 0$

$x = 0$

$9x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3}$

$2x + 3 = 0 \Rightarrow x = \frac{-3}{2}$

soluciones: $x_1 = 0$ $\quad x_3 = \frac{-1}{3}$

$x_2 = \frac{1}{3}$ $\quad x_4 = \frac{-3}{2}$

4 a) $4^{x^2+1} = 2^{5x+5}$

$2^{2(x^2+1)} = 2^{5x+5}$

$2x^2+2 = 5x+5$

$2x^2-5x-3=0$

$x = \frac{5 \pm \sqrt{25-4\cdot 2\cdot(-3)}}{2\cdot 2} = \frac{5 \pm \sqrt{1}}{4}$ $x_1 = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2}$; $x_2 = \frac{5-1}{4} = 1$

Solu: $x_1 = \frac{3}{2}$; $x_2 = 1$

b) $4^{(x-2)^2} = 262144$

$4^{(x-2)^2} = 4^9$

$(x-2)^2 = 9$

$x^2-4x+4-9=0$

$x^2-4x-5=0$

$x = \frac{4 \pm \sqrt{16+20}}{2} =$ $x_1 = \frac{4+6}{2} = 5$; $x_2 = \frac{4-6}{2} = -1$

Sol: $x_1 = 5$
$x_2 = -1$

c) $2^3 \cdot 2^{x-5} = 0'25$

$2^{x-5+3} = \frac{25}{100}$

$2^{x-2} = \frac{1}{4}$

$2^{x-2} = 2^{-2}$

$x-2=-2$

$\boxed{x=0}$

Sol: $x=0$

d) $\left(\frac{2}{5}\right)^{3-5x} = \left(\frac{25}{4}\right)^{3x-1}$

$$\left(\frac{2}{5}\right)^{3-5x} = \left(\frac{5}{2}\right)^{2(3x-1)}$$

$$\left(\frac{2}{5}\right)^{3-5x} = \left(\frac{2}{5}\right)^{-2(3x-1)}$$

$$3-5x = -6x+2$$

$$+6x-5x = 2-3$$

$\boxed{x=-1}$ Sol.

e) $9^x + 5\cdot 3^x - 24 = 0$

$$3^{2x} + 5\cdot 3^x - 24 = 0$$

$t = 3^x$

$$t^2 + 5t - 24 = 0$$

$$t = \frac{-5 \pm \sqrt{25 - 4\cdot 1(-24)}}{2} = \frac{-5 \pm \sqrt{121}}{2} = \frac{-5 \pm 11}{2} \begin{cases} t_1 = 3 \\ t_2 = -9 \end{cases}$$

$t_1 = 3 \Rightarrow 3^x = 3 \Rightarrow x = 1$ solución

$t_2 = -9 \Rightarrow 3^x = -9 \Rightarrow 3^x = -3^2$ No tiene solución

f) $3^{x+2} + 9^{x-1} = 90$

$$3^x \cdot 3^2 + 9^x \cdot 9 = 90$$

$$9\cdot 3^x + 9\, 3^{2x} - 90 = 0$$

$$9\cdot 3^{2x} + 9\cdot 3^x - 90 = 0$$

$t = 3^x$

$$9t^2 + 9t - 90 = 0 \Rightarrow t^2 + t - 10 = 0$$

$$t = \frac{-1 \pm \sqrt{1+40}}{2} = \frac{-1 \pm \sqrt{41}}{2} \begin{cases} t_1 = \frac{-1+\sqrt{41}}{2} \\ t_2 = \frac{-1-\sqrt{41}}{2} \end{cases}$$

No es solución

$$t = \frac{-1+\sqrt{41}}{2}$$

$$3^x = \frac{-1+\sqrt{41}}{2}$$

$$\log_3 3^x = \log_3\left(\frac{-1+\sqrt{41}}{2}\right)$$

$$x = \log_3\left(\frac{-1+\sqrt{41}}{2}\right)$$

g) $9^x + 3^{2x-1} + 3^{x-1} = 111$

h) $3^{2x} + \frac{3^{2x}}{3} + \frac{3^x}{3} = 111$

$$\frac{3\cdot 3^{2x}}{\cancel{3}} + \frac{3^{2x}}{\cancel{3}} + \frac{3^x}{\cancel{3}} - \frac{333}{\cancel{3}} = 0$$

$$4\cdot 3^{2x} + 3^x - 333 = 0$$

$$t = 3^x$$

$$4\cdot t^2 + t - 333 = 0$$

$$t = \frac{-1 \pm \sqrt{1 - 4\cdot 4\cdot(-333)}}{2\cdot 4} = \frac{-1 \pm 73}{8} \begin{cases} t_1 = 9 \\ t_2 = \frac{-74}{8} = \frac{-37}{4} \end{cases}$$

$t_1 = 9$

$t_2 = \frac{-37}{4}$ No es solución por ser negativa.

$$3^x = 3^2$$

$$x = 2$$

i) $2^{2x-4} - 5 \cdot 2^{x-3} + 1 = 0$

$$\frac{2^{2x}}{2^4} - \frac{5 \cdot 2^x}{2^3} + 1 = 0$$

$$\frac{2^{2x}}{\cancel{2^4}} - \frac{2 \cdot 5 \cdot 2^x}{\cancel{2^4}} + \frac{2^4}{\cancel{2^4}} = 0$$

$$2^{2x} - 10 \cdot 2^x + 16 = 0$$

$t = 2^x$

$$t^2 - 10 \cdot t + 16 = 0$$

$$t = \frac{+10 \pm \sqrt{100 - 4 \cdot 1 \cdot 16}}{2 \cdot 1} = \frac{10 \pm 6}{2}$$

$t_1 = 8$

$t_2 = \frac{4}{2} = 2$

$t_1 = 8$

$2^x = 2^3$

$\boxed{x = 3}$

$t_2 = 2$

$2^x = 2$

$\boxed{x = 1}$

j) $9^{x+2} + 3^{x+3} - 810 = 0$

$3^{2(x+2)} + 3^{x+3} - 810 = 0$

$3^4 \cdot 3^{2x} + 3^3 \cdot 3^x - 810 = 0$

$t = 3^x$

$81 \cdot t^2 + 27 \cdot t - 810 = 0$

$3t^2 + t - 30 = 0$

$\Longrightarrow$

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$$t=\frac{-1\pm\sqrt{1-4\cdot 3\cdot(-30)}}{2\cdot 3}=\frac{1\pm 19}{6}=\begin{cases} t_1=\frac{20}{6}=\frac{10}{3} \\ t_2=\frac{-18}{6}=-3 \end{cases}$$

No se puede hacer

$$3^x=t$$

$$t=\frac{10}{3}$$

$$3^x=\frac{10}{3}$$

$$\log_3 3^x=\log_3\frac{10}{3}$$

$$\boxed{x=\log_3\frac{10}{3}}$$

k) $\sqrt[x]{27}=3^{x+2}$

$$3^{3/x}=3^{x+2}$$

$$\frac{3}{x}=x+2$$

$$x^2+2x-3=0$$

$$x=\frac{-2\pm\sqrt{4-4\cdot 1\cdot(-3)}}{2\cdot 1}=\frac{-2\pm 4}{2}\begin{cases} x_1=1 \\ x_2=-3 \end{cases}$$

1) $49^{\frac{x-5}{3}} = \frac{1}{\sqrt{7^{x-1}}}$

$7^{2\left(\frac{x-5}{3}\right)} = 7^{\frac{-(x-1)}{2}}$

$\frac{2x-10}{3} = \frac{-x+1}{2}$

$4x-20 = -3x+3$

$7x = 23$

$x = \frac{23}{7}$