

$$5) f(x) = \frac{ax^2 + 3x + 1}{bx^2 + 1}$$

→ Mx relativo en $x=1$: $f'(1) = 0$

→ Asíntota horizontal : $\lim_{x \rightarrow \infty} f(x) = 2$
en $y = 2$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{ax^2 + 3x + 1}{bx^2 + 1} = \lim_{x \rightarrow \infty} \frac{ax^2}{bx^2} = \frac{a}{b} \Rightarrow \frac{a}{b} = 2 \Rightarrow \boxed{a = 2b}$$

Siempre que $a \neq 0$
y $b \neq 0$

Sustituyendo $a = 2b$ en la función:

$$f(x) = \frac{2bx^2 + 3x + 1}{bx^2 + 1}$$

$$f'(x) = \frac{(4bx + 3) \cdot (bx^2 + 1) - (2bx^2 + 3x + 1) \cdot 2bx}{(bx^2 + 1)^2} =$$

$$= \frac{4b^2x^3 + 4bx + 2bx^2 + 3 - (4b^2x^3 + 6bx^2 + 2bx)}{(bx^2 + 1)^2}$$

$$= \frac{\cancel{4b^2x^3} + 4bx + 3bx^2 + 3 - \cancel{4b^2x^3} - 6bx^2 - 2bx}{(bx^2 + 1)^2} = \frac{-3bx^2 + 2bx + 3}{(bx^2 + 1)^2}$$

$$f'(1) = \frac{-3b + 2b + 3}{(b+1)^2} = \frac{-b+3}{(b+1)^2} > 0 \rightarrow -b+3=0$$

$$\rightarrow -b = -3 \rightarrow \boxed{b = 3}$$

Por lo tanto $a = 2b = 2 \cdot 3 = 6$

SOLUCIÓN: $\boxed{a = 6 \quad b = 3}$

$$f(x) = \frac{x^2-1}{x^2}$$

① Dom = $\mathbb{R} - \{0\}$

② PC FJE $x (y=0) \rightarrow \frac{x^2-1}{x^2} = 0 \rightarrow x^2-1=0 \rightarrow x^2=1 \rightarrow x=\pm 1 \rightarrow (1,0), (-1,0)$
 FJE $y (x=0) \rightarrow f(0) = \text{punto fuera del dominio}$

③ AV $\lim_{x \rightarrow 0} \frac{x^2-1}{x^2} = \frac{-1}{0} = \pm\infty$ AV en $x=0$

④ Monotonía ($f'(x) > 0$)
 $f'(x) = \left(\frac{x^2-1}{x^2}\right)' = \frac{2x \cdot x^2 - (x^2-1) \cdot 2x}{(x^2)^2} = \frac{2x^3 - (2x^3 - 2x)}{x^4} = \frac{2x^3 - 2x^3 + 2x}{x^4} = \frac{2x}{x^4} = \frac{2}{x^3}$

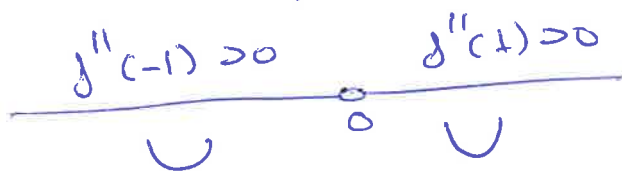
$f'(x) > 0 \rightarrow \frac{2}{x^3} > 0 \rightarrow 2 > 0 \quad \nexists \text{ Sol}$



⑤ Curvatura ($f''(x) = 0$)

$f''(x) = \left(\frac{2}{x^3}\right)' = \frac{0 \cdot x^3 - 2 \cdot 3x^2}{(x^3)^2} = \frac{-6x^2}{x^6} = \frac{-6}{x^4}$

$f''(x) > 0 \rightarrow \frac{-6}{x^4} > 0 \rightarrow -6 > 0 \quad \nexists \text{ Sol}$



⑥ R. infinites $f(x) = \frac{x^2-1}{x^2} \rightarrow \text{Grado 2} / \Rightarrow \text{A. Horizontal}$
 $\rightarrow \text{Grado 2}$

$\lim_{x \rightarrow \infty} \frac{x^2-1}{x^2} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1$. Tiene una Asintota Horizontal de ecuación $y=1$

