5 $f(x) = \dfrac{ax^2+3x+1}{bx^2+1}$

→ Máx relativo en $x=1$: $f'(1)=0$

→ Asíntota horizontal en $y=2$: $\lim_{x\to\infty} f(x) = 2$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{ax^2+3x+1}{bx^2+1} = \lim_{x\to\infty} \dfrac{ax^2}{bx^2} = \dfrac{a}{b} \to \dfrac{a}{b} = 2 \Rightarrow \boxed{a=2b}$

Siempre que $a\neq0$ y $b\neq0$

Sustituyendo $a=2b$ en la función:

$$f(x) = \frac{2bx^2+3x+1}{bx^2+1}$$

$$f'(x) = \frac{(4bx+3)\cdot(bx^2+1) - (2bx^2+3x+1)\cdot 2bx}{(bx^2+1)^2} =$$

$$= \frac{4b^2x^3+4bx+3bx^2+3 - (4b^2x^3+6bx^2+2bx)}{(bx^2+1)^2}$$

$$= \frac{\cancel{4b^2x^3}+4bx+3bx^2+3 \cancel{-4b^2x^3} - 6bx^2 - 2bx}{(bx^2+1)^2} = \frac{-3bx^2+2bx+3}{(bx^2+1)^2}$$

$$f'(1) = \frac{-3b+2b+3}{(b+1)^2} = \frac{-b+3}{(b+1)^2} \to \frac{-b+3}{(b+1)^2} = 0 \to -b+3=0$$

$$\to -b = -3 \to \boxed{b=3}$$

Por lo tanto $a = 2b = 2\cdot 3 = 6$

SOLUCIÓN: $\boxed{a=6 \quad b=3}$

$$f(x) = \frac{x^2-1}{x^2}$$

① $\text{Dom} f = \mathbb{R} - \{0\}$

② PC EJE X $(y=0) \rightarrow \frac{x^2-1}{x^2} = 0 \rightarrow x^2-1=0 \rightarrow x^2=1 \rightarrow \boxed{x=\pm 1} \rightarrow (1,0), (-1,0)$

EJE Y $(x=0) \rightarrow f(0) = \nexists$ (punto fuera del dominio)

③ AV $\lim_{x \to 0} \frac{x^2-1}{x^2} = \frac{-1}{0} = \pm\infty$ AV en $x=0$

④ Monotonía $(f'(x)=0)$

$$f'(x) = \left(\frac{x^2-1}{x^2}\right)' = \frac{2x \cdot x^2 - (x^2-1) \cdot 2x}{(x^2)^2} = \frac{2x^3-(2x^3-2x)}{x^4} = \frac{\cancel{2x^3}-\cancel{2x^3}+2x}{x^4} = \frac{2x}{x^4} = \frac{2}{x^3}$$

$f'(x)=0 \rightarrow \frac{2}{x^3} = 0 \rightarrow 2=0$ $\nexists$ Sol

$f'(-1)<0$ $f'(1)>0$

0

↘ ↗

⑤ Curvatura $(f''(x)=0)$

$$f''(x) = \left(\frac{2}{x^3}\right)' = \frac{\cancel{0 \cdot x^3} - 2 \cdot 3x^2}{(x^3)^2} = \frac{-6x^2}{x^6} = \frac{-6}{x^4}$$

$f''(x)=0 \rightarrow \frac{-6}{x^4} = 0 \rightarrow -6=0$ $\nexists$ Sol

$f''(-1)>0$ $f''(1)>0$

0

∪ ∪

⑥ R. infinitas $f(x) = \frac{x^2-1}{x^2}$ $\begin{matrix} \rightarrow \text{Grado } 2 \\ \rightarrow \text{Grado } 2 \end{matrix} \Big| \Rightarrow$ A. Horizontal

$\lim_{x \to \infty} \frac{x^2-1}{x^2} = \lim_{x \to \infty} \frac{x^2}{x^2} = 1$. Tiene una Asíntota Horizontal de ecuación $y=1$