

- 1) a) $2a + 2b = 2(a + b)$
 b) $h \cdot x^2$
 c) $a \cdot (b + c) = ab + ac$
 d) $\log_a b = c$

2) $P(x) - Q(x) = 2x - 1 - (-2x^2 + 3x - 5) = 2x - 1 + 2x^2 - 3x + 5 = 2x^2 - x + 4$

$P(x) \cdot Q(x) = (2x - 1) \cdot (-2x^2 + 3x - 5) = -4x^2 + 6x^2 - 10x + 2x^2 - 3x + 5 =$
 $= -13x + 5$

- 3) a) $(3 - 4xy)^2 = 9 - 24xy + 16x^2y^2$
 b) $(-b + 7x^2)^2 = (7x^2 - b)^2 = 49x^4 - 14x^2b + b^2$
 c) $(3a + 2\sqrt{b})^2 = 9a^2 + 12a\sqrt{b} + 4b$
 d) $(3x^2 - 4y)(3x^2 + 4y) = 9x^4 - 16y^2$

- 4) a) $9x^2 + 12x + 4 = (3x + 2)^2$
 b) $4x^2 - 20xy + 25y^2 = (2x + 5y)^2$
 c) $4a^2 - 36b^4 = (2a - 6b^2)^2$

- 5) a) $16x^3 + 24x^2 = 4x^2 \cdot (4x + 6)$
 b) $3xy^3 - 27x^3y^2 = 3xy^2 \cdot (y - 9x^2)$
 c) $-3x^2z^2 - 6xz^3 - 9x^2z^4 = -3xz^2(x + 2z + 3xz^2)$
 d) $4xy^5 - 2y^2z + x^2zy^2 = y^2(4xy^3 - 2z + x^2z)$

6) a) $(x^3 + 4x + 6) : (x - 4)$

4	1	0	4	6
	4	16	80	
	1	4	20	86

$x^2 + 4x + 20$ Cociente
 Resto 86

b) $(x^3 - 2x^2 + 4) : (x + 3)$

-3	1	-2	0	4
	-3	-9	-15	-45
	1	-5	15	-41

$x^2 - 5x + 15$ Cociente
 -41 Resto

$$c) (5x^4 - 3x^2) : (x-2)$$

$$\text{Cociente } 5x^3 + 10x^2 + 17x + 34$$

$$\text{Resto } 68$$

$$\begin{array}{r|rrrrr} & 5 & 0 & -3 & 0 & 0 \\ 2 & & 10 & 20 & 34 & 68 \\ \hline & 5 & 10 & 17 & 34 & \underline{68} \end{array}$$

$$d) (x^2 + 2) : (x + 4)$$

$$\text{Resto } 18$$

$$\text{Cociente } x - 4$$

$$\begin{array}{r|rrr} & 1 & 0 & 2 \\ -4 & & -4 & 16 \\ \hline & 1 & -4 & \underline{18} \end{array}$$

$$7) P(-1) = -(-1)^5 + 2 \cdot (-1)^4 + 3 \cdot (-1)^3 - 3 \cdot (-1)^2 + 10 \cdot (-1) - 1 = 1 + 2 - 3 - 3 - 10 - 1 = -14$$

$$8) a) x^2 - 9 = (x-3)(x+3)$$

$$b) 2x^2 - 4x = 2x(x-2)$$

$$c) 2x^2 + 2x - 4 = 2 \cdot (x^2 + x - 2) = 2 \cdot (x+2)(x-1)$$

$$\begin{aligned} x^2 + x - 2 &= 0 \\ x &= \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} \end{aligned} \rightarrow \begin{aligned} x &= \frac{-1-3}{2} = -2 \\ x &= \frac{-1+3}{2} = 1 \end{aligned}$$

$$d) -x^2 + 4x - 4 = -(x^2 - 4x + 4) = -(x-2)^2$$

$$e) x^2 + x + 2 \Rightarrow \text{No se puede factorizar}$$

$$\begin{aligned} x^2 + x + 2 &= 0 \\ x &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-7}}{2} \quad \cancel{\neq} \end{aligned}$$

$$f) -2x^2 + x + 6 = -(2x^2 - x - 6) = -(x-2) \cdot (2x+3)$$

$$\begin{aligned} 2x^2 - x - 6 &= 0 \\ x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-6)}}{2 \cdot 2} = \frac{1 \pm \sqrt{49}}{4} = \frac{1 \pm 7}{4} \end{aligned} \rightarrow \begin{aligned} x &= 2 \\ x &= -\frac{3}{2} \end{aligned}$$

9) a) $4x^2 - 4x - 3 = (2x - 3) \cdot (2x + 1)$

$4x^2 - 4x - 3 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot (-3)}}{2 \cdot 4} = \frac{4 \pm \sqrt{64}}{8} = \frac{4 \pm 8}{8}$$

$$\begin{matrix} \rightarrow x = \frac{3}{2} \\ \searrow x = -\frac{1}{2} \end{matrix}$$

b) $x^4 + 2x^2 - 3 = (x - 1) \cdot (x + 1) \cdot (x^2 + 3)$

	1	0	2	0	-3
1		1	1	3	3
	1	1	3	3	0
-1		-1	0	-3	
	1	0	3	0	

$x^2 + 3 = 0 \Rightarrow x^2 = -3 \Rightarrow x = \pm \sqrt{-3} \notin \mathbb{R}$

c) $2x^3 + 8x^2 - 2x - 8 = 2(x^3 + 4x^2 - x - 4) = 2 \cdot (x - 1)(x + 1)(x - 4)$

	1	4	-1	-4
1		1	5	4
	1	5	4	0

$x^2 + 5x + 4 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{-5 \pm \sqrt{9}}{2} = \frac{-5 \pm 3}{2}$$

$$\begin{matrix} \rightarrow x = \frac{-5 + 3}{2} = -1 \\ \searrow x = \frac{-5 - 3}{2} = -4 \end{matrix}$$

d) $x^3 + 7x^2 - 49x - 55 = (x + 1)(x + 11)(x - 5)$

	1	7	-49	-55
-1		-1	-6	55
	1	6	-55	0

$x^2 + 6x - 55 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-55)}}{2 \cdot 1} = \frac{-6 \pm \sqrt{256}}{2} = \frac{-6 \pm 16}{2}$$

$$\begin{matrix} \rightarrow x = -11 \\ \searrow x = 5 \end{matrix}$$

$$e) 6x^3 + 5x^2 - 3x - 2 = (x+1) \cdot (3x-2) \cdot (2x+1)$$

$$\begin{array}{r|rrrr} & 6 & 5 & -3 & -2 \\ -1 & & -6 & 1 & +2 \\ \hline & 6 & -1 & -2 & \underline{0} \end{array}$$

$$6x^2 - x - 2 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 6 \cdot (-2)}}{2 \cdot 6} = \frac{1 \pm \sqrt{49}}{12} = \frac{1 \pm 7}{12} \begin{cases} \rightarrow x = \frac{1+7}{12} = \frac{2}{3} \\ \rightarrow x = \frac{1-7}{12} = -\frac{1}{2} \end{cases}$$

$$f) x^4 - 2x^3 - 8x^2 + 21x = x(x^3 - 2x^2 - 8x + 21) = x \cdot (x+3) \cdot (x^2 - 5x + 7)$$

$$\begin{array}{r|rrrr} & 1 & -2 & -8 & +21 \\ -3 & & -3 & 15 & -21 \\ \hline & 1 & -5 & 7 & \underline{0} \end{array}$$

$$x^2 - 5x + 7 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} = \frac{5 \pm \sqrt{-3}}{2} \quad \cancel{\neq}$$

$$g) 2x^4 + 7x^3 + 8x^2 + 3x = x \cdot (2x^3 + 7x^2 + 8x + 3) = x \cdot (x+1) \cdot (x+1) \cdot (2x+3) \\ = x \cdot (x+1)^2 \cdot (2x+3)$$

$$\begin{array}{r|rrrr} & 2 & 7 & 8 & 3 \\ -1 & & -2 & -5 & -3 \\ \hline & 2 & 5 & 3 & \underline{0} \end{array}$$

$$2x^2 + 5x + 3 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4} \begin{cases} \rightarrow x = -\frac{3}{2} \\ \rightarrow x = -1 \end{cases}$$

$$h) x^5 - x^4 - x^3 - 2x^2 = x^2 \cdot (x^3 - x^2 - x - 2) = x^2 \cdot (x-2) \cdot (x^2 + x + 1)$$

$$\begin{array}{r|rrrr} & 1 & -1 & -1 & -2 \\ 2 & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & \underline{0} \end{array}$$

$$x^2 + x + 1 = 0 \\ x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} \quad \cancel{\neq}$$

i) $2x^4 + 11x^3 + 11x^2 - 15x - 9 = (x-1)(x+3)(2x+1)(x+3)$

$2x^2 + 7x + 3 = 0$

$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-7 \pm \sqrt{25}}{4} = \frac{-7 \pm 5}{4}$

$\rightarrow x = -3$

$\rightarrow x = -\frac{1}{2}$

	2	11	11	-15	-9
1		2	13	24	9
	2	13	24	9	0
-3		-6	-21	-9	
	2	7	3	0	

ii) $4x^4 + 10x^2 = 2x^2 \cdot (2x^2 + 5)$

$2x^2 + 5 = 0 \Rightarrow 2x^2 = -5 \Rightarrow x^2 = -\frac{5}{2} \Rightarrow \text{no tiene solución}$

k) $2x^3 - 10x^2 + 14x - 6 = 2(x^3 - 5x^2 + 7x - 3) = 2 \cdot (x-1) \cdot (x-1)(x-3) =$

$= 2 \cdot (x-1)^2 \cdot (x-3)$

	1	-5	7	-3
1		1	-4	3
	1	-4	3	0

$x^2 - 4x + 3 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} \rightarrow \begin{matrix} x=3 \\ x=1 \end{matrix}$

10) a) $3x - 2(x+3) = x - 3(x+1) \Rightarrow 3x - 2x - 6 = x - 3x - 3 \Rightarrow 3x - 2x - x + 3x = 6 - 3$

$\Rightarrow 3x = 3 \Rightarrow x = 1$

b) $2x + 7 - 2(x-1) = 3(x+3) \Rightarrow 2x + 7 - 2x + 2 = 3x + 9 \Rightarrow 2x - 2x - 3x = 9 - 7 - 2$

$\Rightarrow -3x = 0 \Rightarrow x = \frac{0}{-3} \Rightarrow x = 0$

c) $\frac{x-3}{5} = \frac{x+1}{3} - 2 \Rightarrow \frac{3x-9}{15} = \frac{5x+5}{15} - \frac{30}{15} \Rightarrow 3x-9 = 5x+5-30 \Rightarrow$

$3x-5x = 9+5-30 \Rightarrow -2x = -16 \Rightarrow x = \frac{-16}{-2} \Rightarrow x = 8$

d) $\frac{2x-4}{3} = 3 - \frac{4+x}{2} \Rightarrow \frac{4x-8}{6} = \frac{18}{6} - \frac{12+3x}{6} \Rightarrow 4x-8 = 18-12-3x \Rightarrow$

$\Rightarrow 4x+3x = 18-12+8 \Rightarrow 7x = 14 \Rightarrow x = \frac{14}{7} \Rightarrow x = 2$

$$e) \frac{5x-16}{6} = \frac{-x+8}{12} + \frac{x+1}{3} \Rightarrow \frac{10x-32}{12} = \frac{-x+8}{12} + \frac{4x+4}{12} \Rightarrow$$

$$\Rightarrow 10x-32 = -x+8+4x+4 \Rightarrow 10x+x-4x = 32+8+4 \Rightarrow 7x = 44 \Rightarrow x = \frac{44}{7}$$

$$f) \frac{x+2}{2} - \frac{x+3}{3} = \frac{-x-4}{4} + \frac{x-5}{5} \Rightarrow \frac{30x+60}{60} - \frac{20x+60}{60} = \frac{-15x-60}{60} + \frac{12x-60}{60}$$

$$30x+60-20x-60 = -15x-60+12x-60 \Rightarrow 10x+15x-12x = -120 \Rightarrow 13x = -120$$

$$\Rightarrow x = \frac{-120}{13}$$

$$g) \frac{6-2(x-3)}{7} = -\frac{8}{4} \Rightarrow \frac{6-2x+6}{7} = -2 \Rightarrow \frac{-2x+12}{7} = \frac{-14}{7} \Rightarrow -2x+12 = -14 \Rightarrow$$

$$\Rightarrow -2x = -14-12 \Rightarrow -2x = -26 \Rightarrow x = \frac{-26}{-2} \Rightarrow x = 13$$

$$h) \frac{x-4}{5} - 4 \cdot (-2x+1) - \frac{(-4x+2)}{10} = 2 \cdot (x-3) + \frac{5x+6}{2} \Rightarrow \frac{x-4}{5} - 8x-4 - \frac{4x+2}{10} = 2x-6 + \frac{5x+6}{2}$$

$$\Rightarrow \frac{2x-8}{10} + \frac{80x}{10} - \frac{40}{10} - \frac{4x+2}{10} = \frac{20x-60}{10} + \frac{25x+30}{10} \Rightarrow 2x-8+80x-40+4x-2 = 20x-60+25x+30$$

$$\Rightarrow 2x+80x+4x-20x-25x = -60+30+8+40+2 \Rightarrow 41x = 20 \Rightarrow x = \frac{20}{41}$$

$$i) \frac{3 \cdot (x+3)}{2} - 2 \cdot (2-3x) = 8x-1 - 2 \cdot (x+3) \Rightarrow \frac{3x+9}{2} - 4+6x = 8x-1-2x-6 \Rightarrow$$

$$\Rightarrow \frac{3x+9}{2} - \frac{8}{2} + \frac{12x}{2} = \frac{16x}{2} - \frac{2}{2} - \frac{4x}{2} - \frac{12}{2} \Rightarrow 3x+9-8+12x = 16x-2-4x-12 \Rightarrow$$

$$\Rightarrow 3x+12x-16x+4x = -2-12-9+8 \Rightarrow 3x = -15 \Rightarrow x = \frac{-15}{3} \Rightarrow x = -5$$

$$j) \frac{3 \cdot (x-2)}{5} + 2 \cdot (-3x+1) - \frac{2}{5} = \frac{16}{3} - \frac{4x-3}{15} \Rightarrow \frac{3x-6}{5} - 6x+2 - \frac{2}{5} = \frac{16}{3} - \frac{4x-3}{15} \Rightarrow$$

$$\Rightarrow \frac{9x-18}{15} - \frac{90x}{15} + \frac{30}{15} - \frac{6}{15} = \frac{80}{15} - \frac{4x-3}{15} \Rightarrow 9x-18-90x+30-6 = 80-4x+3 \Rightarrow$$

$$\Rightarrow 9x-90x+4x = 80+3+18-30+6 \Rightarrow -77x = 77 \Rightarrow x = \frac{77}{-77} \Rightarrow x = -1$$

$$k) 2x - \frac{1}{2}(1+3x) - \frac{3}{5} \cdot (x-2) = \frac{1}{4} \cdot (3-x) \Rightarrow 2x - \frac{1}{2} - \frac{3x}{2} - \frac{3x}{5} + \frac{6}{5} = \frac{3}{4} - \frac{x}{4} \Rightarrow$$

$$\Rightarrow \frac{40x}{20} - \frac{10}{20} - \frac{30x}{20} - \frac{12x}{20} + \frac{24}{20} = \frac{15}{20} - \frac{5x}{20} \Rightarrow 40x-10-30x-12x+24 = 15-5x$$

$$\Rightarrow 40x-30x-12x+5x = 15+10-24 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$

$$\textcircled{11} \text{ a) } 7x^2 - 28 = 0$$

$$7x^2 = 28$$

$$x^2 = \frac{28}{7}$$

$$x^2 = 4$$

$$x = \pm \sqrt{4}$$

$$x = \pm 2$$

$$\text{b) } 7x^2 + 28 = 0$$

$$7x^2 = -28$$

$$x^2 = \frac{-28}{7}$$

$$x^2 = -4$$

$$x = \pm \sqrt{-4}$$

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$$\text{c) } 3x^2 = 4x$$

$$3x^2 - 4x = 0$$

$$x \cdot (3x - 4) = 0$$

$$x = 0$$

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

$$\text{d) } 3x^2 - 12x = 0$$

$$x \cdot (3x - 12) = 0$$

$$x = 0$$

$$3x - 12 = 0$$

$$3x = 12$$

$$x = 4$$

$$\text{e) } 4x^2 - 9 = 0$$

$$4x^2 = 9$$

$$x^2 = \frac{9}{4}$$

$$x = \pm \sqrt{\frac{9}{4}}$$

$$x = \pm \frac{3}{2}$$

$$\text{f) } x - 3x^2 = 0$$

$$x \cdot (1 - 3x) = 0$$

$$x = 0$$

$$1 - 3x = 0$$

$$1 = 3x$$

$$x = \frac{1}{3}$$

$$\text{g) } x^2 + 4x - 21 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-21)}}{2 \cdot 1} = \frac{-4 \pm \sqrt{100}}{2} = \frac{-4 \pm 10}{2}$$

$$x = \frac{-4 - 10}{2} = -7$$

$$x = \frac{-4 + 10}{2} = 3$$

$$\text{h) } x^2 + 9x + 20 = 0$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 1 \cdot 20}}{2 \cdot 1} = \frac{-9 \pm \sqrt{1}}{2} = \frac{-9 \pm 1}{2}$$

$$x = -5$$

$$x = -4$$

$$\text{i) } x^2 + x + 3 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-11}}{2} \quad \text{∅}$$

$$\text{j) } 4x^2 - 20x + 25 = 0$$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \cdot 4 \cdot 25}}{2 \cdot 4} = \frac{+20 \pm 0}{8} = \frac{5}{2}$$

$$\text{k) } x^2 - 2x + 3 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{2 \pm \sqrt{-8}}{2} \quad \text{∅}$$

$$\text{l) } -2x^2 + 3x + 2 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-2)}}{2 \cdot (-2)} = \frac{-3 \pm \sqrt{25}}{-4} = \frac{-3 \pm 5}{-4}$$

$$x = \frac{-3 + 5}{-4} = \frac{2}{-4} = -\frac{1}{2}$$

$$x = \frac{-3 - 5}{-4} = \frac{-8}{-4} = 2$$

$$m) 2 \cdot (x+5)^2 + (x-3)^2 = 14 \cdot (x+4) \Rightarrow 2 \cdot (x^2+10x+25) + (x^2-6x+9) = 14x+56$$

$$\Rightarrow 2x^2+20x+50+x^2-6x+9 = 14x+56 \Rightarrow 2x^2+20x+50+x^2-6x+9-14x-56=0$$

$$\Rightarrow 3x^2+3=0 \Rightarrow 3x^2=-3 \Rightarrow x^2=-1 \Rightarrow x = \pm\sqrt{-1} \quad \cancel{\neq}$$

$$n) (2x+1)(x-3) = (x+1)(x-1) - 8 \Rightarrow 2x^2-6x+x-3 = x^2-1-8 \Rightarrow$$

$$\Rightarrow 2x^2-5x-3-x^2+1+8=0 \Rightarrow x^2-5x+6=0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2} \rightarrow x = \frac{5+1}{2} = 3$$

$$\rightarrow x = \frac{5-1}{2} = 2$$

$$o) (2x-3)(2x+3) - x \cdot (x+1) - 5 = 0 \Rightarrow 4x^2-9-x^2-x-5=0$$

$$3x^2-x-14=0 \Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-14)}}{2 \cdot 3} = \frac{1 \pm \sqrt{169}}{6} =$$

$$= \frac{1 \pm 13}{6} \rightarrow x = \frac{1+13}{6} = \frac{14}{6} = \frac{7}{3}$$

$$\rightarrow x = \frac{1-13}{6} = -2$$

$$p) (2x+1)^2 = 4 + (x+2) \cdot (x-2) \Rightarrow 4x^2+4x+1 = 4+x^2-4 \Rightarrow 4x^2+4x+1-x^2=0$$

$$\Rightarrow 3x^2+4x+1=0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{-4 \pm \sqrt{4}}{6} = \frac{-4 \pm 2}{6} \rightarrow x = \frac{-4+2}{6} = -\frac{1}{3}$$

$$\rightarrow x = \frac{-4-2}{6} = -1$$

$$q) \frac{(x-1)(x+2)}{12} - \frac{(x+1) \cdot (x-2)}{6} - 1 = \frac{x-3}{3} \Rightarrow \frac{x^2-x+2x-2}{12} - \frac{x^2+x-2x-2}{6} - 1 = \frac{x-3}{3}$$

$$\Rightarrow \frac{x^2+x-2}{12} - \frac{2x^2-2x-4}{12} - \frac{12}{12} = \frac{4x-12}{12} \Rightarrow x^2+x-2-2x^2+2x+4-12 = 4x-12$$

$$\Rightarrow -x^2+3x-10-4x+12=0 \Rightarrow -x^2-x+2=0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot (-1)} = \frac{1 \pm \sqrt{9}}{-2} = \frac{1 \pm 3}{-2} \rightarrow x = \frac{1+3}{-2} = -2$$

$$\rightarrow x = \frac{1-3}{-2} = 1$$

$$r) \frac{x+1}{2} - \frac{(x-1)^2}{4} - \frac{x+2}{3} + \frac{(x-2)^2}{6} = \frac{1}{6} \Rightarrow \frac{x+1}{2} - \frac{x^2-2x+1}{4} - \frac{x+2}{3} + \frac{x^2-4x+4}{6} = \frac{1}{6}$$

$$\frac{6x+6}{12} - \frac{3x^2-6x+3}{12} - \frac{4x+8}{12} + \frac{2x^2-8x+8}{12} = \frac{2}{12} \Rightarrow 6x+6-3x^2+6x-3-4x-8+2x^2-8x+8 = 2$$

$$= 2 \Rightarrow -x^2+3-2=0 \Rightarrow -x^2+1=0 \Rightarrow x^2=1 \Rightarrow x = \pm\sqrt{1} \Rightarrow x = \pm 1$$

$$s) \frac{x}{3}(x-1) - \frac{x}{4}(x+1) + \frac{3x+4}{12} = 0$$

$$\frac{x^2}{3} - \frac{x}{3} - \frac{x^2}{4} - \frac{x}{4} + \frac{3x+4}{12} = 0 \Rightarrow \frac{4x^2}{12} - \frac{4x}{12} - \frac{3x^2}{12} - \frac{3x}{12} + \frac{3x+4}{12} = 0$$

$$\Rightarrow 4x^2 - 4x - 3x^2 - 3x + 3x + 4 = 0 \Rightarrow x^2 - 4x + 4 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{4 \pm \sqrt{0}}{2} = \frac{4 \pm 0}{2} = 2$$

$$+) \frac{x \cdot (x-1)}{3} - \frac{x \cdot (x+1)}{4} + \frac{3x+4}{12} = 0 \Rightarrow \frac{x^2-x}{3} - \frac{x^2+x}{4} + \frac{3x+4}{12} = 0$$

$$\Rightarrow \frac{4x^2-4x}{12} - \frac{3x^2+3x}{12} + \frac{3x+4}{12} = 0 \Rightarrow 4x^2 - 4x - 3x^2 - 3x + 3x + 4 = 0 \Rightarrow$$

$$x^2 - 4x + 4 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{4 \pm 0}{2} = 2$$

$$o) \frac{3x^2}{2} - \frac{4x-1}{4} = \frac{2x \cdot (x-3)}{6} + \frac{17}{12} \Rightarrow \frac{3x^2}{2} - \frac{4x-1}{4} = \frac{2x^2-6x}{6} + \frac{17}{12} \Rightarrow$$

$$\frac{18x^2}{12} - \frac{12x-3}{12} = \frac{4x^2-12x}{12} + \frac{17}{12} \Rightarrow 18x^2 - 12x + 3 = 4x^2 - 12x + 17 \Rightarrow$$

$$18x^2 - 4x^2 - 12x + 12x + 3 - 17 = 0 \Rightarrow 14x^2 - 14 = 0 \Rightarrow 14x^2 = 14 \Rightarrow x^2 = \frac{14}{14} \Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm\sqrt{1} \Rightarrow x = \pm 1$$

$$v) 3x^2 - 4x + 5(x^2 - 2) = \frac{3x(x-2)}{2} + 14 \Rightarrow 3x^2 - 4x + 5x^2 - 10 = \frac{3x^2 - 6x}{2} + 14 \Rightarrow$$

$$\Rightarrow \frac{6x^2}{2} - \frac{8x}{2} + \frac{10x^2}{2} - \frac{20}{2} = \frac{3x^2 - 6x}{2} + \frac{28}{2} \Rightarrow 6x^2 - 8x + 10x^2 - 20 = 3x^2 - 6x + 28$$

$$6x^2 - 8x + 10x^2 - 20 - 3x^2 + 6x - 28 = 0 \Rightarrow 13x^2 - 2x - 48 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 13 \cdot (-48)}}{2 \cdot 13} = \frac{2 \pm \sqrt{2500}}{26} = \frac{2 \pm 50}{26}$$

$$\rightarrow x = \frac{2+50}{26} = 2$$

$$\rightarrow x = \frac{2-50}{26} = -\frac{24}{13}$$

$$w) 6x^2 - 1 + \frac{2x \cdot (-x + 3)}{3} = \frac{5x^2 - 2}{6} - 4x^2 + \frac{59}{6} \Rightarrow 6x^2 - 1 + \frac{-2x^2 + 6x}{3} = \frac{5x^2 - 2}{6} - 4x^2 + \frac{59}{6}$$

$$\frac{36x^2 - 6}{6} + \frac{-4x^2 + 12x}{6} = \frac{5x^2 - 2}{6} - \frac{24x^2}{6} + \frac{59}{6} \Rightarrow$$

$$\Rightarrow 36x^2 - 6 - 4x^2 + 12x = 5x^2 - 2 - 24x^2 + 59 \Rightarrow 36x^2 - 4x^2 - 5x^2 + 24x^2 + 12x - 6 + 2 - 59$$

$$51x^2 + 12x - 63 = 0$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 51 \cdot (-63)}}{2 \cdot 51} = \frac{-12 \pm \sqrt{12996}}{102} = \frac{-12 \pm 114}{102}$$

$$\begin{aligned} \nearrow x &= \frac{-12 + 114}{102} = 1 \\ \searrow x &= \frac{-12 - 114}{102} = \frac{-21}{17} \end{aligned}$$

12.)

$$a) -2x^3 + 4x^2 + 18x - 36 = 0$$

$$-2 \cdot (x^3 - 2x^2 - 9x + 18) = 0$$

$$x^3 - 2x^2 - 9x + 18 \begin{cases} \pm 1 & \pm 6 \\ \pm 2 & \pm 9 \\ \pm 3 & \pm 18 \end{cases}$$

1	-2	-9	+18
2	2	0	-18
1	0	-9	0

$x^2 - 9$

$$x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm \sqrt{9} \Rightarrow x = \pm 3$$

$$-2 \cdot (x-3)(x+3) \cdot (x-2) = 0$$

Solución

$$x = 3, \quad x = -3 \quad \text{y} \quad x = 2$$

$$b) 4x^3 - 24x^2 + 48x - 32 = 0 \Rightarrow 4 \cdot (x^3 - 6x^2 + 12x - 8) = 0$$

$$x^3 - 6x^2 + 12x - 8 \begin{cases} \pm 1 & \pm 4 \\ \pm 2 & \pm 8 \end{cases}$$

1	-6	12	-8
2	2	-8	8
1	-4	4	0

$$x^2 - 4x + 4$$

$$x^2 - 4x + 4 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{4 \pm 0}{2} = 2$$

$$4 \cdot (x-2) \cdot (x-2) \cdot (x-2) = 0 \Rightarrow 4(x-2)^3 = 0$$

Solución

$$x = 2$$

$$c) -3x^4 + 3x^3 + 12x^2 - 12x = 0$$

$$-3x \cdot (x^3 - x^2 - 4x + 4) = 0$$

$$x^3 - x^2 - 4x + 4 \quad \begin{array}{l} \uparrow \pm 1 \\ \uparrow \pm 2 \\ \uparrow \pm 4 \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -1 & -4 & 4 \\ 1 & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & \underline{0} \end{array}$$

$$x^2 - 4$$

$$x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

$$-3x \cdot (x-1) \cdot (x-2) \cdot (x+2) = 0$$

Solución

$$x=0 \quad x=1 \quad x=2 \quad x=-2$$

$$d) x^3 + 3x^2 - x - 3 = 0$$

$$\begin{array}{r|rrrr} & 1 & 3 & -1 & -3 \\ 1 & & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & \underline{0} \end{array}$$

$$e) x^4 - 26x^2 + 25 = 0$$

$$\begin{array}{r|rrrrr} & 1 & 0 & -26 & 0 & 25 \\ 1 & & 1 & 1 & -25 & -25 \\ \hline & 1 & 1 & -25 & -25 & \underline{0} \\ -1 & & -1 & 0 & +25 & \\ \hline & 1 & 0 & -25 & \underline{0} & \end{array}$$

$$x^2 - 25 = 0 \Rightarrow x^2 = 25 \Rightarrow x = \pm \sqrt{25} \Rightarrow x = \pm 5$$

$$(x-1) \cdot (x+1) \cdot (x-5) \cdot (x+5) = 0$$

$$\text{Solución } x=1, x=-1, x=5, x=-5$$

$$f) 6x^4 - 5x^3 - 43x^2 + 70x - 24 = 0$$

6	-5	-43	70	-24
2	12	14	-58	24
6	7	-29	12	0
-3	-18	+33	-12	
6	-11	4	0	

$$6x^2 - 11x + 4 = 0$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \cdot 6 \cdot 4}}{2 \cdot 6} = \frac{11 \pm \sqrt{25}}{12} = \frac{11 \pm 5}{12}$$

$\nearrow x = \frac{4}{3}$
 $\searrow x = \frac{11-5}{12} = \frac{1}{2}$

$$(x-2)(x+3)(3x-4) \cdot (2x-1) = 0$$

Solución	x=2	x= $\frac{4}{3}$
	x=-3	x= $\frac{1}{2}$

$$g) x \cdot (9x^2 - 1) \cdot (2x + 3) = 0$$

$$\textcircled{I} \quad x = 0$$

$$\textcircled{II} \quad 9x^2 - 1 = 0$$

$$\textcircled{III} \quad 2x + 3 = 0$$

$$9x^2 = 1$$

$$x^2 = \frac{1}{9}$$

$$x = \pm \sqrt{\frac{1}{9}}$$

$$x = \pm \frac{1}{3}$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

Solución	x=0 , x= $\frac{1}{3}$, x= $-\frac{1}{3}$, x= $-\frac{3}{2}$
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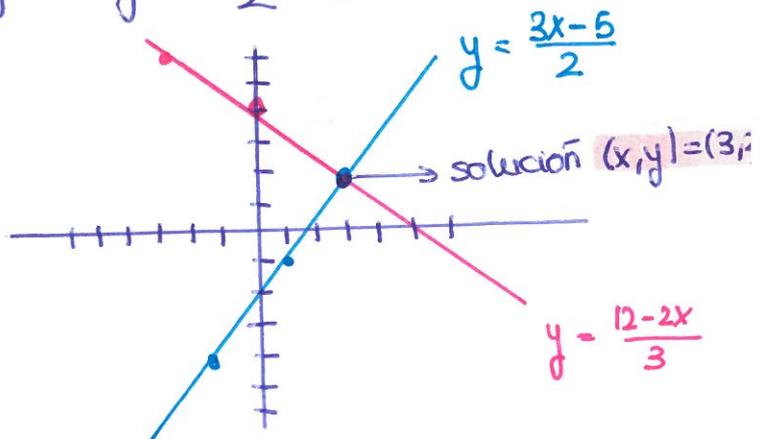
b. a) $\begin{cases} 2x+3y=12 \\ 3x-2y=5 \end{cases} \rightarrow 3y=12-2x \Rightarrow y = \frac{12-2x}{3} \text{ (I)}$
 $\rightarrow 3x-5=2y \Rightarrow y = \frac{3x-5}{2} \text{ (II)}$

(I) $y = \frac{12-2x}{3}$

(II) $y = \frac{3x-5}{2}$

x	y
0	4
3	2
-3	6

x	y
1	-1
3	2
-1	-4



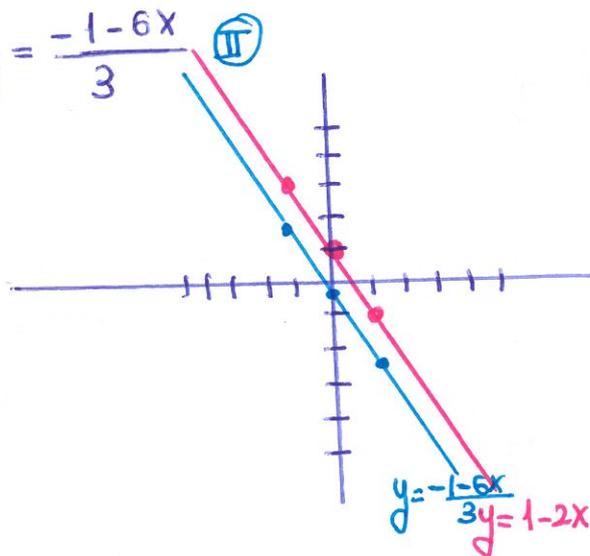
b) $\begin{cases} 2x+y=1 \\ 6x+3y=-1 \end{cases} \rightarrow y=1-2x \text{ (I)}$
 $\rightarrow 3y=-1+6x \Rightarrow y = \frac{-1-6x}{3} \text{ (II)}$

(I) $y = 1-2x$

(II) $y = \frac{-1-6x}{3}$

x	y
0	1
1	-1
-1	3

x	y
0	$-\frac{1}{3} \approx -0,33$
1	$-\frac{7}{3} \approx -2,33$
-1	$\frac{5}{3} \approx 1,67$



SISTEMA INCOMPATIBLE

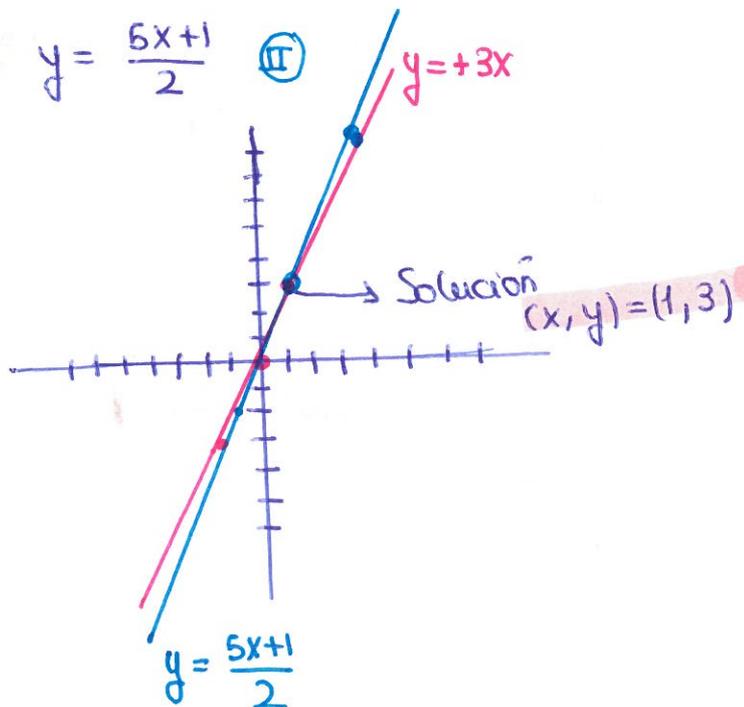
c) $\begin{cases} -3x+y=0 \\ 5x-2y=-1 \end{cases} \rightarrow y=3x \text{ (I)}$
 $\rightarrow 5x+1=2y \Rightarrow y = \frac{5x+1}{2} \text{ (II)}$

(I) $y = 3x$

x	y
1	3
0	0
-1	-3

(II) $y = \frac{5x+1}{2}$

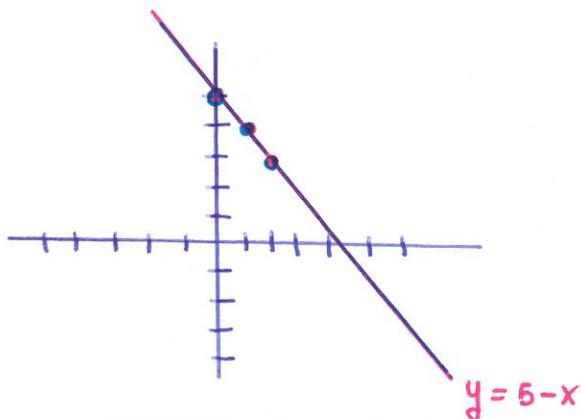
x	y
1	3
+3	8
-1	-2



$$d) \begin{cases} x+y=5 & \rightarrow y=5-x & \textcircled{I} \\ -3x-3y=-15 & \Rightarrow -3x+15=3y & \Rightarrow y=\frac{-3x+15}{3} & \textcircled{II} \end{cases}$$

$$\textcircled{I} \begin{array}{c|c} y=5-x & \\ \hline x & y \\ \hline 1 & 4 \\ 2 & 3 \\ 0 & 5 \end{array}$$

$$\textcircled{II} \begin{array}{c|c} y=\frac{-3x+15}{3} & \\ \hline x & y \\ \hline 1 & 4 \\ 2 & 3 \\ 0 & 5 \end{array}$$



SISTEMA COMPATIBLE INDETERMINADO

$$14) a) \begin{cases} x-3y=4 & \textcircled{I} \\ 2x-3y=1 & \end{cases} \quad \text{Sustitución} \quad \begin{aligned} \textcircled{I} \quad x &= 4+3y \\ 2(4+3y)-3y &= 1 \\ 8+6y-3y &= 1 \\ 3y &= -7 \Rightarrow y = -\frac{7}{3} \end{aligned}$$

$$x = 4 + 3 \cdot \left(-\frac{7}{3}\right) = 4 - 7 = -3$$

$$(x, y) = (-3, -7/3)$$

$$b) \begin{cases} 4x+y=6 & \textcircled{I} \\ -x-y=0 & \textcircled{II} \end{cases} \quad \text{Igualación} \quad \begin{aligned} \textcircled{I} \quad y &= 6-4x \\ \textcircled{II} \quad -x &= y \end{aligned}$$

$$6-4x = -x \Rightarrow 6 = -x+4x \Rightarrow 6 = 3x \Rightarrow x = \frac{6}{3} = 2$$

$$y = -x \Rightarrow y = -2$$

$$(x, y) = (2, -2)$$

$$c) \begin{cases} x-2y=1 \\ 2x-2y=8 \end{cases} \quad \text{Reducción} \quad \begin{aligned} \xrightarrow{\cdot(-1)} \quad -x+2y &= -1 \\ \xrightarrow{\cdot 1} \quad 2x-2y &= 8 \\ \hline x &= 7 \end{aligned}$$

$$7-2y=1 \Rightarrow 7-1=2y \Rightarrow 6=2y \Rightarrow y = \frac{6}{2} = 3$$

$$(x, y) = (7, 3)$$

$$d) \begin{cases} x-3y=4 & \textcircled{I} \\ 2x-3y=1 & \textcircled{II} \end{cases} \quad \text{Igualación} \quad \begin{aligned} \textcircled{I} \quad x &= 4+3y \\ \textcircled{II} \quad 2x &= 1+3y \Rightarrow x = \frac{1+3y}{2} \end{aligned}$$

$$4+3y = \frac{1+3y}{2} \Rightarrow \frac{8+6y}{2} = \frac{1+3y}{2} \Rightarrow 8+6y = 1+3y \Rightarrow 6y-3y = 1-8$$

$$3y = -7 \Rightarrow y = -7/3 \quad x = 4 + 3 \cdot (-7/3) = 4 - 7 = -3 \quad (x, y) = (-3, -7/3)$$

e)
$$\begin{cases} 5x - 3y = 1 \\ 4x + y = 11 \end{cases} \rightarrow y = 11 - 4x$$
 Substitución
$$5x - 3 \cdot (11 - 4x) = 1$$

$$5x - 33 + 12x = 1$$

$$17x = 33 + 1 \Rightarrow 17x = 34 \Rightarrow x = \frac{34}{17} = 2$$

$$y = 11 - 4 \cdot 2 = 3$$

$$(x, y) = (2, 3)$$

f)
$$\begin{cases} 7x - y = -6 \\ 4x + 3y = 3 \end{cases}$$
 Reducción
$$\begin{array}{r} \cdot 3 \rightarrow 21x - 3y = -18 \\ \rightarrow 4x + 3y = 3 \\ \hline 25x = -15 \\ x = \frac{-15}{25} = -\frac{3}{5} \end{array}$$

$$4 \cdot \left(-\frac{3}{5}\right) + 3y = 3 \Rightarrow -\frac{12}{5} + 3y = 3 \Rightarrow -\frac{12}{5} + \frac{15y}{5} = \frac{15}{5} \Rightarrow 15y = 15 + 12$$

$$\Rightarrow 15y = 27 \Rightarrow y = \frac{27}{15} = \frac{9}{5}$$

$$(x, y) = \left(-\frac{3}{5}, \frac{9}{5}\right)$$

15) a)
$$\begin{cases} 3(x-1) + y = 0 \\ 3(x+1) + y = -5 \end{cases} \Rightarrow \begin{cases} 3x - 3 + y = 0 \\ 3x + 3 + y = -5 \end{cases} \Rightarrow \begin{cases} 3x + y = 3 \\ 3x + y = -8 \end{cases}$$

$$\begin{array}{r} \xrightarrow{\cdot 1} 3x + y = 3 \\ \xrightarrow{\cdot (-1)} \frac{-3x - y = 8}{0 = 11} \end{array}$$

\Rightarrow SISTEMA INCOMPATIBLE

b)
$$\begin{cases} x + y = 4 - y \\ 3x - 5 = 7 - 6y \end{cases} \Rightarrow \begin{cases} x + 2y = 4 \\ 3x + 6y = 12 \end{cases} \rightarrow x = 4 - 2y$$

$$3 \cdot (4 - 2y) + 6y = 12$$

$$12 - 6y + 6y = 12$$

$$0 = 0$$

SISTEMA COMPATIBLE INDETERMINADO

c)
$$\begin{cases} 2x + y = 0 \\ 5x - 3 = 9y - 3 \end{cases} \Rightarrow \begin{cases} 2x + y = 0 \\ 5x - 9y = 0 \end{cases} \rightarrow y = -2x$$

$$5x = 9y \Rightarrow y = \frac{5x}{9}$$

$$-2x = \frac{5x}{9} \Rightarrow \frac{-18x}{9} = \frac{5x}{9} \Rightarrow -18x - 5x = 0 \Rightarrow -23x = 0$$

$$x = 0$$

$$y = -2 \cdot 0 = 0$$

$$(x, y) = (0, 0)$$

d)
$$\begin{cases} 2(3x-2) = y-1 \\ 3(x+y) + 2(x-y) = 8 \end{cases} \Rightarrow \begin{cases} 6x - 4 = y - 1 \\ 3x + 3y + 2x - 2y = 8 \end{cases} \Rightarrow \begin{cases} 6x - y = 3 \\ 5x + y = 8 \end{cases}$$

$$\begin{array}{r} \xrightarrow{\cdot 1} 6x - y = 3 \\ \xrightarrow{\cdot 1} \frac{5x + y = 8}{11x = 11} \end{array}$$

$$x = \frac{11}{11} = 1$$

$$6 \cdot 1 - y = 3 \Rightarrow 6 - 3 = y \Rightarrow 3 = y$$

$$(x, y) = (1, 3)$$

$$e) \begin{cases} \frac{x}{3} - \frac{y}{2} = 4 \\ \frac{x}{2} + \frac{y}{4} = 2 \end{cases} \Rightarrow \begin{cases} \frac{2x}{6} - \frac{3y}{6} = \frac{24}{6} \\ \frac{2x}{4} + \frac{y}{4} = \frac{8}{4} \end{cases} \Rightarrow \begin{cases} 2x - 3y = 24 \rightarrow x = \frac{24+3y}{2} \\ 2x + y = 8 \rightarrow x = \frac{8-y}{2} \end{cases}$$

$$\frac{24+3y}{2} = \frac{8-y}{2} \Rightarrow 24+3y = 8-y \Rightarrow 3y+y = 8-24 \Rightarrow 4y = -16 \Rightarrow$$

$$\Rightarrow y = \frac{-16}{4} = -4 \quad x = \frac{8-(-4)}{2} = 6$$

$$(x, y) = (6, -4)$$

$$f) \begin{cases} \frac{x-1}{2} + \frac{y+1}{4} = 1 \\ \frac{2x-1}{2} - \frac{2y+1}{6} = 1 \end{cases} \rightarrow \begin{cases} \frac{2x-2}{4} + \frac{y+1}{4} = \frac{4}{4} \\ \frac{6x-3}{6} - \frac{2y+1}{6} = \frac{6}{6} \end{cases} \Rightarrow \begin{cases} 2x-2+y+1=4 \\ 6x-3-2y-1=6 \end{cases}$$

$$\Rightarrow \begin{cases} 2x+y = 4+2-1 \\ 6x-2y = 6+3+1 \end{cases} \Rightarrow \begin{cases} 2x+y = 5 \xrightarrow{\cdot 2} 4x+2y = 10 \\ 6x-2y = 10 \xrightarrow{\cdot 1} 6x-2y = 10 \end{cases}$$

$$\frac{10x}{10} = 20 \Rightarrow x = \frac{20}{10} = 2$$

$$2 \cdot 2 + y = 5 \Rightarrow y = 5 - 4 = 1$$

$$(x, y) = (2, 1)$$

$$16) a) \begin{cases} x^2 - y^2 = 25 \\ x + y = 25 \rightarrow x = 25 - y \end{cases}$$

$$(25-y)^2 - y^2 = 25$$

$$625 - 50y + y^2 - y^2 = 25$$

$$-50y = 25 - 625$$

$$-50y = -600 \Rightarrow y = \frac{-600}{-50} = 12$$

$$x = 25 - 12 = 13$$

$$(x, y) = (13, 12)$$

$$b) \begin{cases} 2x - y = -1 \Rightarrow 2x + 1 = y \\ y^2 - 2x^2 = 7 \end{cases}$$

$$(2x+1)^2 - 2x^2 = 7$$

$$4x^2 + 4x + 1 - 2x^2 = 7$$

$$2x^2 + 4x - 6 = 0$$

$$x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2} \rightarrow \begin{matrix} x = -3 \\ x = 1 \end{matrix}$$

$$\text{Si } x = -3$$

$$y = 2 \cdot (-3) + 1 = -5$$

$$(x, y) = (-3, -5)$$

$$\text{Si } x = 1$$

$$y = 2 \cdot 1 + 1 = 3$$

$$(x, y) = (1, 3)$$

$$c) \begin{cases} x^2 - xy + y^2 = 7 \\ x + y = 5 \end{cases} \Rightarrow x = (5 - y)$$

$$(5 - y)^2 - (5 - y) \cdot y + y^2 = 7$$

$$25 - 10y + y^2 - 5y + y^2 + y^2 = 7$$

$$3y^2 - 15y + 18 = 0 \Rightarrow y^2 - 5y + 6 = 0$$

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} \begin{cases} \rightarrow y = 3 \\ \rightarrow y = 2 \end{cases}$$

$$\text{si } y = 3 \quad x = 5 - 3 = 2$$

$$(x, y) = (2, 3)$$

$$\text{si } y = 2 \quad x = 5 - 2 = 3$$

$$(x, y) = (3, 2)$$

$$d) \begin{cases} x \cdot y = 8 \\ x + y = 6 \end{cases} \Rightarrow y = 6 - x$$

$$x \cdot (6 - x) = 8$$

$$6x - x^2 = 8 \Rightarrow -x^2 + 6x - 8 = 0$$

$$x^2 - 6x + 8 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2} \begin{cases} \rightarrow x = 4 \\ \rightarrow x = 2 \end{cases}$$

$$\text{si } x = 4 \quad y = 6 - 4 = 2$$

$$(x, y) = (4, 2)$$

$$\text{si } x = 2 \quad y = 6 - 2 = 4$$

$$(x, y) = (2, 4)$$

$$e) \begin{cases} x^2 + 3xy = 22 \\ x + y = 5 \end{cases} \rightarrow x = 5 - y$$

$$(5 - y)^2 + 3 \cdot (5 - y) \cdot y = 22$$

$$25 - 10y + y^2 + 15y - 3y^2 = 22$$

$$-2y^2 + 5y + 3 = 0$$

$$y = \frac{-5 \pm \sqrt{5^2 - 4 \cdot (-2) \cdot 3}}{2 \cdot (-2)} = \frac{-5 \pm \sqrt{25 + 24}}{-4} = \frac{-5 \pm \sqrt{49}}{-4} = \frac{-5 \pm 7}{-4} \begin{cases} \rightarrow y = 3 \\ \rightarrow y = -\frac{1}{2} \end{cases}$$

$$\text{si } y = 3 \quad x = 5 - 3 = 2$$

$$(x, y) = (2, 3)$$

$$\text{si } y = -\frac{1}{2}$$

$$(x, y) = \left(\frac{11}{2}, -\frac{1}{2}\right)$$

$$x = 5 - \left(-\frac{1}{2}\right) = \frac{10}{2} + \frac{1}{2} = \frac{11}{2}$$

$$f) \begin{cases} y = -x^2 + 3x + 4 \\ y = 2x + 3 \end{cases}$$

$$-x^2 + 3x + 4 = 2x + 3$$

$$-x^2 + x + 1 = 0 \Rightarrow x^2 - x - 1 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{1 \pm \sqrt{5}}{2} \begin{matrix} \rightarrow x = \frac{1 + \sqrt{5}}{2} \\ \searrow x = \frac{1 - \sqrt{5}}{2} \end{matrix}$$

$$\text{si } x = \frac{1 + \sqrt{5}}{2}$$

$$y = 2 \cdot \left(\frac{1 + \sqrt{5}}{2} \right) + 3 = 1 + \sqrt{5} + 3 = 4 + \sqrt{5}$$

$$(x, y) = \left(\frac{1 + \sqrt{5}}{2}, 4 + \sqrt{5} \right)$$

$$\text{si } x = \frac{1 - \sqrt{5}}{2}$$

$$y = 2 \cdot \left(\frac{1 - \sqrt{5}}{2} \right) + 3 = 1 - \sqrt{5} + 3 = 4 - \sqrt{5}$$

$$(x, y) = \left(\frac{1 - \sqrt{5}}{2}, 4 - \sqrt{5} \right)$$

17. x: pantalón
y: camisa

$$\begin{cases} x = 5 + y \\ 6x + 4y = 480 \end{cases}$$

$$6 \cdot (5 + y) + 4y = 480$$

$$30 + 6y + 4y = 480$$

$$10y = 450$$

$$y = \frac{450}{10} = 45$$

$$x = 5 + 45 = 50$$

Un pantalón cuesta 50 € y 45 € una camisa

18. x: años que pasarán

Carmen $16 + x$

Hermano 1 $2 + x$

Hermano 2 $3 + x$

$$2 \cdot (2 + x + 3 + x) = 16 + x$$

$$2 \cdot (2x + 5) = 16 + x$$

$$4x + 10 = 16 + x$$

$$4x - x = 16 - 10$$

$$3x = 6$$

$$x = 2$$

Tienen que pasar 2 años

19. x : habitaciones dobles.
 y : habitaciones simples

$$\begin{cases} x + y = 120 \\ 2x + y = 195 \end{cases} \xrightarrow{\begin{matrix} \cdot (-1) \\ \cdot 1 \end{matrix}} \begin{array}{r} -x - y = -120 \\ 2x + y = 195 \\ \hline x = 75 \end{array}$$

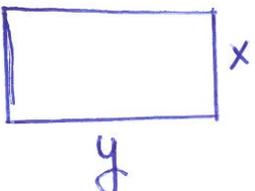
$$75 + y = 120 \Rightarrow y = 120 - 75 = 45$$

Hay 45 habitaciones simples y 75 habitaciones dobles

20. x : edad Antía
 $x + 30$ edad del padre
 $x + 30 - 5$ edad de la madre

$$\begin{aligned} 7 \cdot x &= x + 30 + x + 30 - 5 \\ 7x &= 2x + 55 \\ 7x - 2x &= 55 \\ 5x &= 55 \\ x &= 55/5 = 11 \end{aligned}$$

Antía tiene 11 años

21. 

$$\begin{cases} 2x + 2y = 16 \\ x \cdot y = 15 \end{cases} \Rightarrow \begin{cases} x + y = 8 \Rightarrow x = 8 - y \\ x \cdot y = 15 \end{cases}$$

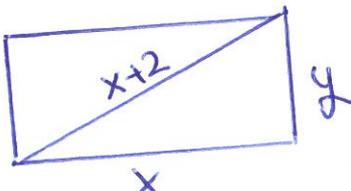
$$(8 - y) \cdot y = 15 \Rightarrow 8y - y^2 = 15 \Rightarrow y^2 - 8y + 15 = 0$$

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 60}}{2} = \frac{8 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2} \rightarrow \begin{array}{l} y = 5 \\ y = 3 \end{array}$$

si $y = 5$ $x = 8 - 5 = 3$

si $y = 3$ $x = 8 - 3 = 5$

La base mide 3 cm y la altura 5 cm

22. 

$$\begin{cases} (x+2)^2 = x^2 + y^2 \\ 2x + 2y = 46 \end{cases} \Rightarrow \begin{cases} x^2 + 4x + 4 = x^2 + y^2 \\ x + y = 23 \end{cases}$$

$$4 \cdot (23 - y) - y^2 = -4 \Rightarrow 92 - 4y - y^2 = -4$$

$$\Rightarrow \begin{cases} 4x - y^2 = -4 \\ x = 23 - y \end{cases}$$

$$y^2 + 4y - 96 = 0$$

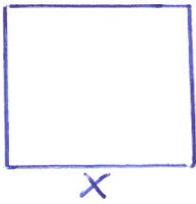
$$y = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-96)}}{2 \cdot 1} = \frac{-4 \pm \sqrt{400}}{2} =$$

$$= \frac{-4 \pm 20}{2} \rightarrow \begin{array}{l} y = 12 \\ y = 8 \end{array} \text{ No puede ser negativo}$$

$$x = 23 - 8 = 15 \text{ cm}$$

Un lado mide 15 cm y el otro 8 cm

23.



$$12x^2 + 28 \cdot 4x = 195$$

$$12x^2 + 112x - 195 = 0$$

$$x = \frac{-112 \pm \sqrt{112^2 - 4 \cdot 12 \cdot (-195)}}{2 \cdot 12} = \frac{-112 \pm \sqrt{21904}}{24} = \frac{-112 \pm 148}{24}$$

→ $x = -65/6$ no puede ser

$$\downarrow x = \frac{36}{24} = \frac{3}{2} = 1,5$$

La ventana mide 1,5 m de lado

24. x: papel de baja calidad
y: papel de alta calidad

$$\begin{cases} x + y = 50 \\ 0,25x + 0,40y = 50 \cdot 0,31 \end{cases} \Rightarrow$$

$$\begin{cases} x + y = 50 \\ 25x + 40y = 1550 \end{cases}$$

$$\rightarrow x = 50 - y$$

$$25 \cdot (50 - y) + 40y = 1550$$

$$\Rightarrow 1250 - 25y + 40y = 1550 \Rightarrow 15y = 300 \Rightarrow y = \frac{300}{15} \Rightarrow y = 20$$

$$x = 50 - 20 = 30$$

Se necesitan 30 kg de la pasta de baja calidad y 20 kg de pasta de alta calidad.

25. x: primer número
y: segundo número

$$\begin{cases} 3x - \frac{y}{4} = 6 \\ x^2 + y^2 = 145 \end{cases} \Rightarrow \begin{cases} 12x - y = 24 \rightarrow 12x - 24 = y \\ x^2 + y^2 = 145 \end{cases}$$

$$x^2 + (12x - 24)^2 = 145 \Rightarrow x^2 + 144x - 576x + 576 = 145 \Rightarrow x^2 - 432x + 431 = 0$$

$$x = \frac{-(-432) \pm \sqrt{(-432)^2 - 4 \cdot 1 \cdot 431}}{2 \cdot 1} = \frac{432 \pm 430}{2}$$

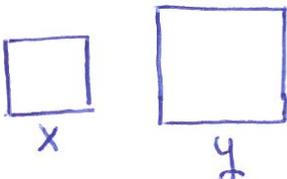
$$\begin{aligned} \rightarrow x &= 431 \text{ No puede ser.} \\ \downarrow x &= 1 \end{aligned}$$

$$\text{Si } x = 1$$

$$y = 12 \cdot 1 - 24 = -12$$

Un número es 1 y el otro es -12

26)



$$\begin{cases} x^2 + y^2 = 90 \\ 4x + 4y = 48 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 = 90 \\ x + y = 12 \rightarrow x = 12 - y \end{cases}$$

$$(12 - y)^2 + y^2 = 90 \Rightarrow 144 - 24y + y^2 + y^2 = 90 \Rightarrow 2y^2 - 24y + 54 = 0$$

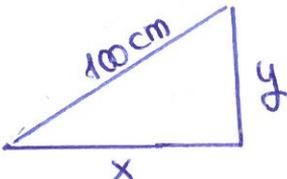
$$\Rightarrow y^2 - 12y + 27 = 0$$

$$y = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 1 \cdot 27}}{2 \cdot 1} = \frac{12 \pm 6}{2} \begin{matrix} \rightarrow y = 9 \\ \rightarrow y = 3 \end{matrix}$$

si $y = 9$ $x = 12 - 9 = 3$ si $y = 3$ $x = 12 - 3 = 9$

Un cuadrado mide 3cm de lado y el otro 9cm

27)



$$\begin{cases} x + y + 100 = 224 \\ x^2 + y^2 = 100^2 \end{cases} \Rightarrow \begin{cases} x + y = 124 \rightarrow x = 124 - y \\ x^2 + y^2 = 10000 \end{cases}$$

$$(124 - y)^2 + y^2 = 10000 \Rightarrow 15376 - 248y + y^2 + y^2 = 10000$$

$$\Rightarrow 2y^2 - 248y + 5376 = 0 \Rightarrow y^2 - 124y + 2688 = 0$$

$$y = \frac{-(-124) \pm \sqrt{(-124)^2 - 4 \cdot 1 \cdot 2688}}{2 \cdot 1} = \frac{124 \pm \sqrt{4624}}{2} = \frac{124 \pm 68}{2} \begin{matrix} \rightarrow y = 96 \\ \rightarrow y = 28 \end{matrix}$$

si $y = 96$ $x = 124 - 96 = 28$

si $y = 28$ $x = 124 - 28 = 96$

Un cateto mide 96 cm y el otro 28 cm

28) x : precio del café barato
 y : precio del café caro

$$\begin{cases} 20 \cdot x + 10 \cdot y = 2 \cdot 30 \\ y = 4x \end{cases}$$

$$20x + 10 \cdot 4x = 60 \Rightarrow 20x + 40x = 60 \Rightarrow 60x = 60 \Rightarrow x = 1$$

$y = 4 \cdot 1$

El café barato vale 1 €/kg y el caro 4 €/kg

- 29) x : tornillos en la caja pequeña
 $x+25$: tornillos en la caja mediana
 $2 \cdot (x+25)$: tornillos en la caja grande.

$$x + x + 25 + 2x + 50 = 375$$

$$4x = 375 - 75$$

$$4x = 300 \Rightarrow x = \frac{300}{4} = 75$$

En la caja pequeña hay 75 tornillos, en la mediana 100 y en la grande 200.

- 30) x € María
 $2x + 30$ Ana

$$x + 2x + 30 = 270$$

$$3x = 270 - 30 \Rightarrow 3x = 240 \Rightarrow x = \frac{240}{3} = 80$$

$$2 \cdot 80 + 30 = 190$$

María tiene 80 € y Ana 190 €

- 31) Repetido

- 32) x : cantidad del 80%
 y : cantidad del 95%

$$\begin{cases} x + y = 5 \\ 0,8x + 0,95y = 0,86 \cdot 5 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 5 - y \\ 0,8x + 0,95y = 4,3 \end{cases}$$

$$0,8 \cdot (5 - y) + 0,95y = 4,3$$

$$4 - 0,8y + 0,95y = 4,3$$

$$0,15y = 0,3$$

$$y = \frac{0,3}{0,15} = 2$$

$$x = 5 - 2 = 3$$

Se necesitan 3 kg del 80% y 2 kg del 95%

- 33.) x : cantidad de café de 34€/kg.

$$34 \cdot x + 50 \cdot 80 = 44 \cdot (x + 80) \Rightarrow 34x + 4000 = 44x + 3520$$

$$4000 - 3520 = 44x - 34x \Rightarrow 480 = 10x \Rightarrow x = \frac{480}{10} = 48$$

Se necesitan 48 kg de 34€/kg

34) x : colorante $2x + 500$: blanca

$$x + 2x + 500 = 5000$$

$$3x = 5000 - 500$$

$$3x = 4500 \Rightarrow x = \frac{4500}{3} = 1500$$

$$2 \cdot 1500 + 500 = 3500$$

Se necesitan 1,5 kg de colorante y 3,5 kg de pintura blanca.

