

Números racionales.

Ejercicio 1.

$$a) \frac{7}{9} + \frac{11}{12} = \frac{28}{36} + \frac{33}{36} = \frac{61}{36}$$

$$9 = 3^2$$

$$12 = 2^2 \cdot 3$$

$$m.c.m(9 \text{ y } 12) = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$$

$$b) \frac{6}{1} - \frac{11}{4} = \frac{24}{4} - \frac{11}{4} = \frac{13}{4}$$

$$c) \frac{3}{1} \cdot \frac{4}{5} = \frac{12}{5}$$

$$d) \frac{6}{1} : \frac{4}{5} = \frac{30}{4} = \frac{15}{2}$$

$$e) \frac{4}{5} : \frac{6}{1} = \frac{4}{30} = \frac{2}{15}$$

$$f) \frac{4}{5} : \frac{1}{6} = \frac{4 \cdot 6}{5 \cdot 1} = \frac{24}{5}$$

Ejercicio 2

$$a) \frac{13}{12} \cdot \frac{84}{65} = \frac{13 \cdot 84}{12 \cdot 65} = \frac{\cancel{13} \cdot \cancel{2} \cdot \cancel{3} \cdot 7}{\cancel{2} \cdot \cancel{3} \cdot 5 \cdot \cancel{13}} = \frac{7}{5}$$

$$b) \frac{90}{35} \cdot \frac{14}{36} = \frac{90 \cdot 14}{35 \cdot 36} = \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{5} \cdot \cancel{2} \cdot 7}{\cancel{5} \cdot \cancel{7} \cdot \cancel{2}^2 \cdot \cancel{3}^2} = \frac{1}{1} = 1$$

$$c) \frac{9}{16} \cdot \frac{20}{27} = \frac{9 \cdot 20}{16 \cdot 27} = \frac{\cancel{3}^2 \cdot \cancel{2}^2 \cdot 5}{2^4 \cdot \cancel{3}^3} = \frac{5}{12}$$

Ejercicio 4

$$a) \left(\frac{3}{4} + \frac{7}{6} - \frac{7}{8} \right) : \frac{25}{12} = \left(\frac{18}{24} + \frac{28}{24} - \frac{21}{24} \right) : \frac{25}{12} = \frac{25}{24} : \frac{25}{12} = \frac{25 \cdot 12}{24 \cdot 25} = \frac{12}{24} = \frac{1}{2}$$

$$b) \left(\frac{13}{15} - \frac{7}{25} \right) \cdot \left(\frac{9}{22} - \frac{13}{33} \right) = \left(\frac{65}{75} - \frac{21}{75} \right) \cdot \left(\frac{27}{66} - \frac{26}{66} \right) =$$

$$= \frac{44}{75} \cdot \frac{1}{66} = \frac{44 \cdot 1}{75 \cdot 66} = \frac{2}{225}$$

$$c) \left(1 + \frac{1}{3} \right) - \left(\frac{3}{4} + \frac{1}{2} \right) \cdot \left(\frac{1}{3} - \frac{1}{4} \right) = \left(\frac{3}{3} + \frac{1}{3} \right) - \left(\frac{3}{4} + \frac{2}{4} \right) \cdot \left(\frac{4}{12} - \frac{3}{12} \right)$$

$$= \frac{4}{3} - \frac{5}{4} \cdot \frac{1}{12} = \frac{4}{3} - \frac{5}{48} = \frac{64}{48} - \frac{5}{48} = \frac{59}{48}$$

$$d) \left(\frac{3}{5} + \frac{1}{3} \right) - \left[1 - \left(\frac{3}{4} - \frac{1}{2} \right) + \frac{2}{3} - \frac{3}{20} \right] = \left(\frac{9}{15} + \frac{5}{15} \right) - \left[1 - \left(\frac{3}{4} - \frac{2}{4} \right) + \frac{2}{3} - \frac{3}{20} \right]$$

$$= \frac{14}{15} - \left[1 - \frac{1}{4} + \frac{2}{3} - \frac{3}{20} \right] = \frac{14}{15} - \left[\frac{60}{60} - \frac{15}{60} + \frac{40}{60} - \frac{9}{60} \right]$$

$$= \frac{14}{15} - \frac{76}{60} = \frac{14}{15} - \frac{19}{15} = \frac{-5}{15} = \frac{-1}{3}$$

$$e) 5 \div \left(\frac{2}{4} + 1 \right) - 3 \div \left(\frac{1}{2} - \frac{1}{4} \right) = 5 \div \left(\frac{1}{2} + \frac{2}{2} \right) - 3 \div \left(\frac{2}{4} - \frac{1}{4} \right) =$$

$$= 5 \div \frac{3}{2} - 3 \div \frac{1}{4} = \frac{10}{3} - \frac{12}{1} = \frac{10}{3} - \frac{36}{3} = \frac{-26}{3}$$

$$f) \frac{2}{3} \cdot \left(\frac{3}{4} - \frac{1}{2} \right)^2 - \frac{1}{6} \cdot \left(\frac{5}{6} - \frac{1}{3} \right)^2 = \frac{2}{3} \cdot \left(\frac{3}{4} - \frac{2}{4} \right)^2 - \frac{1}{6} \cdot \left(\frac{5}{6} - \frac{2}{6} \right)^2 =$$

$$= \frac{2}{3} \cdot \left(\frac{1}{4} \right)^2 - \frac{1}{6} \cdot \left(\frac{3}{6} \right)^2 = \frac{2}{3} \cdot \frac{1}{16} - \frac{1}{6} \cdot \left(\frac{1}{2} \right)^2 = \frac{2 \cdot 1}{3 \cdot 16} - \frac{1}{6} \cdot \frac{1}{4}$$

$$= \frac{1}{24} - \frac{1}{24} = \frac{0}{24} = 0$$

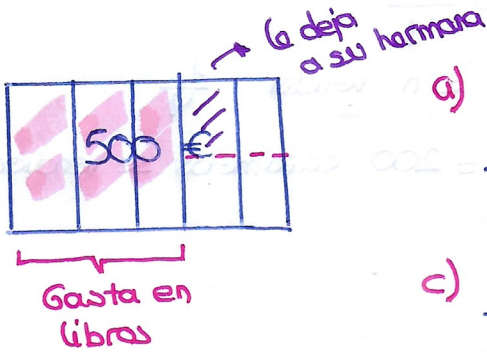
$$g) \frac{1 - \frac{1}{3}}{1 + \frac{3}{5}} - \frac{3 + \frac{1}{6}}{\frac{1}{4} - 2} = \frac{\frac{3}{3} - \frac{1}{3}}{\frac{5}{5} + \frac{3}{5}} - \frac{\frac{18}{6} + \frac{1}{6}}{\frac{1}{4} - \frac{8}{4}} = \frac{\frac{2}{3}}{\frac{8}{5}} - \frac{\frac{19}{6}}{\left(-\frac{7}{4} \right)} = \frac{2 \cdot 5}{3 \cdot 8} + \frac{19 \cdot 4}{6 \cdot 7} =$$

$$= \frac{5}{12} + \frac{38}{21} = \frac{35}{84} + \frac{152}{84} = \frac{187}{84}$$

$$h) \frac{3 \cdot \left(-\frac{2}{5} + 1\right) - \frac{3}{4} \div \frac{1}{2}}{\frac{1}{2} - \frac{1}{3} \div 4} = \frac{3 \cdot \left(-\frac{2}{5} + \frac{5}{5}\right) - \frac{3 \cdot 2}{4 \cdot 1}}{\frac{1}{2} - \frac{1}{12}} = \frac{3 \cdot \frac{3}{5} - \frac{3}{2}}{\frac{6}{12} - \frac{1}{12}} =$$

$$= \frac{\frac{9}{5} - \frac{3}{2}}{\frac{5}{12}} = \frac{\frac{18}{10} - \frac{15}{10}}{\frac{5}{12}} = \frac{\frac{3}{10}}{\frac{5}{12}} = \frac{3 \cdot 12}{10 \cdot 5} = \frac{18}{25}$$

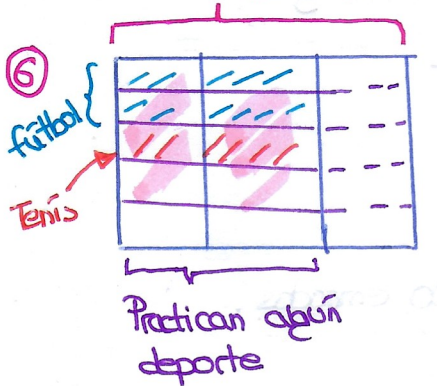
⑤



a) $500 \div 5 = 100 \text{ € cada trozo}$
 $100 \cdot 2 = 200 \text{ € le queda sin gastar.}$
 $100 \cdot 3 = 300 \text{ € ha gastado}$

c) $200 \div 4 = 50 \text{ € le deja a su hermana.}$
 $3 \cdot 50 = 150 \text{ € le quedan}$

⑥



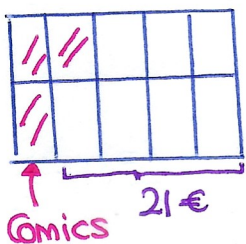
a) $120 \div 3 = 40$
 $40 \cdot 2 = 80 \text{ alumnos practican algún deporte.}$

b) $80 \div 5 = 16$
 $16 \cdot 2 = 32 \text{ alumnos practican fútbol}$

c) $16 \cdot 1 = 16 \text{ alumnos practican tenis}$

d) $2 \cdot 16 = 32 \text{ alumnos practican varios deportes.}$

⑦

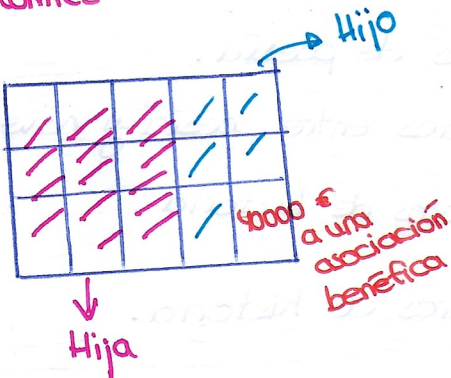


$21 \div 7 = 3 \text{ € cada trozo}$

$3 \cdot 10 = 30 \text{ € tenía al principio}$

$3 \cdot 3 = 9 \text{ € gastó en Comics}$

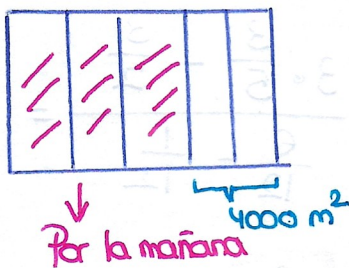
⑧



$40000 \cdot 15 = 600\,000 \text{ €}$

La herencia era de $600\,000 \text{ €}$

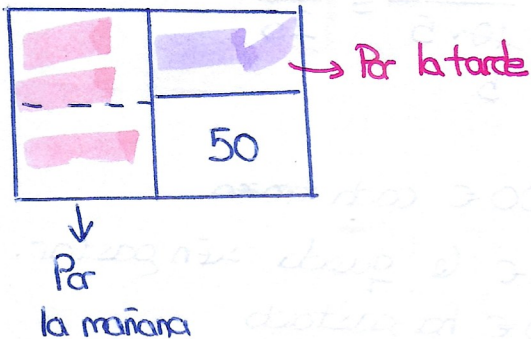
9



$$4000 : 2 = 2000 \text{ m}^2 \text{ cada trozo}$$

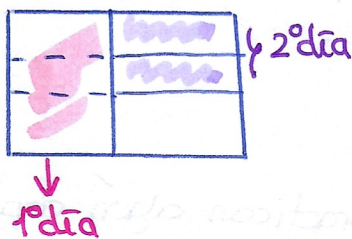
$$2000 \cdot 5 = 10000 \text{ m}^2 \text{ el terreno}$$

10



- a) Vendió $\frac{1}{4}$
- b) Vendió $\frac{3}{4}$
- c) Quedó sin vender $\frac{1}{4}$
- d) $50 \cdot 4 = 200$ caramelos se vendieron.

11



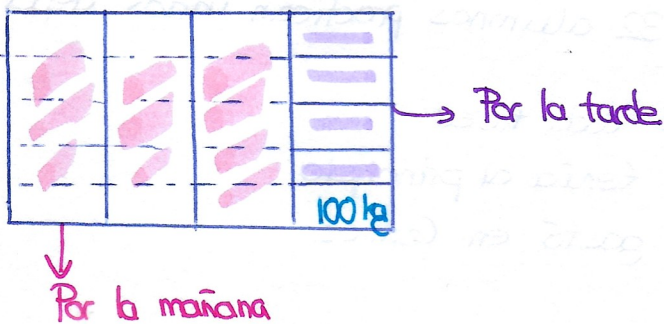
a) $\frac{2}{6} = \frac{1}{3}$

El segundo día se vendieron $\frac{1}{3}$

- b) Entre los dos días $\frac{5}{6}$
- c) Queda por vender $\frac{1}{6}$
- d) $15000 : 6 = 2500$

Quedaron sin vender 2500 entrecos.

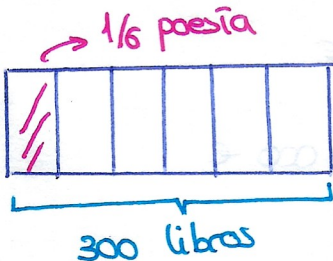
12



$$100 \cdot 20 = 2000 \text{ kg}$$

Tenía 2000 kg.

13

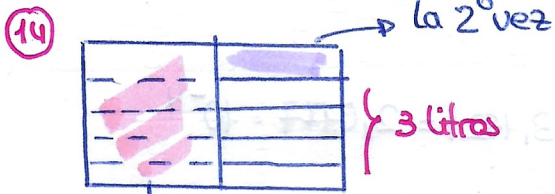


$$300 : 6 = 50 \text{ libros de poesía.}$$

$$180 + 50 = 230 \text{ libros entre poesía y novela.}$$

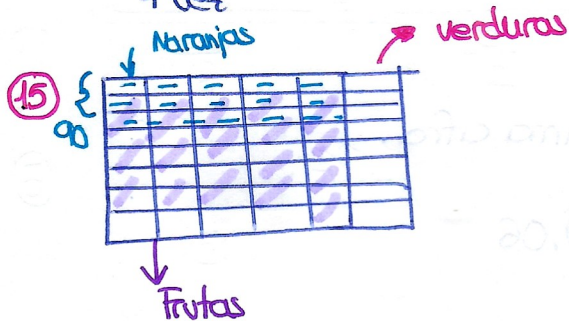
$$300 - 230 = 70 \text{ libros de historia.}$$

$$\frac{70}{300} = \frac{7}{30} \text{ libros de historia.}$$



$$3 : 4 = 0,75 \text{ litros cada trozo}$$

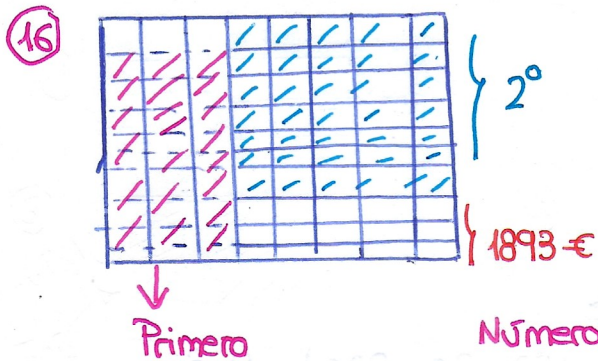
$$0,75 \cdot 10 = 7,5 \text{ litros en total.}$$



$$90 \text{ €} : 15 = 6 \text{ € cada trozo.}$$

$$6 \cdot 48 = 288 \text{ € en total.}$$

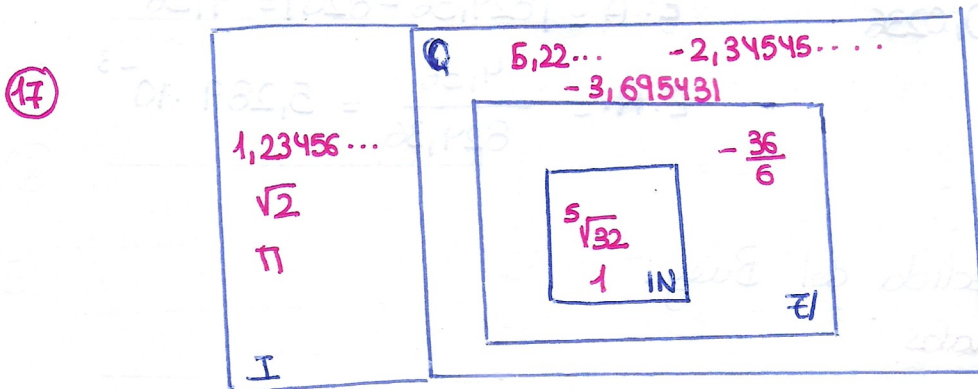
$$6 \cdot 8 = 48 \text{ € en verduras.}$$



$$1893 : 15 = 126,2 \text{ € cada trozo}$$

$$126,20 \cdot 80 = 10096 \text{ € en total}$$

Números reales



18 $\frac{3}{5}$ Q, IR

$-\sqrt{2}$ I, IR

$1,2525 \dots$ Q, IR

$2,010010001 \dots$ I, IR

-4 E, Q, IR

$0,2\bar{6}$ Q, IR

19 a) Falso. Podría ser irracional

b) Falso. Es un número irracional que no puede escribirse como fracción

c) Falso. $-\sqrt{25} = -5$ y $-\sqrt[3]{8} = -2$

20) $\sqrt{10} = 3,16227766 \dots$

$\sqrt{10} \approx 3,162$

$E.A. = |V_{\text{real}} - V_{\text{aprox}}| = |\sqrt{10} - 3,162| = 2,777 \cdot 10^{-4}$

$E.R. = \frac{E.A.}{V_{\text{real}}} = \frac{2,777 \cdot 10^{-4}}{\sqrt{10}} = 8,78 \cdot 10^{-5}$

21) Que la calculadora ha redondeado la ultima cifra.

22) $E.A. = |V_{\text{real}} - V_{\text{aprox}}| = |1,26 - 1,21| = 0,06$

$E.R. = \frac{E.A.}{V_{\text{real}}} = \frac{0,06}{1,26} = 0,0476$

23) Gusano $2,1236 \text{ g} \approx 2,1$

Buey $824,36 \text{ kg} \approx 820 \text{ kg}$

Gusano

$E.A. = |2,1236 - 2,1| = 0,0236$

$E.R. = \frac{0,0236}{2,1236} = 0,0111$

Buey

$E.A. = |824,36 - 820| = 4,36$

$E.R. = \frac{4,36}{824,36} = 5,289 \cdot 10^{-3}$

Es más precisa la medida del Buey.

Intervalos

24)

a) No

b) Dos: -4 y -3

c) Infinitos

d) $[-3, \infty)$

(25)

Intervalo o
Semi recta

Enunciado

Desigualdad

Representación
gráfica

$[1, \infty)$

Números reales mayores o iguales
que 1

$x \geq 1$



$(-\infty, -5)$

Números reales menores que -5

$x < -5$



$[-1, 5)$

Números mayores o iguales a -1
y menores que 5.

$-1 \leq x < 5$



$[-1, 0]$

Números mayores o iguales que -1
y menores o iguales a 0

$-1 \leq x \leq 0$



$(-\infty, 6]$

Números menores o iguales a 6

$x \leq 6$



(26)

a) $x \geq 2$ $[2, \infty)$

b) $-4 \leq x < 8$ $[-4, 8)$

c) $x < 6$ y $x > -7$ $(-7, 6)$

d) $9 > x$ $(-\infty, 9)$

e) $7 < x$ y $x \geq 9$ $[9, \infty)$

f) $x < -2$ y $x \geq 3$ $\emptyset$

(27)

a) $[-2, \infty) = \{x \in \mathbb{R} / x \geq -2\}$



b) $(-3, 0] = \{x \in \mathbb{R} / -3 < x \leq 0\}$



c) $(-\infty, 4) = \{x \in \mathbb{R} / x < 4\}$



d) $[5, 10] = \{x \in \mathbb{R} / 5 \leq x \leq 10\}$



(28) a) $(-3)^3 = -27$

b) $(-2)^4 = 16$

c) $(-2)^{-3} = \frac{1}{8}$

d) $-3^2 = -9$

e) $-2^{-1} = -\frac{1}{2}$

f) $(-1)^{-2} = 1$

g) $(\frac{1}{2})^{-3} = 8$

h) $(-\frac{1}{2})^{-2} = 4$

i) $(\frac{4}{3})^0 = 1$

j) $(\frac{3}{4})^{-3} = \frac{64}{27}$

$$(29) \quad a) 4^3 \cdot 4^4 \cdot 4 = 4^8 \quad b) (5^6)^3 = 5^{18} \quad c) \frac{7^6}{7^4} = 7^2$$

$$d) (6^2)^3 \cdot 3^4 \cdot (2^7 \cdot 2^3) = 6^6 \cdot 3^4 \cdot 2^4 = 6^6 \cdot 6^4 = 6^{10}$$

$$e) 2^{10} \cdot 5^{10} = 10^{10}$$

$$f) \frac{12^5}{3^5 \cdot 4^5} = \frac{12^5}{12^5} = 12^0 = 1$$

$$g) \frac{15^3}{3^3} = 5^3$$

$$h) (a^6 \cdot a^3)^2 : (a^2 \cdot a^4)^3 = (a^9)^2 : (a^6)^3 = a^{18} : a^{18} = a^0 = 1$$

$$i) 4^{-3} \cdot 4^2 : 4^{-1} = 4^{-1} : 4^{-1} = 4^0 = 1$$

$$j) \left(\frac{3}{4}\right)^{-3} \cdot \left(\frac{9}{16}\right)^2 = \left(\frac{3}{4}\right)^{-3} \cdot \left(\left(\frac{3}{4}\right)^2\right)^2 = \left(\frac{3}{4}\right)^{-3} \cdot \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)$$

$$k) 5^{-3} \cdot \left[\left(\frac{1}{5}\right)^{-2}\right]^2 = 5^{-3} \cdot \left(\frac{1}{5}\right)^{-4} = 5^{-3} \cdot 5^4 = 5^1$$

$$(30) \quad a) \left(\frac{1}{5}\right)^2 = \frac{1}{25} \quad b) \left(\frac{1}{5}\right)^{-2} = 5^2 = 25 \quad c) \left(-\frac{1}{5}\right)^{-2} = (-5)^2 = 25$$

$$d) \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9} \quad e) \left(\frac{1}{5} \cdot \frac{1}{2}\right)^{-6} = \left(\frac{1}{10}\right)^{-6} = 10^6 = 1000000$$

$$f) \left(\frac{1}{2}\right)^6 \cdot \left(\frac{1}{5}\right)^6 = \left(\frac{1}{2} \cdot \frac{1}{5}\right)^6 = \left(\frac{1}{10}\right)^6 = \frac{1}{1000000}$$

$$g) \left(\frac{2}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^5 = \frac{32}{243}$$

$$h) \left(\frac{17}{45}\right)^0 = 1$$

$$i) \left[\left(\frac{1}{3}\right)^{-3}\right]^2 = \left(\frac{1}{3}\right)^{-6} = 3^6 = 729$$

$$j) \left(\frac{3}{4}\right)^{-3} : \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^{-5} = \left(\frac{4}{3}\right)^5 = \frac{1024}{243}$$

$$k) \left(\frac{1}{2}\right)^3 : \left(\frac{1}{4}\right)^2 = \left(\frac{1}{2}\right)^3 \cdot \left(\left(\frac{1}{2}\right)^2\right)^2 = \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$$

$$l) \left(\frac{2}{3}\right)^2 \cdot \left(-\frac{3}{2}\right)^4 = \left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{2}\right)^4 = \left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^{-4} = \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$m) \left(-\frac{1}{a}\right)^{-3} \cdot \left(\frac{a}{b}\right)^{-2} = -\left(\frac{1}{a}\right)^{-3} \cdot \left(\frac{a}{b}\right)^{-2} = -a^3 \cdot \left(\frac{b}{a}\right)^2 = -a^3 \cdot \frac{b^2}{a} = -\frac{a^3 \cdot b^2}{a} =$$

$$= -a^2 \cdot b^2 = -(a \cdot b)^2$$

$$n) \left(\frac{a}{b}\right)^{-4} \cdot \frac{a^3}{b^2} = \left(\frac{b}{a}\right)^4 \cdot \frac{a^3}{b^2} = \frac{b^4}{a^4} \cdot \frac{a^3}{b^2} = \frac{b^4 \cdot a^3}{a^4 \cdot b^2} = \frac{b^2}{a}$$

$$\bar{n}) \left(\frac{b}{a}\right)^{-3} \div \frac{(b^2)^{-1}}{a^{-4}} = \left(\frac{a}{b}\right)^3 \div \frac{b^{-2}}{a^{-4}} = \frac{a^3}{b^3} \div \frac{a^4}{b^2} = \frac{a^3 \cdot b^2}{b^3 \cdot a^4} = \frac{1}{b \cdot a}$$

$$(31) a) \frac{3^{-3} \cdot 3^5}{3^{-2} \cdot 3} = \frac{3^2 \cdot 3^5}{3^3 \cdot 3} = \frac{3^7}{3^4} = 3^3 = 27$$

$$b) \frac{2^4 \cdot 2^{-1} \cdot 2^3}{2^5 \cdot 2^{-3}} \cdot (2^2)^{-3} \cdot 2^4 = \frac{2^4 \cdot 2^3 \cdot 2^3}{2^5 \cdot 2} \cdot 2^{-6} \cdot 2^4 = \frac{2^{10}}{2^6} \cdot \frac{1}{2^6} \cdot 2^4 =$$

$$= \frac{2^{10} \cdot 1 \cdot 2^4}{2^6 \cdot 2^6} = \frac{2^{14}}{2^{12}} = 2^2 = 4$$

$$c) \frac{4 \cdot (10^{-2})^3 \cdot 10^2}{12 \cdot 10^{-3}} = \frac{4 \cdot 10^{-6} \cdot 10^2}{12 \cdot 10^{-3}} = \frac{4 \cdot 10^2 \cdot 10^3}{12 \cdot 10^6} = \frac{4 \cdot 10^5}{12 \cdot 10^6} = \frac{4}{12 \cdot 10} =$$

$$= \frac{2^2}{2^2 \cdot 3 \cdot 2 \cdot 5} = \frac{2^2}{2^3 \cdot 3 \cdot 5} = \frac{1}{2 \cdot 3 \cdot 5} = \frac{1}{30}$$

$$d) \frac{25 \cdot (10^2)^{-5} \cdot 121}{11 \cdot 75 \cdot 10^{-9}} = \frac{\cancel{5^2} \cdot 10^{-10} \cdot \cancel{11^2}}{\cancel{11} \cdot \cancel{5^2} \cdot 3 \cdot 10^{-9}} = \frac{10^9 \cdot 11}{3 \cdot 10^{10}} = \frac{11}{3 \cdot 10} = \frac{11}{30}$$

$$e) \frac{27^5 \cdot 243 \cdot \left(\frac{1}{3}\right)^3}{9^2 \cdot 3^5} = \frac{(3^3)^5 \cdot 3^5 \cdot 3^{-3}}{(3^2)^2 \cdot 3^5} = \frac{3^{15} \cdot 3^5}{3^4 \cdot 3^5 \cdot 3^3} = \frac{3^{20}}{3^{12}} = 3^8$$

$$f) \frac{8 \cdot 2^2 \cdot 16^{-2}}{32 \cdot 4^3} \cdot 2^4 = \frac{2^3 \cdot 2^2}{32 \cdot 16^2 \cdot 4^3} \cdot 2^4 = \frac{2^3 \cdot 2^2 \cdot 2^4}{2^5 \cdot (2^4)^2 \cdot (2^2)^3} = \frac{2^9}{2^5 \cdot 2^8 \cdot 2^6} =$$

$$= \frac{2^9}{2^{19}} = \frac{1}{2^{10}}$$

$$g) \frac{24^2 \cdot 15^{-2} \cdot 6^4}{8^4 \cdot 9^{-3} \cdot 3^{10}} = \frac{24^2 \cdot 6^4 \cdot 9^3}{8^4 \cdot 3^{10} \cdot 15^2} = \frac{(2^3 \cdot 3)^2 \cdot (2 \cdot 3)^4 \cdot (3^2)^3}{(2^3)^4 \cdot 3^{10} \cdot (3 \cdot 5)^2} =$$

$$= \frac{2^6 \cdot 3^2 \cdot 2^4 \cdot 3^4 \cdot 3^6}{2^{12} \cdot 3^{10} \cdot 3^2 \cdot 5^2} = \frac{2^{10} \cdot 3^{12}}{2^{12} \cdot 3^{12} \cdot 5^2} = \frac{1}{5^2 \cdot 2^2}$$

$$h) \frac{4^{-2} \cdot 8 \cdot 32^{-1}}{16^3 \cdot \left(\frac{1}{8}\right)^5} = \frac{4^{-2} \cdot 8 \cdot 32^{-1}}{16^3 \cdot 8^{-5}} = \frac{8 \cdot 8^5}{4^2 \cdot 32 \cdot 16^3} = \frac{8^6}{(2^2)^2 \cdot 2^5 \cdot (2^4)^3} =$$

$$= \frac{(2^3)^6}{2^4 \cdot 2^5 \cdot 2^{12}} = \frac{2^{18}}{2^{21}} = \frac{1}{2^3}$$