Exercício 17

$$\begin{cases} \cdot f(x) = x^2 - x - 2 \\ \cdot h(x) = \dfrac{1}{x^2 - 4} \\ \cdot t(x) = 1 - x^2 \end{cases}$$

a) $(f-t)(x) = f(x) - t(x) = x^2 - x - 2 - 1 + x^2 = -x - 3$

b) $(f \circ t)(x) = f[t(x)] = (1-x^2)^2 - (1-x^2) - 2$

$= 1 - 2x^2 + x^4 - 1 + x^2 - 2$

$= x^4 - x^2 - 2$

c) $t^{-1}(x) \Rightarrow$

$t(x) = 1 - x^2$

$y = 1 - x^2$

$x = 1 - y^2$

$y^2 = 1 - x$

$y = \sqrt{1-x}$

$t^{-1}(x) = \sqrt{1-x}$

d) $(t \circ h)(x) = t[h(x)] = 1 - \left(\dfrac{1}{x^2-4}\right)^2 =$

$= 1 - \dfrac{1}{(x^2-4)^2} = 1 - \dfrac{1}{x^4 - 8x^2 + 16} =$

$= \dfrac{x^4 - 8x^2 + 16}{x^4 - 8x^2 + 16} - \dfrac{1}{x^4 - 8x^2 + 16}$

$= \dfrac{x^4 - 8x^2 + 15}{x^4 - 8x^2 + 16}$

d) $h^{-1}(x)$ ⇒

$h(x) = \frac{1}{x^2-4}$

$y = \frac{1}{x^2-4}$

$x = \frac{1}{y^2-4}$

$x \cdot (y^2-4) = 1$

$y^2 - 4 = \frac{1}{x}$

$y^2 = \frac{1}{x} + 4$

$y = \sqrt{\frac{1}{x}+4}$

$y = \sqrt{\frac{1+4x}{x}}$

$h^{-1}(x) = \sqrt{\frac{1+4x}{x}}$

f) $(f \circ t^{-1})(x) = f\left[t^{-1}(x)\right] = \left(\sqrt{1-x}\right)^2 - \sqrt{1-x} - 2$

$= 1 - x - \sqrt{1-x} - 2 = -x - \sqrt{1-x} - 1$

Exercício 18

$\begin{cases} f(x) = \sqrt{x-1} \\ g(x) = \frac{1}{x-3} \end{cases}$

a) $f^{-1}(x)$ ⇒

$f(x) = \sqrt{x-1}$

$y = \sqrt{x-1}$

$x = \sqrt{y-1}$

$x^2 = y - 1$

$y = x^2 + 1$

$f^{-1}(x) = x^2 + 1$

b) $g^{-1}(x)$

$g(x) = \frac{1}{x-3}$

$y = \frac{1}{x-3}$

$x = \frac{1}{y-3}$

$x \cdot (y-3) = 1$

$y - 3 = \frac{1}{x}$

$y = \frac{1}{x} + 3$

$y = \frac{1+3x}{x} \Rightarrow g^{-1}(x) = \frac{1+3x}{x}$

c) $(f \circ g)(x) = f[g(x)] = \sqrt{\frac{1}{x-3} - 1} =$

$= \sqrt{\frac{1-x+3}{x-3}} = \sqrt{\frac{4-x}{x-3}}$

d) $(g \circ f)(x) = g[f(x)] = \frac{1}{\sqrt{x-1} - 3}$

$\frac{1}{\sqrt{x-1} - 3} \cdot \frac{\sqrt{x-1} + 3}{\sqrt{x-1} + 3} = \frac{\sqrt{x-1} + 3}{(\sqrt{x-1})^2 - 3^2} = \frac{\sqrt{x-1} + 3}{x - 10}$

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