

① 1)  $f(x) = \frac{2x^2}{x^2-1}$

①  $\text{Dom} f = \mathbb{R} - \{-1, 1\}$   $x^2-1 > 0 \Rightarrow x^2 > 1 \Rightarrow x = \pm\sqrt{1} \Rightarrow x = \pm 1$

② P. corte ejes  
 Eje X ( $y=0$ )  $\Rightarrow \frac{2x^2}{x^2-1} = 0 \Rightarrow 2x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0 \Rightarrow (0,0)$

Eje Y ( $x=0$ )  $\Rightarrow f(0) = 0 \Rightarrow (0,0)$

③ A. verticales

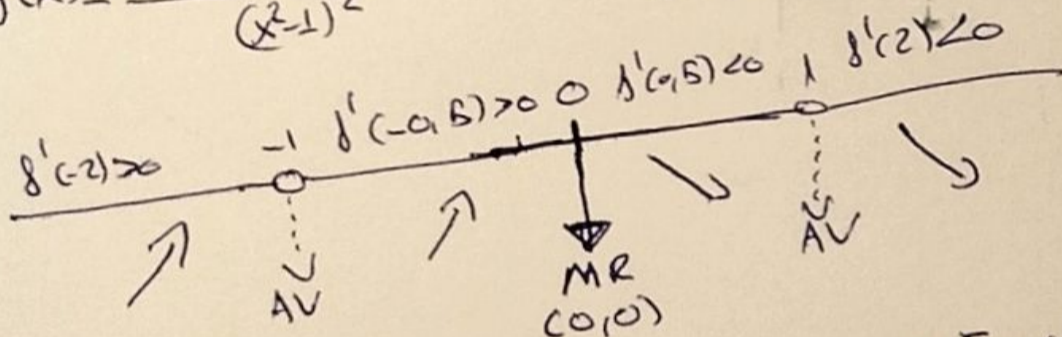
$\lim_{x \rightarrow 1} \frac{2x^2}{x^2-1} = \frac{2}{0} \Rightarrow \text{AV en } x=1$

$\lim_{x \rightarrow -1} \frac{2x^2}{x^2-1} = \frac{2}{0} \Rightarrow \text{AV en } x=-1$

④ Monotonía ( $f'(x)=0$ )

$f'(x) = \frac{4x \cdot (x^2-1) - 2x^2 \cdot 2x}{(x^2-1)^2} = \frac{4x^3 - 4x - 4x^3}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2} \parallel \frac{-4x}{(x^2-1)^2} = 0 \Rightarrow -4x = 0 \Rightarrow x=0$

$f'(x) = \frac{-4x}{(x^2-1)^2}$

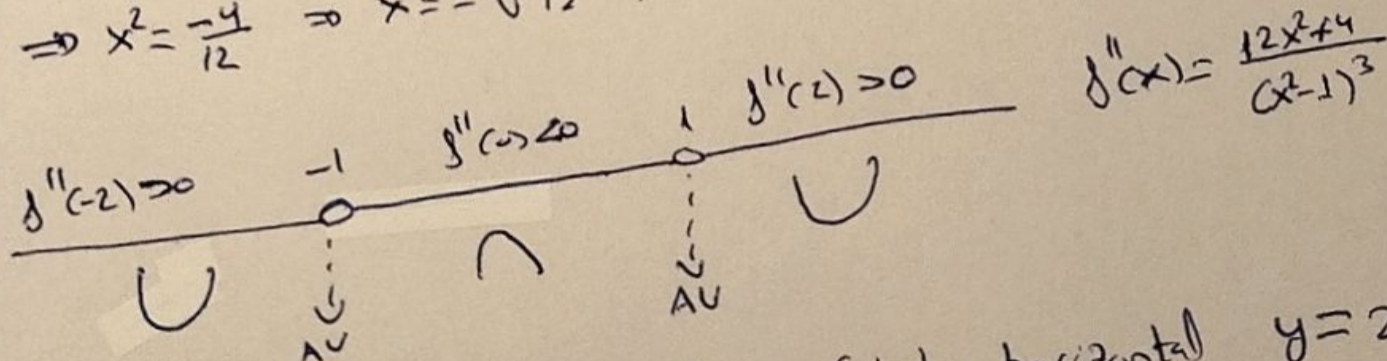


⑤ Curvatura ( $f''(x)=0$ )

$f''(x) = \frac{-4 \cdot (x^2-1)^2 - (-4x) \cdot 2(x^2-1) \cdot 2x}{[(x^2-1)^2]^2} = \frac{(x^2-1) [-4(x^2-1) - (-4x) \cdot 2 \cdot 2x]}{(x^2-1)^4}$

$= \frac{-4x^2 + 4 + 16x^2}{(x^2-1)^3} = \frac{12x^2 + 4}{(x^2-1)^3} \parallel \frac{12x^2 + 4}{(x^2-1)^3} = 0 \Rightarrow 12x^2 + 4 = 0 \Rightarrow 12x^2 = -4$

$\Rightarrow x^2 = \frac{-4}{12} \Rightarrow x = \pm \sqrt{\frac{-1}{3}} \neq \text{solución}$



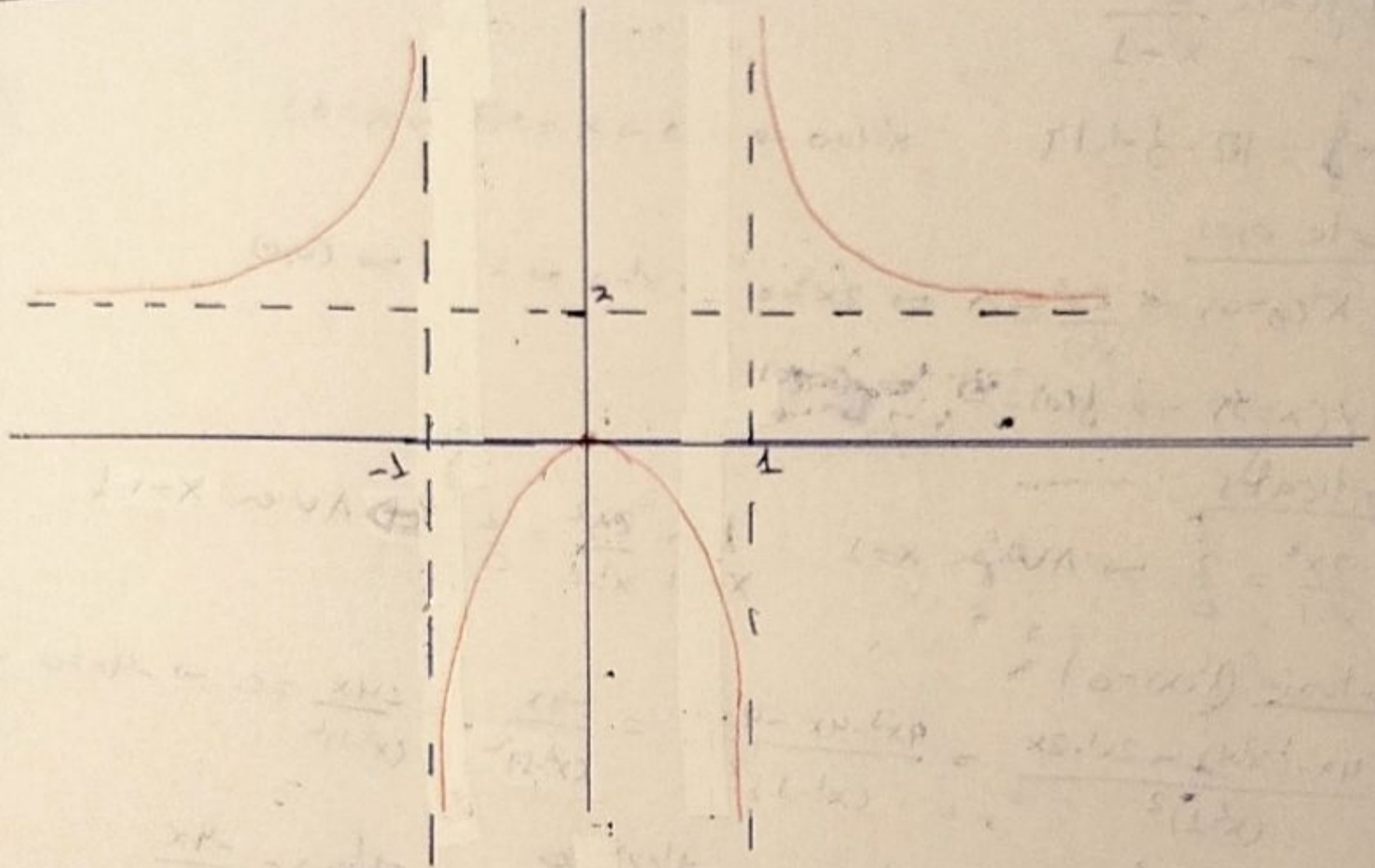
⑥ R. infinitas

$f(x) = \frac{2x^2}{x^2-1} \rightarrow C:2$

$\Rightarrow$  Asíntota horizontal  $y=2$

$\lim_{x \rightarrow \infty} \frac{2x^2}{x^2-1} = \lim_{x \rightarrow \infty} \frac{2x^2}{x^2} = 2$

R. GRÁFICA



$$f(x) = \frac{x^2 - 1}{x^2 + 1} = \frac{(x-1)(x+1)}{(x-1)(x+1)}$$

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$f'(x) = \frac{(2x)(x^2 + 1) - (x^2 - 1)(2x)}{(x^2 + 1)^2}$$

$$f'(x) = \frac{2x(x^2 + 1) - 2x(x^2 - 1)}{(x^2 + 1)^2}$$

$$f'(x) = \frac{2x(x^2 + 1 - x^2 + 1)}{(x^2 + 1)^2}$$

$$f'(x) = \frac{4x}{(x^2 + 1)^2}$$

Critical points:  $f'(x) = 0 \Rightarrow 4x = 0 \Rightarrow x = 0$   
 $f(0) = \frac{0^2 - 1}{0^2 + 1} = -1$

Asymptotes:  
 Horizontal:  $y = 2$   
 Vertical:  $x = -1$  and  $x = 1$

El dominio de la función es  $\mathbb{R} \setminus \{-1, 1\}$ .  
 El rango de la función es  $\mathbb{R}$ .  
 La función es simétrica respecto al eje y.